

## Methodology of Rheological Material Properties Phenomenological Modeling at High Speed Cutting by Reverse Analysis

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**Abstract:** The technique of reverse coefficient analysis concerning the model of the material rheological properties (Johnson-Cook Constitutive Model) at its high-speed milling which allows, regardless of the deformation component to calculate reliably the coefficients of a viscoplastic hardening and thermal softening. The use of this technique reduces the number of calculations with the use of FEM by grouping the experiment results. The example of coefficient calculation for the material with the following chemical composition (C-0,1% Ni-23% Cr-10% Ti-3% Mo-1,3% Al-0.8 rest <1%) is presented.

**Key words:** Cutting temperature, cutting force, rheological properties, the stress-strain state, Johnson-Cook Constitutive Model

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### INTRODUCTION

The main advantages of high-speed processing are well known: the high rates of material removal, the processing period reduction by several times, the cutting force reduction, the withdrawal of most part of the heat with chip removal that help to reduce the workpiece warping as well as to increase the surface accuracy and quality. Nevertheless, the problems associated with the use of high-speed processing are largely determined by the properties of a processed material and the product geometry (Dewes and Aspinwall, 1997). The simulation of the material behavior, especially during high speed cutting is an important at the optimization process of blade processing parameters. The analytical, experimental, numerical simulation methods and hybrid methods are used widely. Some analytical models have been previously proposed by Merchant (1944), Oxley (1963), Arrazola *et al.* (2013) and Molinari and Dudzinski (1992). The last works were adapted for milling processes (Fontaine *et al.*, 2006). The Finite Element Method is one of the most effective tools for the cutting material process study. Finite-Element Models (FEMs) are designed as for orthogonal cutting scheme so as for 2 and 3D FEM. The orthogonal models are used most frequently to investigate the cutting process mechanism. Simoneau *et al.* (2006) studied the chip formation processes for medium carbon steel using the orthogonal cutting model, Jin and Altintas (2012) studied these processes to analyze the cutting force magnitude during

micromilling. In orthogonal setting Mohammadpour *et al.* (2010), the influence of cutting parameters on residual stresses was investigated and Shams and Mashayekhi (2012) developed the model of non-local damage during the cutting process to improve the FEM-simulation quality.

The material properties in the treatment zone are set by defining relations (constitutive models) establishing the connection between the stress, strain state and temperature fields. Since, high deformation speeds are necessary for the processing by cutting then the constant coefficients included in these models are determined depending on the specific processing conditions.

To identify the parameters of material property phenomenological models, several researchers used the method of inverse (reverse) engineering (Shams and Mashayekhi, 2012; Pantale, 1996; Ghouati and Gelin, 1998, 2001; Maurel *et al.*, 2008) based on the finite element method as well as the experimental and analytical method (Haimovich, 2012). The method consists in coefficient determination of the material property model (constitutive models) while minimizing the functional that takes into account the difference between the parameters of the cutting process numerical simulation and the experimental data. The cutting force (or temperature occasionally) as the value that is most easily obtained by measuring is considered as the most frequently selected comparison parameter.

It should be noted that some materials are very susceptible to hardening and softening depending upon the strain rate which varies in the cutting zone within a

wide range (the order makes from 102-105). Unfortunately, the disadvantage of the existing methods concerning reverse engineering is revealed by the fact that at the same time all the coefficients included in the model of defining relations model are determined at the same time. This fact leads to a large number of calculations in CAE systems and to insufficient adequacy of the obtained results. In this regard, the development of the technique that determines the speed hardening of the material in the cutting process, regardless of strain hardening magnitude is considered as relevant as it would significantly improve the defining relations accuracy.

The technique considered in this study allows to reduce the number of calculations using the FEM by grouping the experiment results.

### THEORETICAL PART

The main problem solved by the theoretical part is to develop a method for obtaining the material model coefficients (first approximation) characterizing its rheological properties, depending on the strain rate with a minimum number of resource-intensive numerical analysis of the cutting process using the finite element method. This solution is based on the energy method of the upper estimation, the foundations of which are defined in the following works (Kudo, 1960; Kobayashi and Thomsen, 1965) which allow to get the energy power parameters of the deformation process on the basis of a virtual kinematically possible velocity field. At this approach, a necessary condition in the energy balance equation is the condition of continuity of the normal components of the velocity field and the minimum energy of a plastic deformation corresponds to the values of model coefficients concerning the material properties close to real ones.

In accordance with the upper estimation method (Goun, 1980) at a fixed geometry of the cutting tool in cutting power dissipation obtained on the basis of a virtual kinematically possible velocity field in the Plastic Deformation Hearth (PDH) is described by the following dependence (Eq. 1):

$$I_1^0 = \iiint_{W(V,s,t)} \sigma_s \dot{\epsilon}_2^0 dW + \iint_{A_1(V,s,t)} \frac{\sigma_s}{\sqrt{3}} |V_\gamma^0| dA + \iint_{A_2(V,s,t)} \frac{\sigma_s}{\sqrt{3}} \mu(\sigma_n^0) |V_\sigma^0| dA \quad (1)$$

Where:

W = PDH volume

A<sub>1</sub> = The adhesive friction area along the front surface

A<sub>2</sub> = A<sub>σγ</sub> + A<sub>σα</sub> = The total sliding friction area along the front surface A<sub>σγ</sub> and rear surface A<sub>σα</sub>, determined by the normal pressure magnitude

V<sub>γ</sub>, V<sub>σ</sub> = Chip displacement velocity along the adhesive friction and sliding friction surfaces

μ(σ<sub>n</sub><sup>0</sup>) = 0-1 = Friction coefficient dependent on the normal pressure on the contact surface. The superscript "0" with the variables means that these values were obtained on the basis of the virtual velocity field

V, s, t = The speed, feed and the depth of cut, respectively

In each PDH point the equivalent stresses σ<sub>s</sub> are determined by technological cutting modes:

$$\sigma_s = \sigma_s(\epsilon_2^0, \dot{\epsilon}_2^0, T) \quad (2)$$

The Eq. 2 represents the constitutive relations which characterize the desired properties of the treated material depending on the effective strain  $\epsilon_2^0 = \epsilon_2^0(V, s, t)$ , the effective strain rate  $\dot{\epsilon}_2^0 = \dot{\epsilon}_2^0(V, s, t)$  and the temperature T. In accordance with the extreme principles of continuum mechanics:

$$I_1^0 \geq 1 \quad (3)$$

Where, I is the real cutting power:

$$V_{\gamma_1} = V \frac{\sin \phi}{\cos(\phi - \gamma)}; V_{\gamma_2} = V \cos \alpha \quad (4)$$

Where:

φ = The angle of the conditional plane shear in the chip section

γ = The front angle

α = The rear angle of the tool tooth

Using the mean value theorem in accordance with the extreme principles of continuum mechanics, let's present an upper estimation of the cutting force power:

$$I_2^0 = \sigma_{sm} (\dot{\epsilon}_{2m})_{V=1} \cdot V \cdot W + \frac{\sigma_{sm}}{\sqrt{3}} \cdot V \cdot \frac{\sin \phi}{\cos(\gamma - \phi)} \cdot A_1 + \mu(\sigma_{ny})_m \cdot V \cdot \frac{\sin \phi}{\cos(\gamma - \phi)} \cdot A_{\sigma\gamma} + \mu(\sigma_{n\alpha})_m \cdot V \cdot A_{\sigma\alpha} \cdot \cos \alpha \quad (5)$$

Where:

σ<sub>sm</sub> = The equivalent stress mean value

( $\dot{\epsilon}_{2m}$ )<sub>V=1</sub> =  $\dot{\epsilon}_{2m}/V$  = The average reduced deformation rate

$(\sigma_{ny})_m$  = The average normal stress on the front surface  
 $(\sigma_{na})_m$  = The average normal stress on the rear surface

As in Eq. 5 the plasticity condition is not satisfied which is characteristic for the method of the upper estimation, then we have the following inequality Eq. 6:

$$I_2^0 \geq I_1^0 \geq I \quad (6)$$

On the basis of Eq. 5 we define the following cutting force  $F^0 = I_2^0/V$ :

$$F = \sigma_{sm} (\dot{\epsilon}_{2m})_{V=1} \cdot W \cdot \left[ 1 + \left( \frac{1}{\sqrt{3}} + \mu k_{cy} k_y \right) \frac{1}{k_{A_1}} + \mu \frac{k_{\sigma\alpha}}{k_\alpha} \right] \quad (7)$$

where,  $k_{cy} = (\sigma_{na})_m / (\sigma_{sm})$  is the relatively normal stress on the front surface in the area of sliding friction with the length  $l_{cy}$ .  $k_y = A_{cy}/A_1 = l_{cy}/l_1$  is the ratio of friction section lengths with adhesion and the sliding friction on the front surface:

$$k_{A_1} = \frac{(\dot{\epsilon}_{2m})_{V=1} \cdot W \cdot \cos(\phi - \gamma)}{A_1 \cdot \sin \phi} \cdot c^{-1}$$

is the reduced deformation rate, characterizing PDH shape depending on the front angle and the friction surface value with the adhesion and the length of  $l_1$ :

$$\frac{W}{A_1} = \frac{A}{l_1} = \frac{S_z \cdot t}{l_1}$$

$$k_\alpha = \frac{(\dot{\epsilon}_{2m})_{V=1} \cdot W}{A_{\sigma\alpha} \cdot \cos \alpha}$$

is the reduced deformation rate characterizing the plastic deformation hearth shape depending on the rear angle  $\alpha$ .

The further detailing of dependence Eq. 7 in order to find the values of its variables requires the binding to specific cutting conditions. Let's define the cutoff parameters (the chip cross-section) for side milling.

$$\phi = \arccos \left( 1 - \frac{2t}{d} \right),$$

$$S_{zm} = \frac{A}{d \phi \cos \phi} = \frac{S_z t}{d \phi \cos \phi}, \quad \tau_\Sigma = \frac{z \phi}{2 \pi n} \quad (8)$$

$$\tau_m = \tau_\Sigma \frac{S_{zm}}{S_z} = \frac{z t}{2 \pi n \cos \phi d} = \frac{z t}{2 V \cos \phi} = \frac{z t}{2 V \left( 1 - \frac{2t}{d} \right)}$$

Where:

- $\phi$  = Cutter insertion angle with the cutter diameter  $d$
- $S_{zm}$  = Conditional average slice thickness in the feeding direction based on the constancy cut width condition
- $\tau_\Sigma$  = Cutting time per 1 tooth
- $\tau_m$  = The time at which the slice thickness  $S_{zm}$  value is achieved (Fig. 1)

Then, let's consider the results of the 2D-FEM modeling in Deform environment concerning the side milling of austenitic alloy with the following chemical composition (C-0.1% Ni-23% Cr-10% Ti-3% Mo-1.3% Al-0.8 the rest <1%). Figure 2 shows the distribution of an effective strain rate in the chip cross-section. 15 points of P1-P10 which are defined in the Lagrangian coordinate system, i.e., move together with the local displaceable volume of material around a point are shown as an example. The analysis of strain state parameters at each point allows to evaluate the  $\epsilon_2$ ,  $\dot{\epsilon}$  and T values change during the deformation.

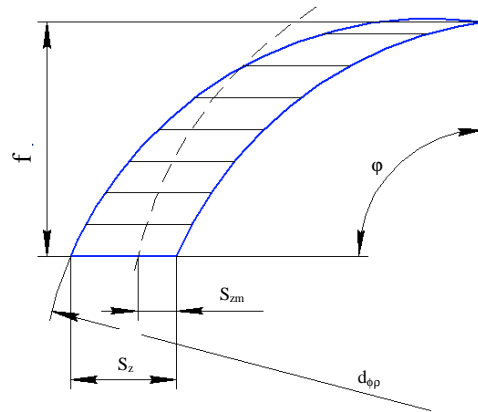


Fig. 1: Section parameters

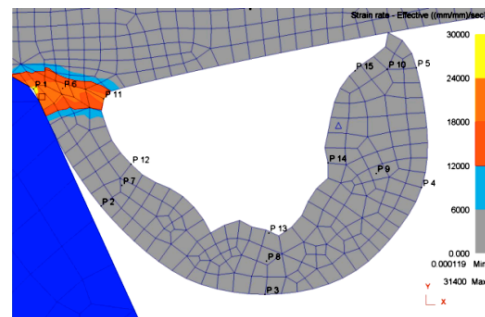


Fig. 2: The effective strain rate and the location of control points P1-P15 (in Lagrangian coordinates) within the chip section during the processing at the following modes  $S_z = 0.2$  mm/tooth;  $t = 0.5$  mm;  $V = 60$  m  $\text{min}^{-1}$

Let's determine the magnitude of the cutting force at the moment  $\tau_m$ . Without the presentation of the detailed evidence let's be limited by the following considerations, based on the numerical analysis of the strain state within the cutting zone.

At high feeds per tooth  $S_z$  the effective strain value at each point of the chip cross section after it came out of the deformation zone at the chip root is changed insignificantly (Fig. 3). Consequently, one may approximately assume that at the steady stage of cutting the effective strain value of the average chip by section is proportional to the volume of material that has passed through a conventional shear plane. Then the following estimate  $\epsilon_{2m}$  obtained for the control points  $I = 1...N$  is a true one:

$$\epsilon_{2m} = \frac{1}{N} \sum_{i=1}^N \epsilon_2(\tau_m)_i \quad (9)$$

The average value of the effective strain, subject to the performance  $(\tau_1)_i \leq \tau_m$  for each point is defined by the following relationship:

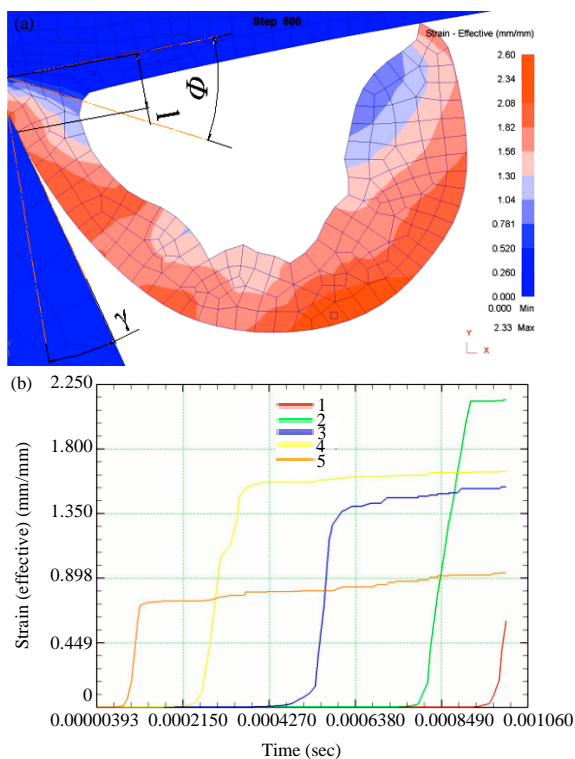


Fig. 3: Effective strain in the chip slice during processing within the following modes;  $S_z = 0.2$  mm/tooth;  $t = 0.5$  mm;  $V = 60$  m  $\text{min}^{-1}$ ; a) the distribution of effective strain and b) variation of the effective strain at the control points P15-P11 on the outer side of the chip cut depending on time

$$\dot{\epsilon}_{2m} = \frac{1}{t_m} \sum_{i=1}^N \int_{(t_0)_i}^{(t_1)_i} (\dot{\epsilon}_2)_i d\tau, (\dot{\epsilon}_{2m})_{V=1} = \frac{\dot{\epsilon}_{2m}}{V} \quad (10)$$

Where:

$(\tau_0)_i$  = The time of  $i$ th point entering in the area severe plastic deformation

$(\tau_1)_i$  = The time of coming out from the deformation zone (Fig. 4)

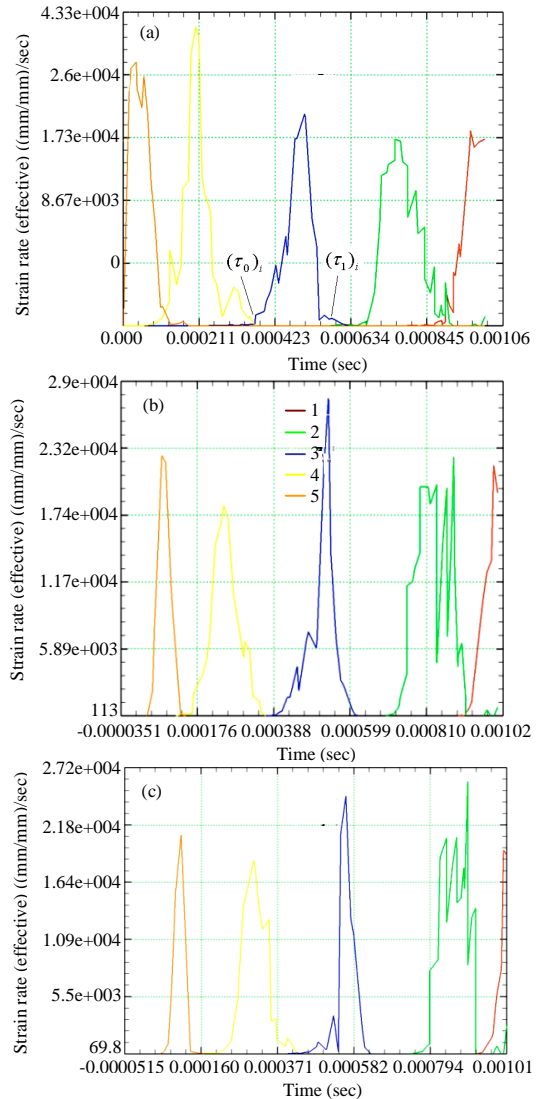


Fig. 4: The dependence of effective strain rate from time at the control points P1-P15; a) successively at the points P5-P1 with the convex inner side of the chip; b) consistently in the points P10-P6 in the mid-section of the chip section; c) successively at the points P15-P11 from the outer side of the chip cut; The volume of cut-off material in time  $\tau_m$

$$(\dot{\epsilon}_{2m})_{V=1} \approx \frac{\dot{\epsilon}_{2m}}{V \cdot \tau_m} = \frac{2\epsilon_{2m}}{zt} \left(1 - \frac{2\tau}{d}\right) \quad (11)$$

The integral in Eq. 10 is equal to the local area of the limited local curve on the graph (Fig. 4):

$$W(\tau_m) = S_{zm} \cdot t \cdot B = \frac{S_z \cdot t \cdot B}{d \left(1 - \frac{2t}{d}\right) \arccos\left(1 - \frac{2\tau}{d}\right)} \quad (12)$$

where, B is the milling width. Let's represent the constitutive relations linking the stress and strain state at the phenomenological relationship:

$$\sigma_s = \sigma_0 \cdot \overline{\sigma(\epsilon_2)} \cdot \overline{\sigma(\dot{\epsilon}_2)} \cdot \overline{\sigma(T)} \quad (13)$$

- $\sigma_0$  = Constant stress coefficient
- $\overline{\sigma(\epsilon_2)}$  = Strain hardening
- $\overline{\sigma(\dot{\epsilon}_2)}$  = Viscoplastic hardening
- $\overline{\sigma(T)}$  = Thermal softening

Equation 12 for example is easily realized in the form of a well-known Johnson-Cook Model (Johnson *et al.*, 1983):

$$\sigma = (A + B\epsilon_2^n) \left[ 1 + C \ln \left( \frac{\dot{\epsilon}_2}{(\dot{\epsilon}_2)_0} \right) \right] \cdot \left( \frac{\dot{\epsilon}_2}{(\dot{\epsilon}_2)_0} \right)^\alpha \cdot (D - E T^m) \quad (14)$$

$$\overline{T} = \frac{T - T_{20}}{T_{melt} - T_{20}}; D = D_0 \exp \left[ k(T - T_0)^\beta \right]$$

Let's consider the full factor plan of an experiment to determine the milling force  $F_{(V, s, t)}$  given for the set of process parameters:

$$\{V\{V_{min}, V_{mid}, V_{max}\}, \{S_{min}, S_{mid}, t_{max}\}, \{t_{min}, t_{mid}, t_{max}\}\}$$

27 experiments, Table 1. Let's consider the following types of relations:

Table 1: Standard experiment plan 3<sup>3</sup>

Exp No.	Cutting rate	Feeding	Cutting depth
1	min	min	min
2	min	min	mid
...	...	...	...
26	max	max	mid
27	max	max	max

$$\overline{F}_i = \frac{F_{(V_i, S = const, t = const)}}{F_{(V_{j \neq i}, S = const, t = const)}} \quad (15)$$

In accordance with the accepted plan of the experiment the Eq. 14 will be the following:

$$\frac{F_i}{F_{i+9}}, \frac{F_i}{F_{i+18}}, \frac{F_{i+9}}{F_{i+18}}, i = 1...9$$

Using Eq. 7-12, Eq. 14 may be transformed into:

$$\overline{F}_i = \frac{[1 + C \ln(\dot{\epsilon}_2)_i] (\dot{\epsilon}_2)_i^\alpha (D - E T_i^m)}{[1 + C \ln(\dot{\epsilon}_2)_j] (\dot{\epsilon}_2)_j^\alpha (D - E T_j^m)} \cdot \left[ \frac{1 + \left( \frac{1}{\sqrt{3}} + \mu(k_{\sigma\gamma})_i k_\gamma \right) \left( \frac{1}{k_{A_i}} \right)_i}{1 + \left( \frac{1}{\sqrt{3}} + \mu(k_{\sigma\gamma})_j k_\gamma \right) \left( \frac{1}{k_{A_j}} \right)_j} \right]^{\gamma \neq i} \quad (16)$$

According to Eq. 15, we temporarily neglected the rear surface friction. The values for the variables  $\epsilon_2, l_1, T, \Phi$  included into Eq. 15 may be determined from the results of the cutting process numerical simulation in CAE system (Fig. 2) for example in Deform. For the 0th iteration the properties of the analog material are taken with the similar rheological properties. The values of D, E, m coefficients for the 0th iteration are also taken from the analog material. Then the unknown coefficients C and  $\alpha$  in Eq. 15 are determined from the condition Eq. 16 of the experimental value comparison with the theoretical results:

$$\overline{F}_i = \frac{F_i}{F_j} = \frac{F_{itest}}{F_{jtest}} \quad (17)$$

where,  $(F_i)_{test}, (F_j)_{test}$  are the cutting force values obtained from the experiment. The method of the material rheological property parameters calculation during milling:

1. Full factor experiment plan {V, S, t} determination with the parameter varying intervals min, mid, max
2. Experimental measurement of the cutting force in accordance with the experiment plan
3. The phenomenological model selection concerning the rheological properties according to the multiplicative principle:

$$\sigma_s = \sigma_0 \cdot \overline{\sigma(\epsilon_2)} \cdot \overline{\sigma(\dot{\epsilon}_2)} \cdot \overline{\sigma(T)} \quad (18)$$

4. The choice of prototype material with similar rheological properties determined by the coefficients (A, B, C, E, D,  $\alpha$ , m, n)<sub>prot</sub> in the case of Johnson-Kudo Model:

$$(\sigma_s)_{prot} = \sigma_0 \cdot \overline{\sigma(\epsilon_2)} \cdot \overline{\sigma(\dot{\epsilon}_2)} \cdot \overline{\sigma(T)} = \sigma_s \{ \epsilon_2, \dot{\epsilon}_2, T (A, B, C, D, E, \alpha, m, n)_{prot} \} \quad (19)$$

5. The iterative calculation of the average values for the deformed state parameters in the chip section according to the numerical simulation results for CAE System, depending on the technological parameters:

$$\begin{aligned} \epsilon_2^l &= \epsilon_2(V, s, t, \sigma_s^l); \dot{\epsilon}_2^l = \dot{\epsilon}_2(V, s, t, \sigma_s^l) \\ T^l &= T(V, s, t, \sigma_s^l) \end{aligned} \quad (20)$$

where, l superscript denotes the iteration number at the initial conditions  $\sigma_s^0 = (\sigma_s)_{prot}$

6. The calculation of the specified model coefficients:

$$\sigma_s^l = \sigma_s \{ \epsilon_2, \dot{\epsilon}_2, T (A, B, C^l, D^l, E, \alpha^l, m, n) \}$$

responsible for viscoplastic hardening {C<sup>l</sup>, D<sup>l</sup>,  $\alpha^l$ } using the experimental data and relationships Eq. 15 and 16

7. The consistent adjustment of the  $\sigma_s$  model in an iterative process which includes the points 5 and 6

### SETTLEMENT AND EXPERIMENTAL PART

The purpose of the experiment performance in accordance with the specified procedure is an accurate definition and the recalculation of the model coefficients which due to the high strain rates typical for the cutting process can not be obtained with conventional plastometers. The technological modes of processing according to full factor experimental plan were appointed according to Table 2. The effective strain, strain rate, temperature T and the angle of the conventional shear plane values were determined by FEM-analysis methods with Deform software package. The rest of the parameters

included in the formula to determine the cutting forces were calculated using the above dependencies. The values of the cutting force for each mode were determined experimentally in accordance with the experiment plan (Table 1) and recorded into Table 2. The workpieces dimensions L×H×D 40×24×6 (mm) were fabricated from the material with the following chemical composition (C-0.1% Ni-23% Cr-10% Ti-3% Mo-1.3% Al-0.8, the rest were <1%). During the experiment, the cutting speed varied from 45-75 m min<sup>-1</sup>, the feed per tooth made 0.05-0.2 mm, the lateral removal  $\alpha_s = t$  made 0.1-0.5 mm. The Seco company cutter with the diameter of d = 12 mm was used as a tool.

The registration of cutting force components F<sub>x</sub>, F<sub>y</sub>, F<sub>z</sub> change in realtime held with Kistler Dynamometer Model 9257V is installed on the machine table ALZMETALL BAZ 15 CNC. To test the above discussed methods for the rheological properties of the processed material determination the analog material with similar properties was chosen.

The Johnson-Cook Model of the analog material is described with an approximating phenomenological relationship:

$$\begin{aligned} \sigma_s &= \sigma_0 \overline{\sigma(\epsilon_2)} \overline{\sigma(\dot{\epsilon}_2)} \overline{\sigma(T)} \\ \sigma_0 &= 1647.61 \\ \overline{\sigma(\epsilon_2)} &= (1-0.1\epsilon_2^{-0.345}) \\ \overline{\sigma(\dot{\epsilon}_2)} &= (1+0.0105 \ln(\dot{\epsilon}_2)) \\ \overline{\sigma(T)} &= \left( 1 - \left( \frac{T-20}{1480} \right)^{2.3} \right) \end{aligned}$$

On the basis of the data presented by Table 2 the dependence was established according to the set technique (Table 3) which helped to clarify the rheological model concerning the Johnson-Cook material properties, responsible for high-speed viscoplastic hardening  $\overline{\sigma(\epsilon_2)}$  and the temperature softening  $\overline{\sigma(T)}$  during the processing by milling.

The unknown coefficients  $\alpha$  and E were calculated by minimizing the mean square deviations with the Gauss-Newton iterative method. The results of these calculations are presented in Table 3.

Thus, the adjusted parameters of the Johnson-Cook Model have the following form:

Table 2: Initial experimental data to develop the phenomenological model of Johnson-Cook rheological properties

Experiment No.	V (m/min)	S <sub>z</sub> (mm/tooth)	t (mm)	$\epsilon_2$	T (°C)	$\Phi$ (grades)	F [N]
...	...	...	...	...	...	...	...
4	45	0.1	0.1	1.92	130	30	36.95
5	45	0.1	0.3	1.79	190	35	89.40
...	...	...	...	...	...	...	...
26	75	0.2	0.3	1.85	300	30	179.85
27	75	0.2	0.5	1.80	320	28	211.98

$\epsilon_2, T, \Phi$ : determined according to calculation results in Deform, F was determined experimentally

Table 3: Statistical results of rheological property model coefficients calculation

Model	Coefficients	Value	Statistical criterion t(2)	Statistical criterion (p)
$\frac{\bar{F}_i}{\bar{F}_j} = \frac{(1+0.01\ln(\dot{\epsilon}_2)_i \cdot (\dot{\epsilon}_2)_i^\alpha) \left(1-E \left(\frac{T_i-20}{1480}\right)^{2.3}\right)}{(1+0.01\ln(\dot{\epsilon}_2)_j \cdot (\dot{\epsilon}_2)_j^\alpha) \left(1-E \left(\frac{T_j-20}{1480}\right)^{2.3}\right)}, (s_2)_i = (s_2)_j, t_1 = t_j$	$\alpha$	0.841	<5	0.036
	E	9.900	<4	0.050

Model reliability: 78%

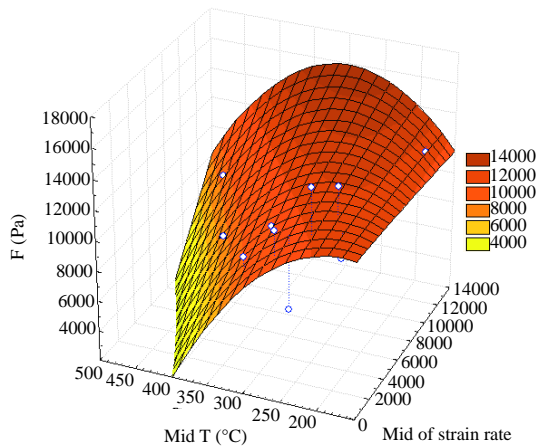


Fig. 5: Dependence of the specific cutting force  $\bar{F}$  from the calculated mean values of strain rate and cutting temperature

$$\begin{aligned} \bar{\sigma}(\dot{\epsilon}_2) &= (1+0.01\ln(\dot{\epsilon}_2))(\dot{\epsilon}_2)^{0.841} \\ \bar{\sigma}(T) &= \left[9.9 - \left(\frac{T-20}{1480}\right)^{2.3}\right] \end{aligned} \quad (21)$$

Dependence of the specific cutting force, equal to the ratio of cutting force to the chip cross-sectional area  $\bar{F} = F/S_2t$  from the parameters of the cutting temperature and the strain state is shown in Fig. 5.

### CONCLUSION

The reverse analysis of the material rheological properties during the cutting processing is tested. This technique combines the experimental determination of the milling force parameters, the analysis of the stress-strain state in the processing area by the finite element methods. The technique allows to calculate and update the coefficients of the rheological property phenomenological model concerning the processed material in the form of the Johnson-Cook law, establishing the connection between the stress and strain state.

Due to the technological cutting modes one may change the average effective strain rates within a wide range (the order from 102-105 s-1). This factor as well as

the result grouping of the full factor experiment concerning the cutting force measurement in the form of relations that are invariant to strain hardening, allows despite the model component deformation concerning rheological properties to calculate its coefficients characterizing the viscoplastic hardening and thermal softening reliably.

The constitutive relation model in the form of Johnson-Cook law is obtained for the heat-treated material with the following chemical composition (C-0.1% Ni-23% Cr-10% Ti-3% Mo-1.3% Al-0.8, the other components are <1%) for the milling conditions with the statistical accuracy certainty of 78%.

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