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# A Novel Project Scheduling Method Based on Fully Fuzzy Linear Programming

Seyyed Mohammad Tabatabaei Mehrizi
Islamic Azad University, Science and Research Branch, Tehran, Iran

**Abstract:** Considering fuzzy duration of activities, this study presents a novel project scheduling approach following Trapezoidal Fuzzy Numbers (TrFN). The purpose of proposed approach is applying a fully fuzzy linear programming with a method to find its optimal solution. In this study, two LP Models are used in order to calculate earliest and latest events time in project scheduling problem and then another method is used to solve proposed fully fuzzy linear programming problem. Proposed method presents a simple integrative LP Model and also results in no negative or in feasible solution in latest time calculations. Finally, to better illustrate the presented method, a numerical example is solved and the results are discussed and compared with existing methods.

**Key words:** Project scheduling, Fully Fuzzy Linear Programming (FFLP), Trapezoidal Fuzzy Number (TrFN), linear, LP Models

## INTRODUCTION

Scheduling is deemed to be one of the most fundamental and essential bases of the project management science. There are several methods for project scheduling such as CPM, PERT and GERT. Since, too many draw backs are involved in methods estimating the duration of activities these methods lack the capability of modeling practical projects. In order to solve these problems, a number of techniques like fuzzy logic, Genetic Algorithm (GA) and Artificial Neural Network (ANN) can be considered. A fundamental approach to solve these problems is applying fuzzy sets. Introducing the fuzzy set theory by Zadeh (1965) opened promising new horizons to different scientific areas such as project scheduling. Fuzzy theory with presuming imprecision in decision parameters and utilizing mental models of expertsis an approach to adapt scheduling models into reality. To this end, several methods have been developed during the last three decades. The first method called FPERT was proposed by Chanas and Kamburowski (1981). They presented the project completion time in the form of a fuzzy set in the time space. Gazdik (1983) developed a fuzzy network of an a priori unknown project to estimate the activity duration and used fuzzy algebraic operators to calculate the duration of the project and its critical path. This work is called FNET. An extension of FNET was proposed by Nasution (1994) and Lorterapong and Moselhi (1996). Following on this, McCahon (1993), Chang *et al.* (1995) and Lin and Yao presented three methodologies to calculate the fuzzy completion project time. Other researchers such as Yao and Lin (2000), Chanas and Zielinski (2001) and Chen (2006) using fuzzy numbers, presented other methods to obtain fuzzy critical paths and critical activities andactivity delay.

Herroelen and Leus (2005) reviewed the fundamental approaches for scheduling under uncertainty: reactive scheduling, stochastic project scheduling, fuzzy project scheduling, robust (proactive) scheduling and sensitivity analysis. Masmoudi and Hait have presented a fuzzy model for project scheduling. Based on this modeling, two project scheduling techniques, resource constrained scheduling and resource leveling are considered and generalized to handle fuzzy parameters. A Greedy algorithm and a Genetic algorithm are provided to solve these two problems and are applied to civil helicopter maintenance domain.

Previous research on network scheduling using fuzzy theory provide methods for scheduling projects. These methods, however do not support backward pass calculations in direct manner similar to that used in the forward pass. This is mainly due to the fact that fuzzy subtraction is not proportionate to the inverse of fuzzy addition. Therefore, these methods are incapable to calculate project characteristics such as the latest times. In this study, a new method is introduced for project scheduling in fuzzy environment. This method is developed based on a number of assumptions and definitions in the fuzzy set and project scheduling. In the fuzzy project network considered in this study, we assume that the duration of activities are Trapezoidal Fuzzy Numbers (TrFN). The project characteristics such as fuzzy earliest and fuzzy latest times of events and fuzzy project completion time are calculated as TrFN by solving two fully fuzzy linear programming.

The proposed approach does not use the fuzzy subtraction operator in its equations. Therefore, the proposed approach at least yields two important advantages. The first advantage of this method is that no negative and in feasible solution generated during calculation of latest times. The second benefit is obtained the optimal solution with solving a simple LP problem.

## PRELIMINARIES

This study introduces some basic definitions of fuzzy numbers and then briefly reviews two LP Model proposed by Zareei *et al*. (2011) and Kumar *et al*. (2011).

**Definitions:** Zadeh (1965) introduced the fuzzy set theory to deal with the uncertainty due to imprecision and vagueness. A major contribution of fuzzy set theory is its capability of representing vague data. The theory also allows mathematical operators and programming apply to the fuzzy domain. In the following, some basic important definitions of fuzzy sets are given.

**Definition 1:** A fuzzy set $\tilde{A}$ in a universe of discourse X is characterized by a membership function $\mu_{\tilde{A}}(x)$ which associates with each element x in X a real number in the interval [0, 1]. The function value $\mu_{\tilde{A}}(x)$ is termed the grade of membership of x in $\tilde{A}$.

**Definition 2:** A trapezoidal fuzzy number $\tilde{A}$ can be defined by (a, b, c, d) and the membership function $\mu_{\tilde{A}}(x)$ is defined:

$$\mu_{\tilde{A}}(x) = \begin{cases} 0 & x \le a \\ \frac{x-a}{b-a} & a < x \le b \\ 1 & b < x \le c \\ \frac{x-d}{c-d} & c < x \le d \\ 0 & x > d \end{cases} \qquad (1)$$

**Definition 3:** Let, $\tilde{A}$ and $\tilde{B}$ be two trapezoidal fuzzy numbers parameterized by $\tilde{A}$ = ($a_1$, $b_1$, $c_1$, $d_1$) and $\tilde{B}$ = ($a_2$, $b_2$, $c_2$, $d_2$), respectively then the operational laws of these two fuzzy numbers are as follows (Zimmerman, 1991):

$$\begin{aligned}\tilde{A} \oplus \tilde{B} &= (a_1, b_1, c_1, d_1) \oplus (a_2, b_2, c_2, d_2) \\ &= (a_1 + a_2, b_1 + b_2, c_1 + c_2, d_1 + d_2)\end{aligned} \qquad (2)$$

$$\begin{aligned}\tilde{A} \odot \tilde{B} &= (a_1, b_1, c_1, d_1) \odot (a_2, b_2, c_2, d_2) \\ &= (a_1 - a_2, b_1 - b_2, c_1 - c_2, d_1 - d_2)\end{aligned} \qquad (3)$$

$$\begin{aligned}\tilde{A} \otimes \tilde{B} &= (a_1, b_1, c_1, d_1) \otimes (a_2, b_2, c_2, d_2) \\ &= (a_1 \times a_2, b_1 \times b_2, c_1 \times c_2, d_1 \times d_2)\end{aligned} \qquad (4)$$

$$\begin{aligned}\tilde{A} \oslash \tilde{B} &= (a_1, b_1, c_1, d_1) \oslash (a_2, b_2, c_2, d_2) \\ &= (a_1/d_2, b_1/c_2, c_1/b_2, d_1/a_2)\end{aligned} \qquad (5)$$

**Definition 4:** The α-cut of a fuzzy set $\tilde{A}$ is a crisp subset of X and is denoted by Lai and Hwang (1992):

$$\left[\tilde{A}\right]_\alpha = \left\{x \middle| \mu_{\tilde{A}}(x) \ge \alpha\right\} \qquad (6)$$

**Definition 5:** A ranking function is a function $\Re$: f(R)→R where f(R) is a set of fuzzy numbers defined on set of real numbers which maps each fuzzy number into the real line where a natural order exists. Let $\tilde{A}$ = (a, b, c, d) be a trapezoidal fuzzy number then:

$$R(A) = \frac{a+b+c+d}{4} \qquad (7)$$

**CPM under linear programming formulation:** Zareei *et al*. (2011) have been proposed two LP Models for calculation of earliest and latest time of events. In this study, these models are reviewed. Consider a project network S = {V, A, T} in which V is a finite set of events and $A \subset V \times V$ is a set of arcs with crisp activity durations and by means of function T, T: $A \to R^+$, the activity durations in the network are determined. The $T_{ij}$ is denoted as the time period of activity (i, j)∈A. LP Model for calculation of earliest events time ($E_i$) in a project network with n nodes is formulated as follows:

$$F = \min(E_1 + E_2 + \ldots + E_n) \qquad (8)$$

s.t: $E_j \ge E_i + T_{ij}$, $E_i$, $E_j$ unrestricted in sign ∀ (i, j)∈A. Where the objective is finding the lowest quantity of events time. The constraints show the preceding relationship between events and denote that in activity ij with tail event i, head event j and activity duration $T_{ij}$, occurrence time of event j is at least $E_i + T_{ij}$ in other words event j occurs at least $T_{ij}$

time after occurrence time of event i. Backward pass is done after forward pass by assignation of earliest event time of node $n(E_n)$ as latest event time of node $n(L_n)$. This equation is added as a constraint to LP Model for calculation of latest times and the objective changes to find the maximum quantity of events time:

$$G = \max(L_1 + L_2 + ... + L_n) \quad (9)$$

s.t: $L_j \geq L_i + T_{ij}$, $E_n = L_n$, $L_i$, $L_j$ = unrestricted in sign $\forall$ (i, j)$\in$A. The activity duration in project network usually has ambiguity, so precise estimation of them is impossible. To consider the ambiguity, fuzzy numbers are applied as activities duration. In this study, we denote activities duration as positive TrFNs. We denote fuzzy activity duration of ij as $T_{ij}$ where we have:

$$T_{ij} = (t_{ij}^1, t_{ij}^2, t_{ij}^3, t_{ij}^4)$$

using fuzzy numbers as activity duration, Eq. 9 will be rearranged as follows:

$$\tilde{F} = \min(\tilde{E}_1 + \tilde{E}_2 + ... + \tilde{E}_n) \quad (10)$$

s.t: $\tilde{E}_j \geq \tilde{E}_i + \tilde{T}_{ij}$, $\forall$ j = 1, 2, ..., n. Similar to Eq. 8, the objective is finding the lowest value of earliest event times. The difference to Eq. 8 is that the activities duration is fuzzy. Also using fuzzy numbers as activity duration, Eq. 6 will be rearranged as follows:

$$\tilde{G} = \max(\tilde{L}_1 + \tilde{L}_2 + ... + \tilde{L}_{(n-1)}) \quad (11)$$

s.t: $\tilde{L}_j \geq \tilde{L}_i + \tilde{T}_{ij}$, $\tilde{E}_n = \tilde{L}_n$, $\tilde{L}_1 = \tilde{0}$. Note that $\tilde{0}$ is a crisp zero ($\tilde{0}$ = (0, 0, 0)$_{LR}$). Zareei *et al.* (2011) have been proposed a procedure to find the membership function of earliest and latest times of events by calculating lower and upper bounds of earliest and latest times considering different $\tilde{\alpha}$ cut of fuzzy duration.

**Solving fully fuzzy linear programming problems:** In this study, we describe the proposed method by Kumar *et al.* (2011) for solving Fully Fuzzy Linear Programming problems (FFLP):

$$\text{Maximize (or minimize)} \left(\tilde{C}^T \otimes \tilde{X}\right) \quad (12)$$

Subject to $\tilde{A} \otimes \tilde{X} = \tilde{b}$, $\tilde{X}$ is a non-negative fuzzy numbers where:

$$\tilde{C}^T = \left[\tilde{c}_j\right]_{1\times n}, \tilde{X} = \left[\tilde{x}_j\right]_{n\times 1}, \tilde{A} = \left[\tilde{a}_{ij}\right]_{m\times n},$$
$$\tilde{B} = \left[\tilde{b}_i\right]_{m\times 1}, \tilde{a}_{ij}, \tilde{c}_j, \tilde{x}_j, \tilde{b}_i \in F(R)$$

The steps of method are as follows:

**Step 1:** Substituting, the above:

$$\tilde{C}^T = \left[\tilde{c}_j\right]_{1\times n}, \tilde{X} = \left[\tilde{x}_j\right]_{n\times 1}, \tilde{A} = \left[\tilde{a}_{ij}\right]_{m\times n}, \tilde{b} = \left[\tilde{b}_i\right]_{m\times 1}$$

The above FFLP problem may be written as:

$$\text{Maximize (or minimize)} \left(\sum_{j=1}^{n} \tilde{c}_j \otimes \tilde{x}_j\right) \quad (13)$$

Subject to:

$$\sum_{j=1}^{n} \tilde{a}_{ij} \otimes \tilde{x}_j = \tilde{b}_i, \forall\ i = 1, 2, ..., m$$

$\tilde{x}_{ij}$ is a non-negative trapezoidal fuzzy number.

**Step 2:** If all parameter $\tilde{c}_j$, $\tilde{x}_j$, $\tilde{a}_{ij}$ and $\tilde{b}_i$ are represented by trapezoidal fuzzy numbers ($p_j$, $q_j$, $r_j$, $t_j$), ($x_j$, $y_j$, $z_j$, $w_j$), ($a_{ij}$, $b_{ij}$, $c_{ij}$, $d_{ij}$) and ($b_i$, $g_i$, $h_i$, $f_i$), respectively then he FFLP problem, obtained in step 1 may be written as:

$$\text{Maximize (or minimize)} \sum_{j=1}^{n} (p_j, q_j, r_j, t_j) \otimes (x_j, y_j, z_j, w_j) \quad (14)$$

Subject to:

$$\sum_{j=1}^{n} (a_{ij}, b_{ij}, c_{ij}, d_{ij})\,(x_j, y_j, z_j, w_j) = (b_i, g_i, h_i, f_i)$$
$$\forall i = 1, 2, ..., m$$

($x_j$, $y_j$, $z_j$, $w_j$) is a non-negative trapezoidal fuzzy number.

**Step 3:** Assuming ($a_{ij}$, $b_{ij}$, $c_{ij}$, $d_{ij}$)$\otimes$($x_j$, $y_j$, $z_j$, $w_j$) = ($m_{ij}$, $n_{ij}$, $o_{ij}$, $p_{ij}$) the FFLP problem, obtained in step 2 may be written as:

$$\text{Maximize (or minimize)}\quad \Re\sum_{j=1}^{n} (p_j, q_j, r_j, t_j) \otimes (x_j, y_j, z_j, w_j) \quad (15)$$

Subject to:

$$\sum_{j=1}^{n} (m_{ij}, n_{ij}, o_{ij}, p_{ij}) = (b_i, g_i, h_i, f_i)$$
$$\forall i = 1, 2, ..., m$$

($x_j$, $y_j$, $z_j$, $w_j$) is a non-negative trapezoidal fuzzy number.

**Step 4:** Using arithmetic operations defined in study the fuzzy linear programming problem, obtained in step 3 is converted into the following CLP problem:

Maximize (or minimize)

$$\Re\sum_{j=1}^{n}(p_j, q_j, r_j, t_j)\otimes(x_j, y_j, z_j, w_j) \quad (16)$$

$$\sum_{j=1}^{n} m_{ij} = b_i, \forall\, i = 1, 2, ..., m$$

$$\sum_{j=1}^{n} n_{ij} = g_i, \forall\, i = 1, 2, ..., m$$

$$\sum_{j=1}^{n} o_{ij} = h_i, \forall\, i = 1, 2, ..., m$$

$$\sum_{j=1}^{n} p_{ij} = f_i, \forall\, i = 1, 2, ..., m$$

$$y_i - x_j \geq 0,\ z_j - y_j \geq 0,\ w_j - z_j \geq 0\ \ \forall\, i = 1, 2, ..., m$$

**Step 5:** Find the optimal solution $x_j$, $y_j$, $z_j$ and $w_j$ by solving the CLP problem obtained in step 4.

**Step 6:** Find the optimal solution by putting the values of $x_j$, $y_j$, $z_j$ and $w_j$ in $\tilde{X}_j = (x_j, y_j, z_j, w_j)$.

## PROPOSED METHOD

In this study, a new method for project scheduling in fuzzy environment is introduced. This method is developed based on a number of assumptions and definitions in the fuzzy set which defined in study. First we combine two LP Models to calculate the earliest and latest time of events in fuzzy sense proposed by Zareei *et al.* (2011) which described in study then, we use the proposed method by Kumar *et al.* (2011) which describe in study to solve Fully Fuzzy Linear Programming problems (FFLP).

In sample fuzzy project network considered in this study, we assume that the duration of activities are Trapezoidal Fuzzy Numbers (TrFN). The project characteristics such as fuzzy earliest and fuzzy latest times of events and fuzzy project completion time are calculated as TrFN by solving two fully fuzzy linear programming. The steps of the proposed method are as follows:

**Step 1:** Specification the activity duration as Trapezoidal Fuzzy Numbers (TrFN).

**Step 2:** Depiction the project network according to executive logic of projects.

**Step 3:** Writing Eq. 10 to obtain the earliest times of events with considering the trapezoidal fuzzy number as to activity duration specified in step 1 and the project network specified in step 2.

**Step 4:** Solving the obtained model in step 3 by using the presented method by Kumar *et al.* (2011) and specify the earliest times of events.

**Step 5:** Writing Eq. 11 to obtain the latest times of events with considering the trapezoidal fuzzy number as to activity duration specified in step 1 and the project network specified in step 2.

**Step 6:** Solving the obtained model in step 5 by using the presented method by Kumar *et al.* (2011) and specify the latest times of events.

## NUMERICAL EXAMPLE

In this study to confirm the validity of proposed method, a numerical example which was studied by Soltani and Haji (2007) is experimented. The network representing a structure of project is given in Fig. 1.

The durations of activities are positive TFNs (Table 1). The fuzzy start time of this example is (0, 0, 0, 0). According to proposed method structure, the step 1 is presented in Table 1 and the project network is shown in Fig. 1.

In step 3 using the proposed method in study, we calculate the earliest event time of events by solving the Model (10), so the problem is formulated as follows to calculate the earliest event time:

$$\tilde{F} = \min\left(\tilde{E}_1 + \tilde{E}_2 + ... + \tilde{E}_n\right)$$

Table 1: Duration of activities

| Activity | Duration |
|---|---|
| (1, 2) | (25, 28, 32, 35) |
| (1, 3) | (40, 55, 65, 70) |
| (2, 4) | (32, 37, 43, 48) |
| (3, 4) | (20, 25, 35, 40) |
| (2, 5) | (35, 38, 42, 45) |
| (3, 6) | (42, 45, 55, 60) |
| (4, 7) | (60, 65, 75, 85) |
| (5, 7) | (65, 75, 85, 90) |
| (6, 7) | (15, 18, 22, 26) |

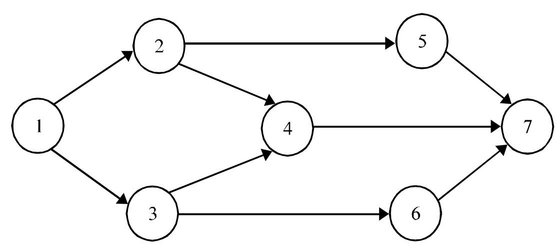


Fig. 1: The project network (Soltani and Haji, 2007)

Subject to:

$$\tilde{E}_2\text{-}\tilde{E}_1\text{-}\tilde{S}_1 = \tilde{T}_{12},\ \tilde{E}_3\text{-}\tilde{E}_1\text{-}\tilde{S}_2 = \tilde{T}_{13}$$
$$\tilde{E}_4\text{-}\tilde{E}_2\text{-}\tilde{S}_3 = \tilde{T}_{24},\ \tilde{E}_4\text{-}\tilde{E}_3\text{-}\tilde{S}_4 = \tilde{T}_{34}$$
$$\tilde{E}_5\text{-}\tilde{E}_2\text{-}\tilde{S}_5 = \tilde{T}_{25},\ \tilde{E}_6\text{-}\tilde{E}_3\text{-}\tilde{S}_6 = \tilde{T}_{36}$$
$$\tilde{E}_7\text{-}\tilde{E}_4\text{-}\tilde{S}_7 = \tilde{T}_{47},\ \tilde{E}_7\text{-}\tilde{E}_5\text{-}\tilde{S}_8 = \tilde{T}_{57}$$
$$\tilde{E}_7\text{-}\tilde{E}_6\text{-}\tilde{S}_9 = \tilde{T}_{67},\ \tilde{E}_i \text{ s are positive TFNs}$$

Let $\tilde{E}_j = (e_j^1, e_j^2, e_j^3, e_j^4)$ and $\tilde{S}_k = (s_k^1, s_k^2, s_k^3, s_k^4)$ the above model may be written as:

$$\text{Minimize: } R(e_1^1+e_2^1+e_3^1+e_4^1+e_5^1+e_6^1+e_7^1,\ e_1^2+e_2^2+ e_3^2+e_4^2+e_5^2+e_6^2+e_7^2,\ e_1^3+e_2^3+e_3^3+e_4^3+ e_5^3+e_6^3+e_7^3,\ e_1^4+e_2^4+e_3^4+e_4^4+e_5^4+e_6^4+e_7^4)$$

Subject to:

$$\tilde{e}_2^1\tilde{e}_1^1 s_1^1\tilde{e}_2^2\tilde{e}_1^2 s_1^2\tilde{e}_2^3\tilde{e}_1^3 s_1^3\tilde{e}_2^4\tilde{e}_1^4 s_1^4 = (25, 28, 32, 35)$$
$$\tilde{e}_3^1\tilde{e}_1 s_2^1\tilde{e}_3^2\tilde{e}_1^2 s_2^2\tilde{e}_3^3\tilde{e}_1^3 s_2^3\tilde{e}_3^4\tilde{e}_1^4 s_2^4 = (40, 55, 65, 70)$$
$$\tilde{e}_4^1\tilde{e}_2^1 s_3^1\tilde{e}_4^2\tilde{e}_2^2 s_3^2\tilde{e}_4^3\tilde{e}_2^3 s_3^3\tilde{e}_4^4\tilde{e}_2^4 s_3^4 = (32, 37, 43, 48)$$
$$\tilde{e}_4^1\tilde{e}_3^1 s_4^1\tilde{e}_4^2\tilde{e}_2^2 s_3^2\tilde{e}_4^3\tilde{e}_2^3 s_3^3\tilde{e}_4^4\tilde{e}_2^4 s_3^4 = (20, 25, 35, 40)$$
$$\tilde{e}_5^1\tilde{e}_2^1 s_5^1\tilde{e}_5^2\tilde{e}_2^2 s_5^2\tilde{e}_5^3\tilde{e}_2^3 s_5^3\tilde{e}_5^4\tilde{e}_2^4 s_5^4 = (35, 38, 42, 45)$$
$$\tilde{e}_6^1\tilde{e}_3^1 s_6^1\tilde{e}_6^2\tilde{e}_3^2 s_6^2\tilde{e}_6^3\tilde{e}_3^3 s_6^3\tilde{e}_6^4\tilde{e}_3^4 s_6^4 = (42, 45, 55, 60)$$
$$\tilde{e}_7^1\tilde{e}_4^1 s_7^1\tilde{e}_7^2\tilde{e}_4^2 s_7^2\tilde{e}_7^3\tilde{e}_4^3 s_7^3\tilde{e}_7^4\tilde{e}_4^4 s_7^4 = (60, 65, 75, 85)$$
$$\tilde{e}_7^1\tilde{e}_5^1 s_8^1\tilde{e}_7^2\tilde{e}_5^2 s_8^2\tilde{e}_7^3\tilde{e}_5^3 s_8^3\tilde{e}_7^4\tilde{e}_5^4 s_8^4 = (65, 75, 85, 90)$$
$$\tilde{e}_7^1\tilde{e}_6^1 s_9^1\tilde{e}_7^2\tilde{e}_6^2 s_9^2\tilde{e}_7^3\tilde{e}_6^3 s_9^3\tilde{e}_7^4\tilde{e}_6^4 s_9^4 = (15, 18, 22, 26)$$

$\tilde{E}_i$s are positive TFNs. The above fuzzy linear problem is converted into the simple CLP problem. The optimal solution of this CLP problem is: $e_1^1 = 0$; $e_1^2 = 0$; $e_1^3 = 0$; $e_1^4 = 0$; $e_2^1 = 25$; $e_2^2 = 28$; $e_2^3 = 32$; $e_2^4 = 35$; $e_3^1 = 52$; $e_3^2 = 57$; $e_3^3 = 67$; $e_3^4 = 70$; $e_4^1 = 90$; $e_4^2 = 92$; $e_4^3 = 92$; $e_4^4 = 92$; $e_5^1 = 70$; $e_5^2 = 70$; $e_5^3 = 70$; $e_5^4 = 70$; $e_6^1 = 112$; $e_6^2 = 112$; $e_6^3 = 112$; $e_6^4 = 112$; $e_7^1 = 164$; $e_7^2 = 167$; $e_7^3 = 171$; $e_7^4 = 175$.

Finally, the fuzzy optimal solution of earliest time is given by:

$$\tilde{E}_1 = (0, 0, 0, 0)$$
$$\tilde{E}_2 = (25, 28, 32, 35)$$
$$\tilde{E}_3 = (52, 57, 67, 70)$$
$$\tilde{E}_4 = (90, 92, 92, 92)$$
$$\tilde{E}_5 = (70, 70, 70, 70)$$
$$\tilde{E}_6 = (112, 112, 112, 112)$$
$$\tilde{E}_7 = (164, 167, 171, 175)$$

And the fuzzy optimal solution of latest event time is given by:

$$\tilde{L}_1 = (0, 0, 0, 0)$$
$$\tilde{L}_2 = (48, 48, 48, 48)$$
$$\tilde{L}_3 = (63, 66, 70, 70)$$
$$\tilde{L}_4 = (90, 95, 101, 103)$$
$$\tilde{L}_5 = (85, 86, 90, 93)$$
$$\tilde{L}_6 = (149, 149, 149, 149)$$
$$\tilde{L}_7 = (164, 167, 171, 175)$$

## COMPARATIVE ANALYSIS

In this study, the proposed method is compared with Soltani and Haji's Method (Table 2) in finding the earliest and latest event times.

This comparison shows that the results of proposed method are so close to those of Soltani and Haji presented, although in some cases the proposed method presents the closed intervals which justify the validity of proposed method. In this study, also the optimal solution of numerical example using the Zareei *et al*. (2011) method is obtained as follows. Review of this result for $\alpha = 1$ show that the interval (145-175) is obtained in the earliest and latest time for final event and it's very close to the result of proposed method (Table 3-6).

Table 2: The comparison of earliest and latest event times

| | $\tilde{E}_i$ | | $\tilde{L}_i$ | |
|---|---|---|---|---|
| Event (i) | Proposed approach | Soltani and Haji (2007) | Proposed approach | Soltani and Haji (2007) |
| 1 | (0, 0, 0, 0) | (0, 0, 0, 0) | (0, 0, 0, 0) | (0, 0, 0, 0) |
| 2 | (25, 28, 32, 35) | (25, 28, 32, 35) | (48, 48, 48, 48) | (25, 32, 48, 60) |
| 3 | (52, 57, 67, 70) | (45, 55, 65, 70) | (63, 66, 70, 70) | (45, 55, 65, 70) |
| 4 | (90, 92, 92, 92) | (60, 80, 100, 110) | (90, 95, 101, 103) | (65, 80100, 110) |
| 5 | (70, 70, 70, 70) | (60, 66, 74, 80) | (85, 86, 90, 93) | (60, 7090, 105) |
| 6 | (112, 112, 112, 112) | (82, 100, 120, 130) | (149, 149, 149, 149) | (110, 127, 153, 169) |
| 7 | (164, 167, 171, 175) | (125, 145, 175, 195) | (164, 167, 171, 175) | (125, 145, 175, 195) |

Table 3: Calculation results of lower bound of earliest times

| α | $E_1^L$ | $E_2^L$ | $E_3^L$ | $E_4^L$ | $E_5^L$ | $E_6^L$ | $E_7^L$ |
|---|---|---|---|---|---|---|---|
| 0.1 | 0 | 25.3 | 41.5 | 62 | 60.6 | 83.8 | 126.1 |
| 0.2 | 0 | 25.6 | 43.0 | 64 | 61.2 | 85.6 | 127.2 |
| 0.3 | 0 | 25.9 | 44.5 | 66 | 61.8 | 87.4 | 128.3 |
| 0.4 | 0 | 26.2 | 46.0 | 68 | 62.4 | 89.2 | 130.0 |
| 0.5 | 0 | 26.5 | 47.5 | 70 | 63.0 | 91.0 | 133.0 |
| 0.6 | 0 | 26.8 | 49.0 | 72 | 63.6 | 92.8 | 135.0 |
| 0.7 | 0 | 27.0 | 50.0 | 74 | 64.2 | 94.6 | 137.0 |
| 0.8 | 0 | 27.4 | 52.0 | 76 | 64.8 | 96.4 | 140.0 |
| 0.9 | 0 | 27.7 | 53.5 | 78 | 65.4 | 98.2 | 142.5 |
| 1.0 | 0 | 28.0 | 55.0 | 80 | 66.0 | 100.0 | 145.0 |

Table 4: Calculation results of upper bound of earliest times

| α | $E_1^U$ | $E_2^U$ | $E_3^U$ | $E_4^U$ | $E_5^U$ | $E_6^U$ | $E_7^U$ |
|---|---|---|---|---|---|---|---|
| 0.1 | 0 | 34.0 | 69.0 | 109.0 | 79.4 | 129 | 193 |
| 0.2 | 0 | 34.0 | 69.0 | 108.5 | 78.8 | 128 | 191 |
| 0.3 | 0 | 34.1 | 68.5 | 107.0 | 78.2 | 127 | 189 |
| 0.4 | 0 | 33.8 | 68.0 | 106.0 | 77.6 | 126 | 187 |
| 0.5 | 0 | 33.5 | 67.5 | 105.0 | 77.0 | 125 | 185 |
| 0.6 | 0 | 33.2 | 67.0 | 104.0 | 76.4 | 124 | 183 |
| 0.7 | 0 | 32.9 | 66.5 | 103.0 | 75.8 | 123 | 181 |
| 0.8 | 0 | 32.6 | 66.0 | 102.0 | 75.2 | 122 | 179 |
| 0.9 | 0 | 32.3 | 65.5 | 101.0 | 74.6 | 121 | 177 |
| 1.0 | 0 | 32.0 | 65.0 | 100.0 | 74.0 | 120 | 175 |

Table 5: Calculation results of lower bound of latest times

| α | $L_1^L$ | $L_2^L$ | $L_3^L$ | $L_4^L$ | $L_5^L$ | $L_6^L$ | $L_7^L$ |
|---|---|---|---|---|---|---|---|
| 0.1 | 0.0 | 25.3 | 45.1 | 65.6 | 60.6 | 110.8 | 126.1 |
| 0.2 | 0.0 | 25.6 | 45.2 | 66.2 | 61.2 | 111.6 | 127.2 |
| 0.3 | 0.0 | 25.9 | 45.3 | 66.8 | 61.8 | 112.4 | 128.3 |
| 0.4 | 0.0 | 26.8 | 46.0 | 68.0 | 63.0 | 113.8 | 130.0 |
| 0.5 | 0.5 | 29.0 | 48.0 | 70.5 | 65.5 | 116.5 | 133.0 |
| 0.6 | 0.0 | 30.2 | 49.0 | 72.0 | 67.0 | 118.0 | 135.0 |
| 0.7 | -0.5 | 31.4 | 50.0 | 73.5 | 68.5 | 119.0 | 137.0 |
| 0.8 | 0.0 | 33.6 | 52.0 | 76.0 | 71.0 | 122.6 | 140.0 |
| 0.9 | 0.0 | 35.3 | 53.5 | 78.0 | 73.0 | 124.0 | 142.5 |
| 1.0 | 18.9 | 44.2 | 64.0 | 84.0 | 79.0 | 129.0 | 145.0 |

Table 6: Calculation results of upper bound of latest times

| α | $L_1^U$ | $L_2^U$ | $L_3^U$ | $L_4^U$ | $L_5^U$ | $L_6^U$ | $L_7^U$ |
|---|---|---|---|---|---|---|---|
| 0.1 | 0 | 58.8 | 69.5 | 109 | 103.5 | 167.4 | 193 |
| 0.2 | 0 | 57.6 | 69.0 | 108 | 102.0 | 165.0 | 191 |
| 0.3 | 0 | 56.4 | 68.5 | 107 | 100.0 | 164.0 | 189 |
| 0.4 | 0 | 55.2 | 68.0 | 106 | 99.0 | 162.6 | 187 |
| 0.5 | 0 | 54.0 | 67.0 | 105 | 97.0 | 161.0 | 185 |
| 0.6 | 0 | 52.8 | 67.0 | 104 | 96.0 | 159.0 | 183 |
| 0.7 | 0 | 51.6 | 66.5 | 103 | 94.0 | 157.8 | 181 |
| 0.8 | 0 | 50.4 | 66.0 | 102 | 93.0 | 156.0 | 179 |
| 0.9 | 0 | 49.2 | 65.5 | 101 | 91.0 | 154.0 | 177 |
| 1.0 | 0 | 48.0 | 65.0 | 100 | 90.0 | 153.0 | 175 |

## CONCLUSION

In this study, we developed a linear programming formulation for calculating earliest and latest time of events assuming activity duration time as TrFN fuzzy number. The optimal solution of earliest and latest times is generated by solving the proposed models in case of fuzzy CPM problem. Applying this approach, no infeasible and negative solution is generated. The second benefit is obtaining optimal solution with solving a simple LP problem.

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