

## Development of Squeeze Film Damper Characteristics Calculation Methods Which Take into Account a Liquid Inertia Forces

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**Abstract:** Research of characteristics of Squeeze Film Damper (SFD) is presented in this study. These characteristics depend on convective forces of inertia which were obtained for models of short and long dampers by averaging of velocity on a gap thickness by Slezkin Targ Method. It is obtained that for both of long and short SFD in a case of full coverage a tangential force depends only of oil viscous properties and equal to force calculated without taking into account an inertia forces. Radial force depends only on inertia properties of the layer. In a case of half-coverage both of components of hydrodynamic force include the viscous and inertia components and a damping reduces. It is shown that it is necessary to take inertia forces into account because they come to 30% of viscous forces and may even exceed the viscous forces for large gaps.

**Key words:** Squeeze film damper, convective inertia forces, viscosity, characteristics, layer

### INTRODUCTION

Dampers of different types are widespread now for supports of turbo-machines (Belousov *et al.*, 2009). In the present time dry friction dampers and squeeze film dampers are used (Novikov *et al.*, 1996; Shabaev *et al.*, 2010). A liquid flow in a damper gap is described by equations of Navier-Stokes and continuity. If a liquid with little viscosity is used in damper, for high frequency convective members of inertia have significant influence in equations of Navier-Stokes. Zhang and Roberts (Zhang and Roberts, 1997; Zhang, 1997) researched this question. They solved a problem by superposition method; coefficients of stiffness, damping and attached mass were obtained for a case of full coverage. Tichy (1987) solved the problem in analogous way by introduction of flow functions; it allowed him to obtain a modified Reynolds equation. In a research (Belousov *et al.*, 1985), an experimental data about distribution of pressure in a gap are presented and study (Jiang *et al.*, 2005; Novikov and Zhangrh, 2004; Novikov, 2014) contains a method to take into account of influence of seals made of Metal Rubber material on SFD characteristics. An algorithm of choice of damper type for “stiff” rotors is presented by Belousov and Novikov (1986), Falaleev *et al.* (2014), Chaadaev and Novikov (2009) and Shabliy *et al.* (2014).

### DEVELOPMENT OF CALCULATION MODEL

In a long damper an axial flow of liquid is absent and it is possible to present the Navier-Stokes equations as:

$$\rho \left( V_x \frac{\partial V_x}{\partial x} + V_y \frac{\partial V_x}{\partial y} \right) = -\frac{\partial P}{\partial x} + \mu \frac{\partial^2 V_x}{\partial y^2}, \quad (1)$$

$$\frac{\partial P}{\partial y} = 0; \quad \frac{\partial V_x}{\partial x} + \frac{\partial V_y}{\partial y} = 0$$

Here:

- $V_x, V_y$  = Components of liquid movement velocity along x and y-axis, respectively (Fig. 1)
- $P$  = A dynamic pressure in the oil layer
- $\rho$  = Density of oil
- $\mu$  = A coefficient of dynamic viscosity

Equation 1 is solved with boundary conditions by velocity (Fig. 1):

$$V_x = V_y = 0 \text{ for } y = y\delta, \quad V_x = -e\Omega \cos \varphi, \quad (2)$$

$$V_y = -e\Omega \sin \varphi \text{ for } y = 0$$

Here:

- $\delta = \delta_0(1 + \varepsilon \cos \varphi)$  = A gap on an angle  $\varphi$
- $\delta_0$  = A value of radial gap in the damper
- $\Omega$  = Angular velocity of vibrator precession
- $\varepsilon = e/\delta_0$  = A relative amplitude of vibration
- $e$  = A displacement of vibrator sleeve (Fig. 1)

It is possible to present a first equation of the system Eq. 1 as:

$$\rho \frac{\partial V_x^2}{\partial x} + \rho \frac{\partial (V_x V_y)}{\partial y} = -\frac{\partial P}{\partial x} + \mu \frac{\partial^2 V_x}{\partial y^2} \quad (3)$$

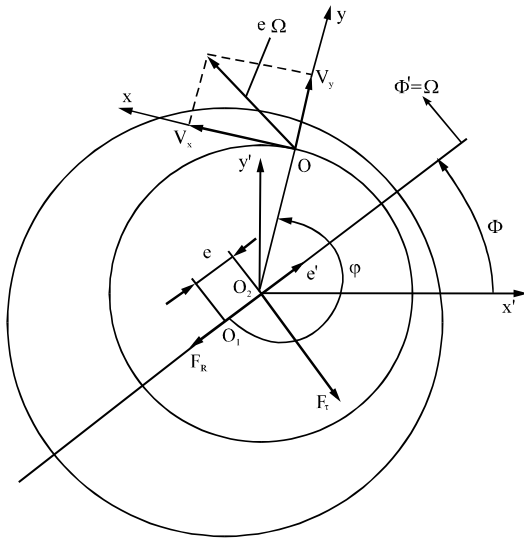


Fig. 1: Scheme of SFD

Let integrate an Eq. 3 in limits from 0 to \$\delta\$ and to take into account:

$$V_x V_y \Big|_0^\delta = -e^2 \Omega^2 \cos \phi \sin \phi; \frac{\partial V_x}{\partial y} \Big|_0^\delta = -\frac{12}{\delta} \bar{V}_x$$

here, \$\bar{V}\_x\$ is a middle velocity from pressure gradient (flow of Poiseuille):

$$\int_0^\delta \frac{\partial V_x^2}{\partial x} dy = \frac{\partial}{\partial x} \left( \int_0^\delta V_x^2 dy \right) - V_x^2 \Big|_0^\delta \frac{\partial \delta}{\partial x}$$

If to take into account the first boundary condition of Eq. 2, it is possible to obtain:

$$\int_0^\delta \frac{\partial V_x^2}{\partial x} dy = \frac{\partial}{\partial x} \left( \int_0^\delta V_x^2 dy \right)$$

It is possible to assume approximately that:

$$\int_0^\delta V_x^2 dy = \delta \alpha \bar{V}_m^2$$

here, \$\alpha\$ is correction factor which takes into account an influence of profile of velocity:

$$\bar{V}_m = \bar{V}_x + \frac{V_x}{2}$$

is common middle velocity of liquid flow equal to sum of middle velocities of Poiseuille flow \$\bar{V}\_m\$ and Couette flow \$V\_x\$.

For the case it is possible to assume that \$\alpha = 1.2\$. If to insert to the result of integration an equation for expenditure:

$$Q = \delta \bar{V}_m \tag{4}$$

it is possible to obtain an equation of movement of liquid as:

$$\frac{2\delta}{D} \frac{\partial P}{\partial \phi} + \frac{2\rho\alpha}{D} \frac{\partial}{\partial \phi} \left( \frac{Q^2}{\delta} \right) + 12 \frac{\mu}{\delta} \left( \frac{Q}{\delta} + \frac{e\Omega}{2} \sin \phi \right) - \rho e^2 \Omega^2 \cos \phi \sin \phi = 0 \tag{5}$$

here \$D\$ is diameter of damper. Let find the expenditure:

$$Q = \int_0^\delta V_x dy$$

For this let integrate an equation of continuity in limits from 0 to \$\delta\$ and take into account the boundary conditions Eq. 2:

$$\int_0^\delta \frac{\partial V_x}{\partial x} dy = \frac{\partial}{\partial x} \left( \int_0^\delta V_x dy \right) - V_x \Big|_0^\delta \frac{\partial \delta}{\partial x}$$

Thus, an equation for expenditure will be as:

$$Q = 0.5e\Omega D \cos \phi + C_2 \tag{6}$$

An arbitrary constant \$C\_2\$ depends on \$\rho, \mu, \Omega, e, \alpha\$. It is possible to find this constant by a help of boundary condition on pressure. It should meet the condition of equality of expenditure to zero for \$\Omega = 0\$ that is:

$$C_2(\rho, \mu, \Omega, e, \alpha) = 0 \text{ for } \Omega = 0 \tag{7}$$

It is more convenient for a further solution to present an Eq. 5 as dimensionless. It is possible to use for it a dimensionless pressure:

$$\bar{P} = \frac{1}{3\mu\Omega} \left( \frac{\delta_0}{D} \right)^2 P \tag{8}$$

If to insert Eq. 6 into Eq. 4 and to throw off parts with order \$\delta\_0/D\$, a result will be as:

$$\frac{\partial \bar{P}}{\partial \phi} = f'(\phi, \bar{C}_2) - \epsilon \frac{\cos \phi}{h^3} - \frac{\bar{C}_2}{h^3} \tag{9}$$

here \$\bar{C}\_2\$ is dimensionless constant of integration:

Table 1: Equations for forces in SFD which take into account the convective inertia forces

Calculation	Equations
<b>Full coverage</b>	
Long damper	$F_{Rf}^{(l)} = \frac{\pi}{6} \alpha \sigma \frac{1-\sqrt{1-\epsilon^2}}{\epsilon}, F_{\tau}^{(l)} = 2\pi \frac{\epsilon}{(2+\epsilon^2)\sqrt{1-\epsilon^2}}$
Short damper	$F_{Rf}^{(s)} = \frac{2\pi\alpha\sigma\lambda^2}{9\epsilon} \left( \frac{2-\epsilon^2}{\sqrt{1-\epsilon^2}} - 2 \right), F_{\tau}^{(s)} = \frac{2\pi\epsilon\lambda^2}{3(1-\epsilon^2)\sqrt{1-\epsilon^2}}$
<b>Half coverage</b>	
Long damper	$F_{Rh}^{(l)} = F_{Rj}^{lh} + F_{R\mu}^{lh}; F_{Rj}^{lh} = \frac{\pi\alpha\sigma}{24\epsilon} \left[ 2 \frac{\epsilon^2(1-\bar{C})^2 + 2(1-\epsilon^2)^2}{(1-\epsilon^2)^{1.5}} \right]; F_{R\mu}^{lh} = 2 \frac{\bar{C}-\epsilon^2}{(1-\epsilon^2)^2}; F_{\tau}^{(l)} = F_{\tau}^{lh} + F_{\tau\mu}^{lh}; F_{\tau}^{lh} = \frac{\alpha\sigma}{12} \left[ 2 \left[ 1 + \epsilon^2 \left( \frac{1-\bar{C}}{1-\epsilon^2} \right)^2 \right] \frac{1}{\epsilon} \ln \frac{1+\epsilon}{1-\epsilon} \right]; F_{\tau\mu}^{lh} = \frac{1+2\epsilon^2-3\bar{C}}{(1-\epsilon^2)^{2.5}}$
Short damper	$F_{Rh}^{(s)} = F_{Rj}^{sh} + F_{R\mu}^{sh}; F_{Rj}^{sh} = \frac{\pi\alpha\sigma\lambda^2}{9\epsilon} \left( \frac{2-\epsilon^2}{\sqrt{1-\epsilon^2}} - 2 \right); F_{R\mu}^{sh} = \frac{4\lambda^2\epsilon^2}{3(1-\epsilon^2)^2}; F_{\tau}^{(s)} = F_{\tau}^{sh} + F_{\tau\mu}^{sh}; F_{\tau}^{sh} = \frac{2\alpha\sigma\lambda^2}{9\epsilon} \left( \ln \frac{1+\epsilon}{1-\epsilon} - 2\epsilon \right); F_{\tau\mu}^{sh} = \frac{\pi\epsilon\lambda^2}{3(1-\epsilon^2)\sqrt{1-\epsilon^2}}$

$$\bar{C}_\theta = \frac{2C_\theta}{\delta_\theta \Omega D}$$

$$f'(\varphi, \bar{C}_\theta) = \frac{\alpha\sigma\epsilon \sin \varphi}{12h^2} \left( \frac{2\epsilon \cos \varphi - \epsilon^2 \frac{\cos^2 \varphi}{h} +}{2\bar{C}_\theta \epsilon - 2\bar{C}_\theta \epsilon \frac{\cos \varphi}{h} - \frac{\bar{C}_\theta^2}{h}} \right) \quad (10)$$

here,  $h = 1 + \epsilon \cos \varphi$ . Parameter  $\sigma$  is determined by equation  $\sigma = \rho \Omega \delta_\theta^2 / \mu$ . This parameter  $\sigma$  is a ratio of inertia force and viscosity force. A part  $f'(\varphi, \bar{C}_\theta)$  in Eq. 9 determines a contribution of convection members of inertia. If it doesn't take into account ( $\sigma = 0$ ), a well-known equation for purely viscous flow will be obtained. If to integrate an Eq. 9, it is possible to find a distribution of pressure:

$$\bar{P} = f(\varphi, \bar{C}_\theta) - \epsilon J_3^{01} - \bar{C}_\theta J_3^{00} + C'_\theta \quad (11)$$

Here:

$$f(\varphi, \bar{C}_\theta) = \frac{\alpha\sigma\epsilon}{12} \left( 2\epsilon J_2^{11} - \epsilon^2 J_3^{12} + 2\bar{C}_\theta J_2^{10} - 2\bar{C}_\theta \epsilon J_2^{11} - \bar{C}_\theta^2 J_3^{10} \right) \quad (12)$$

$$J_N^j = \int \frac{\sin^j \varphi \cos^i \varphi d\varphi}{h^N}$$

are integrals of theory of lubrication. Dimensionless hydrodynamic forces are  $F_R, F_\tau$  that is coefficients of loading, it is possible to obtain as integrals from the dimensionless pressure. Model of calculation of short SFD is analogous. Results of calculation are presented in Table 1. Indexes in Table 1 means: R and  $\tau$  are force direction, radial and tangential respectively; (l) is long damper, (s) is short one, h is half coverage, f is full coverage; j is inertia component,  $\mu$  is viscous one.  $\lambda = L/D$ , here L is length of damper.

### ANALYSIS OF CALCULATION RESULTS

Let consider a dependency of damper characteristics on inertia force. Ratio of radial and tangential forces for short SFD, full coverage and little  $\epsilon$  it is possible to obtain from Table 1:

$$\frac{F_{Rf}^{(s)}}{F_{\tau}^{(s)}} = \frac{\alpha\sigma\epsilon^2}{24} \quad (13)$$

As it is seen from Eq. 13 for little vibration amplitude the inertia force is less than viscous force. Radial force depends on parameter linearly. For vibration amplitude  $\epsilon \leq 0.6$  inertia force considerably less than viscosity force: for  $\sigma < 10$  they are not  $>10\%$  of viscous force and only for  $\sigma = 40$  they approach to it.

In a case of half coverage it is convenient to introduce the relative coefficients of load. These coefficients are characteristics of ratio of inertia force and total force. They are equal to:

$$f_{Rh}^{(s)} = \frac{F_{Rj}^{sh}}{F_{Rh}^{(s)}}, f_{\tau}^{(s)} = \frac{F_{\tau}^{sh}}{F_{\tau}^{(s)}} \quad (14)$$

Here, components of Eq. 14 should be obtained from Table 1. It is possible to use analogous coefficients for long SFD too.

If to analyze dependencies of these coefficients on  $\sigma$  and  $\epsilon$  (Fig. 2), it is possible to see that for its increasing an influence of inertia force becomes significant. For example, for  $\epsilon = 0.6$  already for  $\sigma = 10$  inertia forces are about 25% on radial component and 40% on tangential one. If  $\sigma$  continue to increase, an influence of inertia forces increases much more. For long damper and full coverage it is possible to obtain from Table 1:

$$\frac{F_{Rh}^{(l)}}{F_{\tau}^{(l)}} = \frac{\alpha\sigma}{12} \quad (15)$$

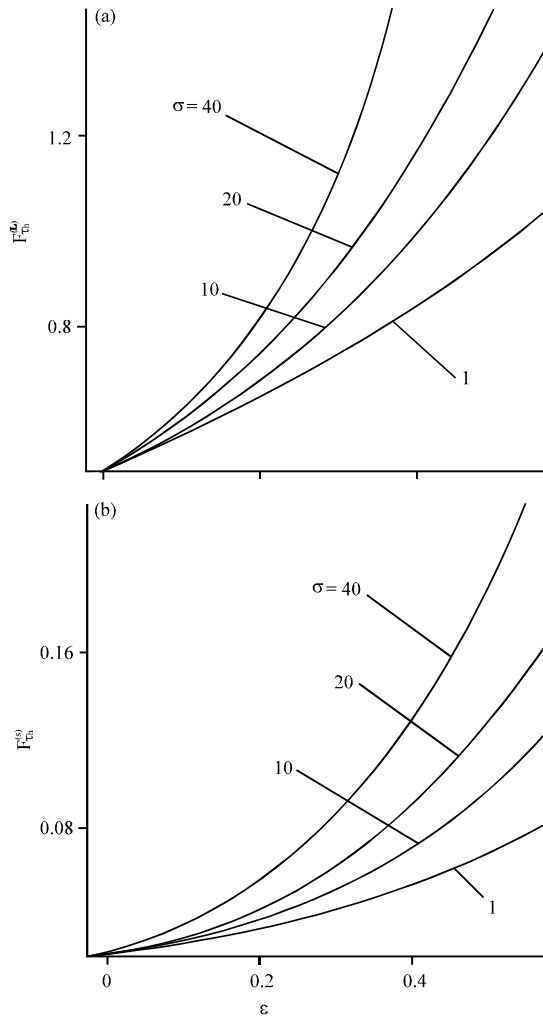


Fig. 2: Characteristics of SFD; a) short and b) long

If to compare Eq. 13 and 15, it is possible to see that in long SFD for little inertia forces have much more large influence than in the short one. If  $\epsilon \geq 0.6$  already for  $\sigma = 10$  inertia forces are about 30% from viscous one. Thus, it is impossible to neglect them in this case.

It is follow from Fig. 2 that hydrodynamic forces till  $\epsilon \leq 0.1$  are approximately linear relatively to displacement. After  $\epsilon \geq 0.2$  an exfoliation of curves begins. It depends on parameter  $\sigma$  that is an influence of inertia forces begins and a type of this influence is as for the case of full coverage: the non-linearity increases when  $\sigma$  increases.

### CONCLUSION

The taking into account of local inertia members requires a research of transient processes in a layer of oil. These processes have no significant influence

on dynamic of rotors of turbo-machines, thus it is possible to limit the research by obtaining of convective members of inertia, however to estimate the influence of amplitude more correctly. To obtain solution for any vibration amplitude, a method of averaging by thickness of velocity layer developed by Slezkin and Targ is used. A case of direct synchronous precession is considered. In the beginning, the problem of infinitely long damper is solved.

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