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# Determination of Extreme Capabilities for Vibration Isolation Systems with One Degree of Freedom

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**Abstract:** The system of inequalities describing the solvability area for the task of the Vibration Isolation System design that satisfies the specified requirements. The optimum type of the deformation limiter characteristic is specified. This limiter is applied to increase the capabilities of a vibration isolation system to protect against shock loading.

**Key words:** Vibration Isolation System, impact protection, optimization, optimum, design

## INTRODUCTION

An important task of the Vibration Isolation Systems (VIS) design is to define the limits of their capacities concerning the shock and vibration protection in the framework of the chosen model with elastic-damping properties of the vibration isolator, the given loads and set constraints. The solution to this problem allows to clarify whether it is possible to create the required VIS and to determine the best characteristics of elastic-damping shock absorbers for a given draft proposal.

An insufficient attention is provided for VIS optimization issues. Ping (2007), Zhang *et al*. (2012) and Zhao *et al*. (2005) consider only the impact effect while the vibration loads also influence the VIS. The effects of shock and vibration are taken into account but the issue of characteristic optimality is not posed (Yang and Li, 2006). Bubulis *et al*. (2011) considers the hydraulic isolator which is suitable for large-size equipment protection. This study is not applied for the isolators with dry friction, high damping, high strength and a significant life-time often used to protect machinery and equipment from vibration and shock. The VIS, considered by Yang *et al*. (2006) and Yang (2005) are too complex (it contains quadratic damping, linear damping, Coulomb friction, non-linear elasticity together) so their optimization is too difficult problem. The best approach for the problem solution is the which defines the area of the system optimal parameters for vibration protection system with two degrees of freedom but the issues of its extreme capacities concerning the vibration and impact isolation are not taken into account. It should be noted that the real VIS of mechanical objects and their design schemes may contain from one to six or more freedom degrees. Therefore, the development of shock absorbers with optimal elastic-damping characteristics for such VIS is a complex, time-consuming, highly personalized task. Therefore, the world practice of vibration isolation for various technical objects uses the vibration isolators developed for a wide range of mass and dynamic loads (for example, VIBROCHOC isolators, Russian firm isolators including ONIL1-SSAU isolators).

During the development of such modular series of vibration isolators a number of formal requirements for the VIS is used with one degree of freedom, synthesized ones, as a rule, based on the technical specifications for vibration and shock isolation of a particular class of technical objects (aircraft, rocket and space equipment, shipbuilding, railway etc.).

## DETERMINATION OF VIBRATION-PROOF SYSTEM AREA

One may define two groups of parameters for the existing terms of reference to develop a certain VIS: mechanical loads affecting VIS and the restrictions imposed on it. The mechanical loads in turn may be divided into vibration, shock and permanent ones. The vibration and shock loads tend to affect the VIS in a kinematic manner. According to current standards the vibration loads have a harmonic character and the acceleration amplitude is given as the following function:

$$J_B = a_0\omega^2$$

Where:
$a_0$ = Vibration amplitude
$\omega$ = Excitation frequency

The shock loads are usually given in the form of a half wave sinusoid acceleration at least in the form of a

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triangular or trapezoidal acceleration impulse, characterized by a peak value of acceleration $J_s$ and the duration $\tau_y$. Constant loads affecting VIS are set with the linear acceleration $J_{П}$.

The mechanical parameters restrictions imposed on VIS are divided usually into current acceleration limit at the VIS oscillations (short term ones $[J_y]$, under impact loading, long term ones $[J_B]$, at vibration loading), according to VIS shift $[x_g]$ and by resonant frequency $f_p$ which should be in the range of $[f_1, f_2]$.

The selection of Elastic-Damping Characteristic Model for the vibration isolator is performed from the possibility of their simple mathematical description and the maximum actual physical properties of the vibration isolator projected for a model. Thus, the vibration isolators with dry friction may use the elastic-damping property model developed by Sorokin (1960) and Volk (1962). In this case, the resonant modes of VIS oscillations have the expression determining the vibration isolator deformation:

$$A_B = a_0\sqrt{1+\left(\frac{2\pi}{\psi}\right)^2}$$

and the absolute acceleration:

$$W_B = k^2 a_0\left[1+\left(\frac{\psi}{2\pi}\right)^2\right]\frac{2\pi}{\psi}$$

here, $\Psi$ is energy dissipation coefficient; $k=\sqrt{C/M}$ is angular frequency of VIS free oscillations without damping; M is VIS weight; C is vibration isolator stiffness (Sorokin, 1960). The static VIS displacement from the permanent acceleration is calculated according to the equation $y = J_{П}/k^2$. In order to perform the restrictions according to permissible strain $[x_g]$ and acceleration $[J_B]$ it is necessary to require the following inequalities:

$$\frac{J_{П}}{k^2}+a_0\sqrt{1+\left(\frac{2\pi}{\psi}\right)^2}\le[x_g] \quad (1)$$

$$k^2 a_0\left[1+\left(\frac{\psi}{2\pi}\right)^2\right]\sqrt{1+\left(\frac{2\pi}{\psi}\right)^2}\le[J_B] \quad (2)$$

Given that the resonant VIS frequency is defined according to:

$$f_p = \frac{k}{2\pi}\sqrt{1+\left(\frac{\psi}{2\pi}\right)^2}$$

let's write the condition of constraint performance according to the resonance frequency:

$$k\sqrt{1+\left(\frac{\psi}{2\pi}\right)^2}\ge 2\pi f_1 \quad (3)$$

$$k\sqrt{1+\left(\frac{\psi}{2\pi}\right)^2}\ge 2\pi f_2 \quad (4)$$

The use of vibration isolators with $\Psi>3$ is not reasonable due to the significant deterioration of vibration protection quality within the resonance frequency area so, we take the following:

$$\psi\le 3 \quad (5)$$

Let's describe the conditions of shock loading limitation performance. First we note that the sign-constant impact load F(t) may be represented as a rectangular impulse with a peak acceleration value:

$$J_y = \frac{1}{\tau_y}\int_0^{\tau_y}F(t)dt$$

However, under the impact loading of VIS the influence of the energy dissipation coefficient of the usual vibration isolator structures ($\Psi\le3$) may be neglected. Therefore, the maximum deformation of the vibration isolator may be found according to the equation (Sorokin, 1960):

$$A_y\begin{cases}J_y\tau_y/k & \forall\ k\tau_y\le 2\\ 2J_y/k & \forall\ k\tau_y>2\end{cases}$$

and absolute acceleration:

$$W_y\approx\begin{cases}J_y k\tau_y & \forall\ k\tau_y\le 2\\ 2J_y & \forall\ k\tau_y>2\end{cases}$$

Thus, it is necessary to satisfy the following inequalities to implement restrictions on the VIS shock loading:

$$\begin{cases}\frac{1}{k^2}\left(J_{П}+J_y k\tau_y\right)\le[x_g] & \forall\ k\tau_y\le 2\\ \frac{1}{k^2}\left(J_{П}+2J_y\right)\le[x_g] & \forall\ k\tau_y>2\end{cases} \quad (6)$$

$$\begin{cases}J_y k\tau_y\le[J_y] & \forall\ k\tau_y\le 2\\ 2J_y\le[J_y] & \forall\ k\tau_y>2\end{cases} \quad (7)$$

By solving the inequalities Eq. 1-7 one may obtain within the coordinates ($\Psi$, k) the domain Q of VIS existence, satisfying the set requirements. In this domain, one may solve various optimization problems (according to the lowest acceleration during the vibration, according to the lowest acceleration at impact, etc.).

Figure 1 shows the Q domain, developed for the following conditions: M = 5 kg, $a_0$ = 0.5 mm, $J_s$ = 120 m $sec^2$, $\tau_y$ = 0.005 sec, half-sinusoidal impulse, $J_{\text{П}}$ = 40 m/$sec^2$, [$x_g$] = 12 mm, [$J_B$] = 100 m/$sec^2$, [$J_y$] = 130 m/$sec^2$, $f_1$ = 15 HZ, $f_2$ = 30 HZ. For example, let's optimize the VIS according to the vibration deformation (taking into account the effect of constant acceleration):

$$H = A_B + y = \frac{J_{\text{П}}}{k^2} + a_0\sqrt{1+\left(\frac{2\pi}{\psi}\right)^2}$$

at that determining the following $H_{min} = \min H(\Psi, k)$; $(\Psi, k) \in Q$ in O point with the coordinates $\Psi = \Psi'$, $k = k'$. While $\partial H/\partial\Psi<0$ and $\partial H/\partial k<0$ then the O point is located on (AB) section (Fig. 1) which is the upper boundary of the Q domain by $\Psi$, k. The (AB) section is the part of the curve, set by the equation:

$$k = \frac{2\pi f_2}{\sqrt{1+\left(\frac{\psi}{2\pi}\right)^2}}$$

By inserting the k value into the equation for H and having explored $H(\Psi)$ for the least value on (AB) we will obtain the optimal $\Psi' = 3$, $k' = 170\ c^{-1}$.

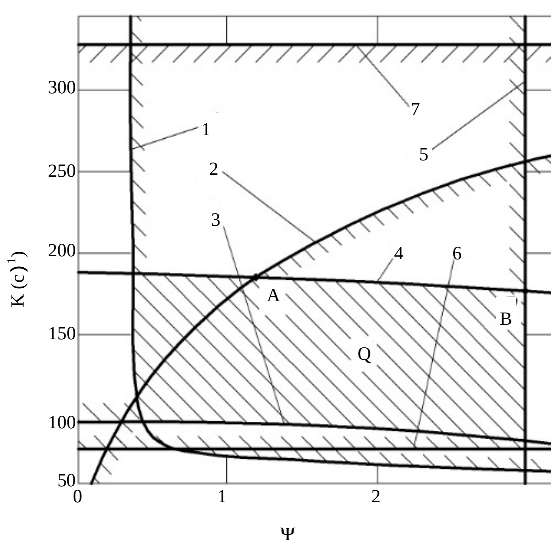


Fig. 1: Q domain of vibration isolation system existence that satisfies the set requirements. The curve numbers correspond to the number of inequalities defining these curves

## DEFORMATION LIMITER APPLICATION

The system of inequalities Eq. 1-7 may be unsolvable due to excessive deformation or acceleration at a shock loading of VIS. Then we must try to fulfill the requirements for VIS by applying the deformation limiter with the optimal characteristic P(x) in the vibration isolator design. This characteristic provides the minimum possible value of the deformation when the limit acceleration [$J_y$] is performed. In this case, the elastic characteristics of the vibration isolator contain two explicit areas (Fig. 2). One area of the characteristic is described at $|x|<\Delta$ within the previously selected model and provides the performance of inequalities. Thus, the choice of the vibration isolator free running value $\Delta$ shall correspond to the condition $\Delta \geq H$. The description of the other area is obtained from the solution concerning the VIS impact loading by rectangular impulse.

Let's take into account the case of $\tau_y>\tau_\Delta$ (where $\tau_\Delta$ is time when $x = \Delta$). Then the equation of VIS motion for $t\in[\tau_\Delta; \tau_y]$ takes the following form $M\ddot{x} + P(x) = MJ_y$. Having divided the both parts by M and by integrating twice by t, we obtain the following:

$$x(t_1) = \dot{x}_\Delta(t_1-\tau_\Delta) + J_y\frac{(t_1-\tau_\Delta)^2}{2} + \Delta - \frac{1}{M}\int_{\tau_\Delta}^{t_1}\left[\int_{\tau_\Delta}^{t_1} P(x(t))dt\right]dt$$

where, $\dot{x}_\Delta$ is VIS speed value at the moment $\tau_\Delta$. Changing the order of integration and substituting $t_1 = \tau_y$, we obtain the following:

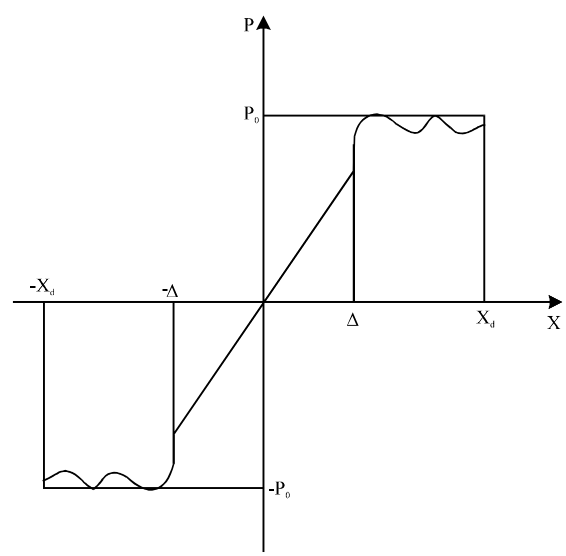


Fig. 2: Elastic characteristics of the vibration isolator with deformation limiters

$$x(\tau_y) = \dot{x}_\Delta(\tau_y-\tau_\Delta)+J_y\frac{(\tau_y-\tau_\Delta)^2}{2}+\Delta-\frac{1}{M}\int_{\tau_\Delta}^{\tau_y}P(x(t))(\tau_y-t)dt$$

The limiter reaction at $x\in[\Delta;\, x(\tau_y)]$ has the maximum value $P_0$:

$$c\Delta \le P_0 \tag{8}$$

while $P(x)\le P_0\ \forall\ x\in[\Delta;\, x(\tau_y)]$ then:

$$x(\tau_y) = \dot{x}_\Delta(\tau_y-\tau_\Delta)+J_y\frac{(\tau_y-\tau_\Delta)^2}{2}+\Delta-\frac{1}{M}\int_{\tau_\Delta}^{\tau_y}P(x(t))(\tau_y-t)dt \le \frac{P_0}{M}\frac{(\tau_y-\tau_\Delta)^2}{2}$$

and the minimum value:

$$x(\tau_y)_{min} = \dot{x}(\tau_y-\tau_\Delta)+J_y\frac{(\tau_y-\tau_\Delta)^2}{2}+\Delta-\frac{P_0}{M}\frac{(\tau_y-\tau_\Delta)^2}{2}$$

is achieved at $P(x)$ = const = $P_0$. Thus, the optimal characteristics of a limiter has the following form $P(x) = P_0$. For the case $\tau_y<\tau_\Delta$ и $t\in[\tau_\Delta;\, \tau_g]$ where, $\tau_g$ is the time when the deformation reaches its maximum value $x_g$ taking into account $J_y = 0$ we shall obtain the same result. The optimal characteristics of a vibration isolator with a limiter has the following general view:

$$R(x) = \begin{cases} cx & \forall x \in (-\Delta;\Delta) \\ P_0 & \forall x \in (-\infty;-\Delta] \cup [\Delta;+\infty) \end{cases} \tag{9}$$

## OPTIMIZATION OF VIBRATION PROTECTION SYSTEM WITH DEFORMATION LIMITER

In order to perform the acceleration limit during an impact the following equation is necessary:

$$P_0 \le [J_y]M \tag{10}$$

Let's determine the optimal parameters $P_0$, $\Delta$ and C (or $k = \sqrt{C/M}$ ) the characteristics of Eq. 9 type at which $x_g$ reaches its minimum. Let's consider the following case: $\tau_y<\tau_\Delta$. Having divided the VIS motion time from the impact start till the moment $\tau_g$ into three stages: $[0;\, \tau_y]$; $(\tau_y;\, \tau_\Delta)$; $[\tau_\Delta;\, \tau_g)$ having integrated the linear differential equations of VIS motion during these stages, we obtain the following:

$$x(\tau_y) = \frac{2J_y}{k^2}\sin^2\frac{k\tau_y}{2}$$

$$x_g = \Delta+\frac{Mk^2}{P_0}\left(4\frac{J_y^2\sin^2(k\tau_y/2)}{k^4}-\Delta^2\right)$$

The solution performance domain during the second stage:

$$\Delta \le 2\frac{J_y}{k^2}\sin\frac{k\tau_y}{2} \tag{11}$$

From $\tau_y<\tau_\Delta$ we obtain:

$$x(\tau_y)<\Delta \tag{12}$$

To make the desired optimal parameter k satisfy the vibration isolation requirements, it shall be located within $Q_1$ domain, determined by the inequalities Eq. 1-5:

$$k_1 \le k \le k_2 \tag{13}$$

where, $k_1$ = min k; (k, $\Psi$)$\in Q_1$; $k_2$ = max k; (k, $\Psi$)$\in Q_1$. The range of $\Delta$ changes will be obtained on the basis of deformation limit condition performance and the absence of limiter impact:

$$H_1 \le \Delta \le [x_g] \tag{14}$$

where, $H_1$ is the least H value within $Q_1$ domain, obtained similarly to $H_{min}$ within Q domain. The inequalities Eq. 8, 10-14 determine the problem solution domain within $\Delta$, k, $P_0$ coordinates. Having replaced the variables $k\tau_y/2 = u$, $2\Delta u^2/J_y\tau^2_y = z$ we obtain the inequality system S:

$$z \le \frac{P_0}{2J_y}M;\ P_0 \le [J_0]M;$$
$$z \le \sin u;\ z \ge \sin^2 u;$$
$$u_1 \le u \le u_2;\ \alpha_1 u^2 \le z \le \alpha_2 u^2$$

where, $\alpha_1 = 2H_1/J_y\tau^2_y$; $\alpha_2 = 2[x_g]/J_y\tau^2_y$. The expression for $x_g$ by taking into account the new variables will be the following:

$$x_g(u, z, P_0) = \frac{J_y\tau_y^2}{2u^2}\left(z+\frac{MJ_y}{2P_0}(\sin^2 u-z^2)\right)$$

Let's determine the optimal point where, $x_g$ value is the least one, $(u^*, z^*, P^*_0)$. While $z\le\sin u$ and $\partial x_h/\partial P_0<0$, then $P^*_0 = [J_y]M$ and the problem is reduced to a two-dimensional optimization problem min (u, z)$x_g$ (u, z, $P^*_0$). Figure 3 shows the domain D, determined by an inequality system S at the same numerical values of the VIS loads and limits as described above (except for the value $J_s$, increased up to 720 m/sec$^2$).

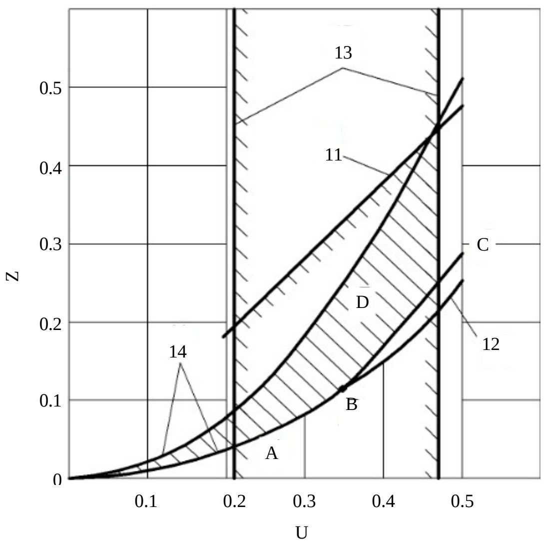


Fig. 3: Domain D of vibration isolator existence when the deformation limiter satisfying the set requirements. The curve numbers correspond to the numbers of inequalities defining these curves

While $\partial x_g/\partial_z = J_y\tau^2_y/2u^2\,(1\text{-}MJ_y z/P_0)$ and $z \le P_0/2J_yM$, then $\partial x_g/\partial z > 0$ and $\min x_g$; $(u, z \in D)$ is achieved by the least value z. Therefore, $\min x_g\,(u, z, P^*_0)$; $(u, z \in D) = \min x_g(u, z \in \Gamma)$ where $\Gamma$ is ABC line (Fig. 3). Having divided $\Gamma$ line into spots $\Gamma_1$ $(z = \sin^2 u)$ and $\Gamma_2(z = \alpha_1 u^2)$ and inserting these z values in Eq. 15 we obtain that $\partial x_g/\partial u < 0$ in all points of $\Gamma$ line. Thus, the C point with the coordinates $u^* = u_2 = k_2\tau_y/2$, $z^* = \alpha_1 u^2_2 = H_1 k^2_2/2J_y$ is an optimal one. Thus, the optimal values are $\Delta^* = H_1$, $k^* = k_2$, $P^*_0 = [J_y]M$.

Having explored BC motion at $\tau_y > \tau_\Delta$, we obtain the following:

$$x_g = \frac{\left(A_1 + J_y\tau_y\right)^2 M}{2P_0} - \frac{J_y}{2}\tau^2_y + B_1$$

Where:

$$A_1 = \sqrt{\Delta\left(2J_y - \Delta k^2\right)} - \left(J_y - \frac{P_0}{M}\right)\tau_\Delta;\ \tau_\Delta = \frac{1}{k}\arccos\left(1 - \frac{\Delta k^2}{J_y}\right)$$

$$B_1 = \Delta + \frac{MJ_y - P_0}{2M}\tau^2_\Delta - \sqrt{\Delta\left(2J_y - \Delta k^2\right)}\tau\Delta$$

The optimal values are $P^*_0 = [J_y]M$, $\Delta^* = H_1$. The optimal value k* due to its complex exploration $\partial x_g/\partial k$ is reasonable to define by a numerical method on the interval $[k_1; k_2]$ at $\Delta = \Delta^*$, $P_0 = P^*_0$.

## CONCLUSION

The obtained results may be used during the VIS design for the objects of different techniques. They make possible to evaluate quickly the feasibility of the requirements for the projected VIS, select the type of vibration isolator elastic-damping characteristics and their optimal parameters.

Having replaced the equations of mathematical description for dry friction shock absorbers with the equations for the shock absorbers with elastic-damping elements based on elastomers one may obtain in a similar manner the VIS existence area with the vibration isolators based on elastomers.


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