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Definition of the Stress-Strain State of Masonry Including Technological Factors of its Work

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Abstract: The study considers the basis of the new method of definition of the stress-strain state of masonry including technological factors of its research. Shortcomings of the existing masonry design codes in Russia and worldwide were considered and method of determining the irregularity of mortar consistency is shown. The proposed algorithms allow to determine the stress-strain state of the masonry of arbitrary geometry, strength and geometric properties of masonry materials.

Key words: Masonry, stress-strain state, technology of masonry, Fourier series implementation, geometry

INTRODUCTION

Not with standing the fact that more and more non-typical civil construction continues to increase with the use of brickwork as the basic material of the bearing structures and continuous scientific interest concerning the issue of these construction, the method used currently in the designing codes in Russia and abroad for the calculations of the bearing capacity has not changed much since, then middle of the last century (Onishchik, 1937) and there are more diverse experimental data towards the unreasonable huge allowance of bearing capacity of masonry with the use of modern bricks and mortars (Donchenko and Degtev, 2013).

Despite the fact that the load resistance of the materials composing masonry (brick and mortar) has been studied into details, the brickwork still remains one of the least rational construction materials, the strength of the masonry composite is far less stronger than that of the bricks and mortars separately. To a lesser extent than others, the masonry increases as the construction material. If in the 50 sec of the last century, the average strength of the concrete was about 20 MPa, at the end of century it was up to 50 MPa and currently it is possible to obtain more than 100 MPa but at the same time the highest strength of the unreinforced masonry has increased from 4-8 MPa (Belentsov *et al*., 2010). In particular, it concerns the most widespread types of masonry made with general purpose cement-sand mortars.

MATERIALS AND METHODS

On the whole, the principal problem of the masonry designing codes updating is characteristic as for the countries of the Eastern and Western Europe. The common principles of designing and calculation of masonry, underlying the European Standards (BS EN, 1996) are also based on the empirical dependencies which mostly idealize the conditions of the masonry structure to the detriment of the authenticity of the definition of its strain-stressed state. Eurocode 6 (BS EN, 1996) contains the simplified method of calculation for the most widespread space-planning decisions of civil brick buildings: The span of the floors and roof supported by the walls shall not exceed 7 m, the clear storey height shall not exceed 3.2 m and values of the variable actions on the overlapping shall not exceed 5.0 kPa. The characteristic compressive strength of masonry f_k is determined according to Eurocode depending on the parameters of the mortars bed joints: For masonry made with general purpose mortar:

$$f_k = Kf_b^{0.7} f_m^{0.3} \quad (1)$$

For masonry made with thin layer mortar, in bed joints of thickness 0.5-3 mm:

$$f_k = Kf_b^{0.85} \quad (2)$$

Where:

f_b = Normalized compressive strength of the brick
f_m = Compressive strength of the mortar, corresponding to its strength class

The K constant depends on the type of masonry units and mortars and changes within the range of 0.25-0.80. In particular for the layer of clay and calcium silicate units with general purpose mortar value of K is 0.4-0.5.

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According to the Russian Building codes (SP) 15.13330.2012 (2012) Masonry and reinforced masonry structures as well as based on them the building codes of masonry in the Eastern Europe, the compressive strength of masonry is determined via the tables made on the basis of the formula of prof. L.I. Onishchik (Russia, 1895-1968), specified experimentally (Onishchik, 1937):

$$R_u = AR_1\left(1-\frac{a}{b+\dfrac{R_2}{2R_1}}\right)\eta \tag{3}$$

Where:

R_1 and R_2 = Compressive strength of brick and mortar
η = Capacity reduction factor for weak mortar
A = Constructive index determined by Eq. 4

$$A = \frac{100+R_1}{100m+nR_1} \tag{4}$$

The a, b, m, n, the empirical constants depending on the type of masonry. Thus, the compressive strength of masonry by Eurocode 6 as well as by the methods based on the Russian Building codes directly depends only on the strength of brick and mortar and indirectly, via the various adjustment constants on a number of additional factors, e.g., mortar bed joints thickness. The use of methods with such restrictions does not allow a full consideration of the deformability, the geometry of masonry construction and what is more important, the technological factors of masonry works which contradicts the principles of rational construction designs.

Even the superficial analysis of masonry design building codes allows the determination of two main potential directions of their improvement. First, it is the introduction of the quantitative clarity amendment into the influence of geometrical size of units and mortar bed joints into its bearing capacity when building codes provide only average calculation value of masonry strength for a certain number of height ranges of the brick having scope of 200-300% (SP 15.13330, 2012). Secondly, it is reasonable quantitative calculation not only seasonal (summer/winter) conditions and arrangements of brick walls (external/internal) but also other technology factors connected with the unit geometry imperfection and the quality of masonry works (uneven height, filling and consistency of mortar bed joints, masonry pattern, etc.) which still do exist only fragmentarily in different scientific works (e.g., the coefficient of the bricklayer's hand mentioned by Rayzer (1990)).

Currently, the world industry of construction materials produces a large number of various masonry units significantly differing by sizes, strength and deformability. The production technologies of masonry works likewise considerably differ, the degree of readiness of mortar for use, the methods of bricklaying, the unit geometry imperfection and other factors result in the most diverge technological defects in the idealized design model of masonry composite. In this regard, the strength determination of the masonry from units of various size and deformability and also taking into account the real technological errors of bricklaying should be based on of mathematical modeling of the deformation and destruction of the loaded bricks, depending on the geometry, deformability and the general physical imperfection of mortar bed joints.

The set of technological imperfections of masonry composite, in the opinion can be generally expressed through the physical heterogeneity of the horizontal mortar bed joints characterized by variable passive pressure on the overlapping unit (Naumov and Ezhechenko, 2007). The researches of researchers allow to presuppose that this integrated defect which has been evenly distributed across the bed face, in this connection despite the spatial loading of masonry the stress problem could be considered as plane (Naumov, 2010).

The design model of single unit of masonry with generalized technological imperfections of mortar bed joints is shown in Fig. 1.

Separate evaluation of brick and joints loadings allows to set the plane stress problem of masonry units (Fig. 1) as plates interlinked by contact stresses and deformations, possessing a set of characteristics: geometrical l, h_b и h_m; physical E_b, ν_b и E_m (x), ν_m; external loads: axial $q_1^m(x), q_2^m(x), q_1^b(x), q_2^b(x)$ and lateral $t_1^m(x), t_2^m(x), t_1^b(x), t_2^b(x)$.

The greatest commonness for this type of stress problem are the results received on the basis of solutions of plane stress problem of theory of elasticity which are presented using expression of the external load and stresses in the form of infinite trigonometric Fourier series (Alexandrov and Potapov, 1990).

Determination of stress-strain state of a plate is based on entering stress function φ (x, y) and approximating of external loading by periodic functions both for a masonry unit and for a mortar bed joints (Fig. 2):

$$\begin{aligned} &\phi(x,y)=\sum_{k=1}^{\infty}\phi_k(x,y);\\ &\phi_k(x,y)=F_k(y)\sin(\alpha_k x);\alpha_k=k\pi/l,\\ &q=\sum_{k=1}^{\infty}q_k^m\sin(\alpha_k x), t=t_{0k}+\sum_{k=1}^{\infty}t_k^m\cos(\alpha_k x)\end{aligned} \tag{5}$$

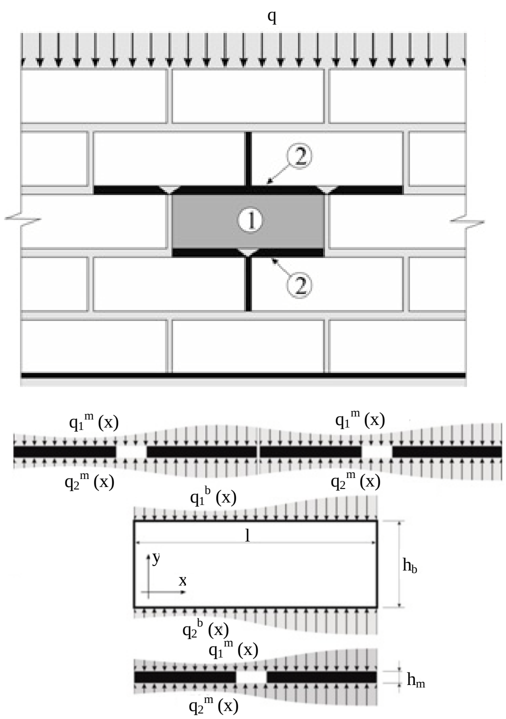

Fig. 1: The design model of single unit of masonry with generalized technological imperfections of mortar bed joints (plane stress problem; mortar loads signed with "m" index, brick loads with "b" index): 1: masonry unit, 2: physically uneven mortar bed joints (X-axis loads are omitted)

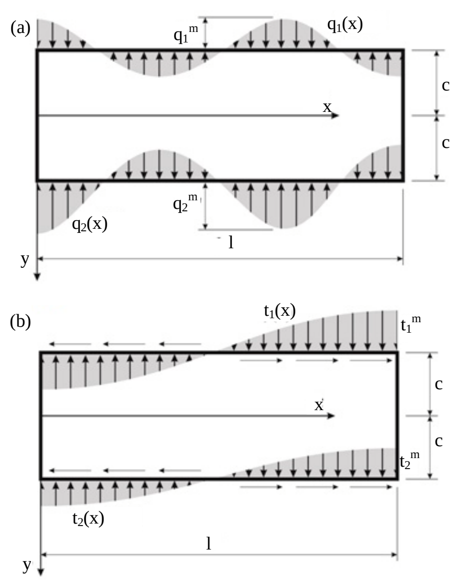

Fig. 2: External loads of the plate, presented in trigonometric functions; a) axial load and b) lateral loads

where, $F_k(y)$ function for y-axis corresponds to kth harmonic. The accurate solution of the task is defined by the number of accepted harmonics (terms of series) (Eq. 5).

The substitution of φ (x, y) from Eq. 5 in the Maxwell-Erie's biharmonic Eq. 6:

$$\nabla^2\left(\frac{\partial^2\phi}{\partial x^2}+\frac{\partial^2\phi}{\partial y^2}\right)=0$$

results in:

$$F_k(y)\alpha_k^4\cos(\alpha_k x)-2\alpha_k^2F_k^{//}(y)\cos(\alpha_k x)+ F_k^{IV}(y)\cos(\alpha_k x)=0 \tag{6}$$

Or:

$$F_k^{IV}(y)-2\alpha_k^2F_k^{//}(y)+F_k(y)=0 \tag{7}$$

The common solution of Eq. 7 is (Alexandrov and Potapov, 1990):

$$F_k(y)=A_ke^{a_ky}+B_kye^{a_ky}+C_ke^{-a_ky}+D_kye^{-a_ky} \tag{8}$$

Or:

$$F_k(y)=C_{1k}ch(\alpha_k y)+C_{2k}\alpha_k y\,sh(\alpha_k y)+ C_{3k}sh(\alpha_k y)+C_{4k}\alpha_k y\,ch(\alpha_k y)=F_k^{sym}(y)+F_k^{asym}(y) \tag{9}$$

Where the first and second segments in the right part represent the amplitude of stress function for symmetric and asymmetric parts of the solution of the task, respectively. On condition of omitting the volume loads:

$$\sigma_x=\frac{\partial^2\phi}{\partial y^2}=\sum_{k=1}^{\infty}\sigma_x^m\sin(\alpha_k x)$$
$$\sigma_x=\frac{\partial^2\phi}{\partial x^2}=\sum_{k=1}^{\infty}s_y^m\sin(\alpha_k x) \tag{10}$$
$$\tau=\frac{\partial^2\phi}{\partial x\partial y}=\sum_{k=1}^{\infty}\tau^m\cos(\alpha_k x)$$

where for the symmetric part of solution:

$$\sigma_x^m=[C_{1k}ch(\alpha_k y)+C_{2k}(2ch(\alpha_k y)+a_k y\,sh(\alpha_k y))]$$
$$\sigma_y^m=-\alpha_k^2[C_{1k}ch(\alpha_k y)+C_{2k}a_k y\,sh(\alpha_k y)]$$
$$\tau^m=\alpha_k^2[C_{1k}sh(\alpha_k y)+C_{2k}(sh(\alpha_k y)+a_k y\,ch(\alpha_k y))] \tag{11}$$

For the asymmetric part (in which "sh" replaces "ch", and C_{3k}-C_{1k} and Ha C_{4k}-C_{2k}):

$$\sigma_x^m=\alpha_k^2[C_{3k}sh(\alpha_k y)+C_{4k}\ (2sh(\alpha_k y)+\alpha_k y\,ch(\alpha_k y))]$$
$$\sigma_y^m=-\alpha_k^2[C_{3k}sh(\alpha_k y)+C_{4k}\alpha_k y\,ch(\alpha_k y)]$$
$$\tau^m=\alpha_k^2[C_{3k}ch(\alpha_k y)+C_{4k}(ch(\alpha_k y)+\alpha_k y\,sh(\alpha_k y))] \tag{12}$$

The deformations of the plates boundaries where $y = \pm c$ are determined by Eq. 11 and 12 in the generalized Hooke's law:

$$\begin{aligned} \varepsilon_x &= \frac{\partial u}{\partial x} = \frac{\sigma_x^m - \nu\sigma_y^m}{E}\sin(\alpha_k x) \\ \varepsilon_y &= \frac{\partial v}{\partial y} = \frac{\sigma_y^m - \nu\sigma_x^m}{E}\sin(\alpha_k x) \end{aligned} \tag{13}$$

The integration of Eq. 13 gives for kth harmonic:

$$\begin{aligned} u &= \int \varepsilon_x dx = u_k^m \cos(\alpha_k x) \\ v &= \int \varepsilon_y dy = v_k^m \sin(\alpha_k x) \end{aligned} \tag{14}$$

where for symmetric part of solution:

$$\begin{aligned} u_k^m &= -\alpha_k/E\,[C_{1k}(1+\nu)\,ch(\alpha_k y) + \\ &\quad C_{2k}\{2ch(\alpha_k y) + (1+\nu)\,\alpha_k y\,sh(\alpha_k y)\}] \\ v_k^m &= \alpha_k/E\,[C_{1k}(1+\nu)sh(\alpha_k y) + \\ &\quad C_{2k}\{(1+\nu)\,a_k y\,ch(\alpha_k y) - (1-\nu)\,sh(\alpha_k y)\}] \end{aligned} \tag{15}$$

for the asymmetric part:

$$\begin{aligned} u_k^m &= -\alpha_k/E\,[C_{3k}(1+\nu)sh(\alpha_k y) + \\ &\quad C_{4k}\{2sh(\alpha_k y) + (1+\nu)\,\alpha_k y\,ch(\alpha_k y)\}] \\ v_k^m &= -\alpha_k/E[C_{3k}\,(1+\nu)\,ch(\alpha_k y) + \\ &\quad C_{4k}\{(1+\nu)\,\alpha_k y\,sh(\alpha_k y) - (1-n)\,ch(\alpha_k y)\}] \end{aligned} \tag{16}$$

The external loads presented in the form of trigonometric Fourier series (Eq. 5) have the amplitude values of q^m и t^m, determined on plate boundaries if $y = \pm c$:

$$\begin{aligned} q_k^m &= \frac{2}{\alpha_k c}\int_0^l q_k(x)\sin(\alpha_k x)\,dx \\ t_k^m &= \frac{2}{\alpha_k c}\int_0^l t_k(x)\cos(\alpha_k x)\,dx \end{aligned} \tag{17}$$

Equation 10-12 provide the possibility to define all components of stress tensor of the plane stress problem, (Eq. 14-16) linear displacements under the indicated by equations of Eq. 5 and 17 external loads for kth harmonic of series (Eq. 5). Constants C_1-C_4 are determined from the boundary conditions of the plane problem.

The physical interaction of the brick and mortar can be determined by considering two interrelated plates-unit and bed joint under boundary conditions (external loads) as seen in scheme in Fig. 2. Equation 10-16 for each analyzed plates contain 8 unknown constants C_1^b-C_4^b and C_1^m-C_4^m for stress function where the index "b" belongs to the plate of unit whilst "m" belongs to the plate of mortar bed joint. Similarly, for the external loads there are 8 unknown constants $q_{1b}^m, q_{2b}^m, q_{1m}^m, q_{2m}^m$ и $t_{1b}^m, t_{2b}^m, t_{1m}^m, t_{2m}^m$.

To determine their values we will define the boundary conditions on the horizontal boundaries of plates. The unit plate is between two joint plates, the top and bottom joint plates are displaced across from each other at a size of half of their length (0.5l). From this, for the bottom joint plate, necessary to define the trigonometric functions from x which are used in Eq. 5, 10 and 14 through the basis of x-0.5l. The corresponding transformations for trigonometric functions have the following:

$$\cos\left(\alpha_k\left(x - \frac{1}{2}\right)\right) = \begin{cases} (-1)^{\frac{k}{2}}\cos(\alpha_k x) & k-\text{even} \\ (-1)^{\frac{k-1}{2}}\sin(\alpha_k x) & k-\text{odd} \end{cases}$$

$$\sin\left(\alpha_k\left(x - \frac{1}{2}\right)\right) = \begin{cases} (-1)^{\frac{k}{2}}\sin(\alpha_k x) & k-\text{even} \\ (-1)^{\frac{k+1}{2}}\cos(\alpha_k x) & k-\text{odd} \end{cases}$$

In this regard, the amplitude values of stresses and deformations in the equations for the bottom unit plate ($y = c$) consist of two separate equations for each of the given representations.

The assumption of the equality of the displacements of adjoining sides of plates allows to equate the amplitude values of the vertical and horizontal displacements determined by the equations of Eq. 15 and 16. For the bottom edge of the joint plate and the top edge of the unit plate:

$$v_m^m(c) = v_b^m(-c);\ u_m^m(c) = u_b^m(-c) \tag{18}$$

For the top edge of the joint plate and the bottom edge of the unit plate:

$$v_b^m(c) = v_m^m(-c);\ u_b^m(c) = u_m^m(-c) \tag{19}$$

Similarly, it is possible to equate the amplitude values of the normal and shear stresses which is set by the Eq. 11 and 12. For the bottom edge of the joint plate and the top edge of the unit plate:

$$\begin{aligned} \sigma_{xm}^m(c) &= \sigma_{xb}^m(-c) = 0 \\ \sigma_{ym}^m(c) &= \sigma_{yb}^m(-c) = q_{2m}^m(c) = q_{1b}^m(-c) \\ \tau_m^m(c) &= \tau_b^m(-c) = t_{2m}^m(c) = t_{1b}^m(-c) \end{aligned} \tag{20}$$

For the top edge of the joint plate and the bottom edge of the unit plate:

$$\begin{aligned}
&\sigma_{xb}^{m}(c)=\sigma_{xm}^{m}(-c)=0\\
&\sigma_{yb}^{m}(c)=\sigma_{ym}^{m}(-c)=q_{1m}^{m}(-c)=q_{2b}^{m}(c) \qquad (21)\\
&\tau_{b}^{m}(c)=\tau_{m}^{m}(-c)=t_{1m}^{m}(-c)=t_{2b}^{m}(c)
\end{aligned}$$

Having the boundary conditions of Eq. 18-21 and setting the vertical loads equal to external vertical loads (at the first estimation) and passive pressure of bed joints, through the method of iterations it is possible to receive constants of C_1-C_4 for both plates and at the same time the analytical notion of stress-strained state of these elements.

The determination of values of passive pressure of the bed joints, distributed across the bed face $q_2(x)$ is possible using the theory of calculation of short beams on an elastic bed of the soil layer of final depth which is justified for majority of the mortars having ν_0 = 0.2-0.5 and E_0 = 500-8000 MPa (changing in these ranges with load growth up to the layer destruction), silicate and ceramic bricks and stones with constant ν_1 = 0.1-0.15 and E_1= 10000-15000 MPa. In this case passive pressure r_i depends on modulus of subgrade reaction c_i and on value of the foundation (mortar bed joint) yielding w_i:

$$r_i = c_i w_i \qquad (22)$$

Prof. M.I. Gorbunov-Posadov (Russia, 1908-1991) (Gorbunov-Posadov *et al*., 1984) offered the formula for determination of Winkler's coefficient c, considering the geometrical and deformative characteristics of the elastic bed, presented in the case of the task in Eq. 23:

$$c=\frac{(1-\nu_m)E_m}{(1+\nu_m)(1-2\nu_m)h_m} \qquad (23)$$

where, E_m, ν_m, h_m are Young's modulus, Poisson's ratio of mortar, thickness of the foundation (mortar bed joint), respectively.

By Gorbunov-Posadov *et al*. (1984), the formula of (Eq. 23) is experimentally well confirmed at h_m (10-20 mm) <0.5 b (30 mm) where b is semi-width of the masonry unit.

In the horizontal bed joints, surrounding each unit areas, the geometrical (cavities, cracks) and physical (increased deformability) unevenness of mortar bed joints present. In most cases, the heterogeneities of increased by 1.5-2 times deformability are localized across the bed face within l_2 length in the zone of adjoining of horizontal and vertical mortar joints. Taking into account the arrangement of the units into masonry this allows to offer for the Young's modulus of mortar E_m the distribution function which is indicated in Fig. 3. The zones of increased deformability of mortar joints are formed in the central part of the bottom bed face and at the edges of the upper bed face. Referring to analogical heterogeneity of the soil base and representing its character by trigonometric functions, it is possible to present distribution of the uneven Young's modulus of mortar bed joint on the lower edge of the unit as follows (Fig. 3a):

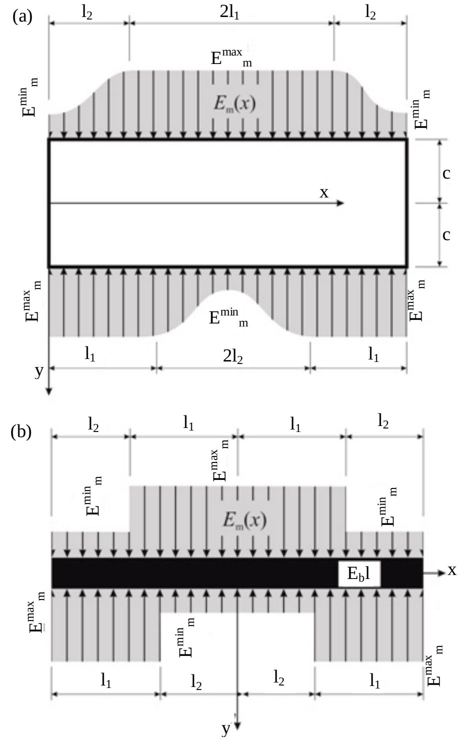

Fig. 3: Design models of distribution of Young's modulus of mortar bed across the; a) bed face and b) theoretical simplified

$$E_m(x)=\begin{cases}E_m^{min}+\left(E_m^{max}-E_m^{min}\right)\sin\left(\dfrac{\pi x}{l_2}\right),\ \text{at } x\in\left(0;\dfrac{l_2}{2}\right);\\[2mm] E_m^{max},\ \text{at } x\in\left(\dfrac{l_2}{2};\ 2l_1+\dfrac{l_2}{2}\right);\\[2mm] E_m^{min}+\left(E_m^{max}-E_m^{min}\right)\sin\left(\dfrac{\pi\left(x-2l_1-l_2\right)}{l_2}\right),\\[2mm] \text{at } x\in\left(2l_1+\dfrac{l_2}{2};\ 2l_1+l_2\right]\end{cases} \qquad (24)$$

Equation 24 reflects in Eq. 13 and 14 and also in boundary conditions of Eq. 18 and 19, entering the functions of the stress-strained state of the unit and bed joint plates.

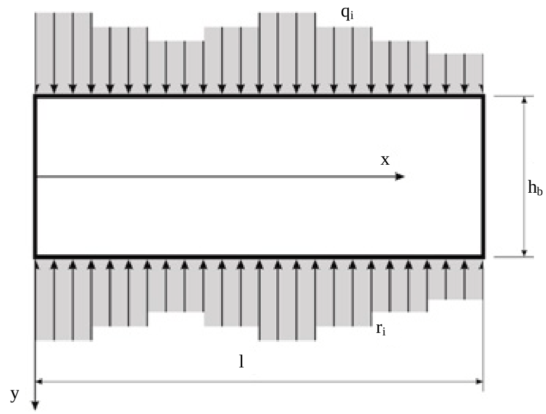

Fig. 4: Model of a masonry unit as a short rigid beam on an finite thickness elastic layer; h_b: unit height, q_i: uneven loads, r_i: passive pressure of the uniform bed

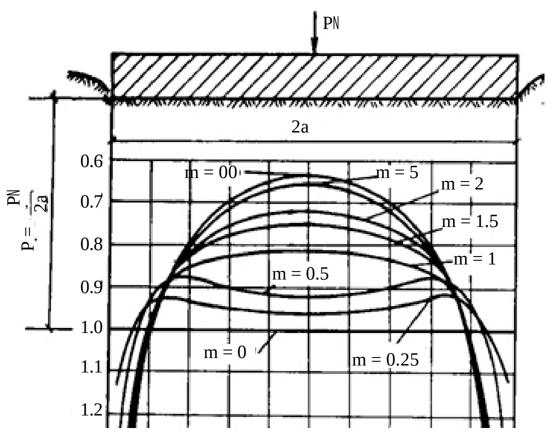

Fig. 5: Dimensionless profile of passive pressure under short rigid beam on an finite thickness elastic layer [12]: m = H/a, where H: thickness of a bed layer, a: beam semi-length

The existing in soil mechanics problem of infinite values of passive pressure at the edges of a beam on an elastic bed in not essential at definition of the stress-strained state of the beam on a finite thickness elastic layer (Fig. 4). As K.E. Egorov (Russia, 1911-2001) (Egorov, 1961) showed for a rigid beam unevenness of distribution of passive pressure of the uniform bed even in case of extremely uneven external loads (concentrated force) is significantly eliminated at reduction of thickness of a bed layer. For sizes m = H/a (H; thickness of a bed layer, a; beam semi-length), being in limits 0.1-0.15 (for a standard brick on mortar bed joint of 10-20 mm), the volumes of the passive pressure of the uniform mortar bed could be considered as a finite and rather evenly distributed on all length of the unit bed face (Fig. 5).

Numerical simulation allows to draw a conclusion that deformations of general purpose mortar does not change significantly if as a distribution of Eq. 24 offer the simplified piecewise constant function presented in Fig. 3b and to replace a unit plate with a short beam of E_mI rigidity with generalization of deformations of a plate by the equation of the elastic curve of a beam.

In these conditions it is possible to identify the bed yielding w, required for Eq. 22 with deflections of the beam y and to receive the equation of the elastic curve of a beam by integration of the following differential equation (Sobolev and Gagin, 1989):

$$\frac{d^4y(\xi)}{d\xi^4}+4k^4y(\xi)=\bar{q} \tag{25}$$

where, $\xi = x/L$; $4k^4 = cL^4/E_bI$; $\bar{q} = qL^4/E_bI$; $L = l_1+l_2$ (Fig. 3b and c) by Eq. 23. The general solution to the differential Eq. 25 is as follows:

$$\begin{aligned}y(\xi)=\;&C_1ch(k\xi)+C_2\left(ch(k\xi)\sin(k\xi)+sh(k\xi)\cos(k\xi)\right)+\\&C_3sh(k\xi)\sin(k\xi)+C_4\left(ch(k\xi)\sin(k\xi)+sh(k\xi)\cos(k\xi)\right)\end{aligned} \tag{26}$$

As uneven Young's modulus E_m is set by simplified piecewise constant function (Fig. 3b) that having the beam splitted into five zones ($-L\le x<-l_1$; $-l_1\le x<-l_2$; $-l_2\le x\le l_2$; $l_2<x\le l_1$; $l_1<x\le L$) of constant Young's modulus E_{mi} on bottom and top sides of a bed face, we will receive the decision of (Eq. 26) as follows. For the elastic curve of a beam y:

$$\begin{aligned}y(\xi)=\sum_{i=1}^{6}\Big[&C_ich(k_i\xi)+C_{i+1}\left(ch(k_i\xi)\sin(k_i\xi)+\right.\\&\left.sh(k_i\xi)\cos(k_i\xi)\right)+C_{i+2}sh(k_i\xi)\sin(k_i\xi)+\\&C_{i+3}\left(ch(k_i\xi)\sin(k_i\xi)+sh(k_i\xi)\cos(k_i\xi)\right)\Big]\end{aligned} \tag{27}$$

For small slopes of the elastic curve [fi] (tg [fi] equal to [fi]), the bending moments M and shear forces Q in a beam:

$$\begin{aligned}&\phi(\xi)=\frac{y(\xi)}{L};\ M(\xi)=-E_bI\frac{y^{//}(\xi)}{L^2};\\&Q(\xi)=-E_bI\frac{y^{///}(\xi)}{L^3}\end{aligned} \tag{28}$$

The Eq. 27 contains 24 constants of integration defined from following boundary conditions: Condition of continuity of deformations and stresses on borders of zones:

$$
\begin{aligned}
&y(-\xi_2-0)=y(-\xi_2+0);\ y(\xi_2-0)=y(\xi_2+0);\\
&y(0-0)=y(0+0);\\
&\varphi(-x_1-0)=\varphi(-x_1+0);\ \varphi(x_1-0)=\varphi(x_1+0)\\
&\varphi(-x_2-0)=\varphi(-x_2+0);\ \varphi(x_2-0)=\varphi(x_2+0)\\
&\varphi(0-0)=\varphi(0+0);\\
&M(-\xi_1-0)=M(-\xi_1+0);\ M(\xi_1-0)=M(\xi_1+0);\\
&M(-\xi_2-0)=M(-\xi_2+0);\ M(\xi_2-0)=M(\xi_2+0);\\
&M(0-0)=M(0+0);\\
&Q(-\xi_1-0)=Q(-\xi_1+0);\ Q(\xi_1-0)=Q(\xi_1+0);\\
&Q(-\xi_2-0)=Q(-\xi_2+0);\ Q(\xi_2-0)=Q(\xi_2+0);\\
&Q(0-0)=Q(0+0)
\end{aligned}
\qquad (29)
$$

where, $\xi_i = l_i/L$; condition of absence of internal stresses at beam edges:

$$M(-1)=M(1)=0;\ Q(-1)=Q(1)=0 \qquad (30)$$

Thus, the volume of passive pressure q_2 (x) of masonry bed of h_m thickness, with characteristics E_m and ν_m, after action of load q_i by a rigid unit beam with characteristics h_b, E_b and ν_b can be found by Eq. 22-24. As the minimum and the maximum of $q_1(x)$ are equal to the same values of passive pressure $q_2(x)$, the task becomes recurrent and is decided by the method of consecutive approximations with acceptable convergence after 3-4 iterations.

The strength criterion which is reliable and reasonably describes a limit states of the majority of masonry materials is the generalized strength criterion by prof. G.S. Pisarenko (Russia, 1910-2001) (Pisarenko and Lebedev, 1976) which is the piecewise continuous function containing the principal stresses of a plane stress problem (σ_1 and σ_3) and two constants of a material defined experimentally (strength on compression σ_c, and on tension σ_t):

$$
\begin{aligned}
&\sigma_1-\chi\sigma_3\le\sigma_t\\
&\chi^2(\sigma_1-\sigma_3)+\sigma_p(1-\chi^2)\,\sigma_1\le\sigma_t^2\\
&\chi^2(\sigma_1-\sigma_3)+(1-\chi^2)\,\sigma_1^2\le\sigma_t^2
\end{aligned}
\qquad (31)
$$

where, $\chi = \sigma_t/\sigma_c$.

RESULTS AND DISCUSSION

Considering the above, we will formulate the general algorithm to determine the parameters of the stress-strain state of an average masonry unit: set the strength and deformative characteristics of masonry materials: compression (R_c) and bending tension (R_t) strength, Young's modulus, Poisson's ratio by standard tests. Set the unevenness of deformability of the mortar bed joints in the form of the piecewise constant function $E_m(x)$ (Fig. 3).

Make the solution of nonlinear plane problem for unit plate consistently defining the stress-strain state parameters on each of loading stages, setting the corresponding amplitude values of q^m and t^m for each of withheld harmonics of a trigonometric Fourier series (Eq. 5).

Check for limit state conditions in unit plate by Pisarenko criterion (Eq. 31). Given of the manufacturing characteristics of the unit, the moment of achievement by the equivalent stresses σ_{red} of the volumes R_t^{min} could be considered as the moment of beginning of the cracking and achievement of the volumes, R_t^{max} the moment of finishing of the cracking, splitting of a masonry on separate substructures by main cracks.

While masonry remains undestroyed (main cracks at stage No. 4 split the masonry on separate columns of cross-sections with width and height of semi-unit), repeat the calculation for stages No. 3, 4 for the next iterative step with accordingly changing of volumes of external loads, passive pressure and the deformability of masonry materials. Formation of the main cracks corresponding to the splitting of a masonry on separate columns changes the design model of plane task by shortening of plate length with l on 0.5l (Fig. 1) with acceptance of uniform deformability of the truncated mortar bed joint. For the changed design model repeat the calculation for stages No. 3, 4. The moment for half-length plates of achievement by the equivalent stresses σ_{red} of the volumes R_t^{max} is considered as the moment of final fracture of the masonry.

CONCLUSION

The presented algorithm could be realized as a software application, in spreadsheet applications (MS Excel, etc.) in computer algebra systems (Maple, etc.). Programming allows to automate the stated calculation procedures with the organization of the phased solution of the plane stress task for any range of arguments by means of two iterative cycles the external, checking of convergence of the solution and internal, setting the current recurrent values of passive pressure of mortar bed and external loads on each iterative step. Computer calculation of the presented algorithms allows the carry out the both general and differentiated quantitative analysis of influence of various technological factors on the stress-strain state of masonry of an arbitrary materials, geometry, technology and quality of production.

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