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Economic Reliability GASP for Pareto Distribution of the 2nd Kind Using Poisson and Weighted Poisson Distribution

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Abstract: This research study elaborate an economic reliability group acceptance sampling plan using the Poisson and Weighted Poisson distributions when the lifetime of the product follows the Pareto distribution of 2nd kind. The tables and examples of this research analysis provides the minimum termination time necessary to assure a certain mean or average life, operating characteristic values of the sampling plans and the producer's risk. The benefits and comparative study is made between the proposed plan and existing plan.

Key words: Group acceptance sampling, operating characteristic values, producer's risk, Pareto distribution of the 2nd kind, Poisson and Weighted Poisson distribution

INTRODUCTION

Various approaches of inspection exist in Statistical Quality Control (SQC) to improve the quality of an item to required quality standards. Acceptance sampling plan is one of the most useful techniques of SQC to protect the quality of the item and examine the quality of the item inspected in a sample taken from the lot and on the basis of this information make an inference to accept or reject a submitted lot by the vendor. Accepting sampling is very helpful when the inspection of the item is too costly and also minimizes cost, time, energy and labor. It is implicitly considered in the ordinary acceptance sampling plans that only a single item is put in a tester. Acceptance sampling on the basis of single item by using the various lifetime distribution are discussed by Epstein (1954), Kantam and Rosaiah (1998), Kantam *et al*. (2006), Tsai and Wu (2006), Balakrishnan *et al*. (2007) and Radhakrishnan and Priya (2008) also proposed the CRGS plans using Poisson and Weighted Poisson distribution. However, the tester wants to inspect multiple numbers of items at a time because cost and time can be saved by testing these items simultaneously. The acceptance sampling plan under this type of tester will called a Group Acceptance Sampling Plan (GASP) under the truncated life test. Aslam *et al*. (2011) developed a comparison of GASP for Pareto distribution of the 2nd kind using Poisson and Weighted Poisson distributions. The GASP introduced for specified consumer's risks at the specified quality level and a lot of product is rejected if more than pre-assumed failures are observed in any group. Pareto (1897) proposed the Pareto distribution as a model for income. The Pareto distribution of the second kind are also called Lomax or Pearson's type VI distribution. Bain and Engelhardt have been discussed several applications of Pareto distribution of the 2nd kind in survival and biomedical sciences. Aslam *et al*. (2010) and Baklizi (2003) proposed an ordinary and group acceptance sampling plan for Pareto distribution of the 2nd kind, respectively. Recently, Mughal and Ismail (2013) developed an economic reliability efficient group acceptance sampling plans for family Pareto distributions. The cumulative distribution function and the probability density function of the Pareto distribution of the 2nd kind are as follow, respectively:

$$F(t;\sigma,\lambda) = 1-\left(1+\frac{t}{\sigma}\right)^{-\lambda} \quad t>0,\ \sigma>0,\ \lambda>0 \tag{1}$$

$$f(t;\sigma,\lambda) = \frac{\lambda}{\sigma}\left(1+\frac{t}{\sigma}\right)^{-(\lambda+1)} \quad t>0,\ \sigma>0,\ \lambda>0 \tag{2}$$

where, 'σ' and 'λ' denote the scale and shape parameters, respectively. The mean Pareto distribution of the 2nd is:

$$\mu = \frac{\sigma}{\lambda-1},\ \lambda>1 \tag{3}$$

for validation of mean, the value of shape parameter should be >1.

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CONSTRUCTION OF THE PLANS

In the literature, Aslam *et al.* (2011) proposed the group acceptance sampling plan and made a decision on the basis that a lot under inspection will be not acceptable if the number of defective items in each group is greater than specified number, otherwise the same lot is accepted. Let μ represent the true average life of an item and μ_0 denote the specified average life. A product is stated as good and accepted for consumer use if the average life μ is higher than a specified average life μ_0. The proposed acceptance sampling plan under GASP is stated below:

- Draw the random sample of size n from a lot, distributed r items to each of g group. The required sample size in the life test is $n = r\times g$
- Determine the acceptance number c for every group and specify the termination time of the life test t_0 and accept the lot if at most c failed items are found in every group by the termination time, otherwise not acceptable

Consider the consumer's risk and producer's in the completion of proposed acceptance sampling plans. So, the probability of rejecting a good lot is called the producer risk and the probability of accepting a defective lot is called the consumer risk denoting by α and β, respectively. As mentioned earlier, the researcher is striving to suggest a new lifetime distribution in area of acceptance sampling to find less sample size. Therefore, the main objective of this research is to introduce the economic reliability GASP using the Poisson and weighted Poisson distributions when the lifetime of the product follows the Pareto distribution of 2nd kind. The Poisson distribution is used when small values of p (non-perfects per unit in a lot) and large values of n (sample size), then binomial distribution is approximate case of Poisson distribution having parameter $\lambda = np$. The weighted distribution theory and application are used when we examine the biased data. In life testing recorded data will be biased and do not follow the parent information unless every observation is explained by giving equal probability of being chosen. Weighted distribution theory expresses a clarify path for describing the biased data. The probability mass function of Poisson distribution and the probability mass function of weighted Poisson distribution can be written in the form, respectively:

$$P(i: \lambda) = \frac{e^{-np}(np)^i}{i!},\ i = 1, 2, ... \quad (4)$$

$$P(i: \lambda, \alpha) = \frac{i^{\alpha}P(i, \lambda)}{\sum i^{\alpha}P(i, \lambda)},\ i = 0, 1, ... \quad (5)$$

where, $\lambda = np$. If $\alpha = 1$, the probability mass function of weighted Poisson distribution (Eq. 5) is:

$$P(i: \lambda) = \frac{e^{-np}(np)^{i-1}}{(i-1)!},\ i = 1, 2, ... \quad (6)$$

For more justification, one may refer to Radhakrishnan and Priya (2008). The probability of lot acceptance for the given plans using Poisson distribution and Weighted Poisson distribution are given, respectively:

$$L(p) = \left[\sum_{i=0}^{c}\frac{e^{-np}(np)^i}{i!}\right]^g \quad (7)$$

$$L(p) = \left[\sum_{i=0}^{c}\frac{e^{-np}(np)^{i-1}}{(i-1)!}\right]^g \quad (8)$$

The termination time as a multiple of the pre assumed constant a then $t_0 = a\mu_0$ and p can be evaluated as:

$$p = F(t; \sigma, \lambda) = 1-\left[1+\frac{a}{(\lambda-1)(\mu/\mu_0)}\right]^{-\lambda} \quad (9)$$

For given producer's risk, the minimum termination time can be determined when the following two inequalities are satisfied for both Poisson and weighted Poisson distributions, respectively:

$$L(p) = \left[\sum_{i=0}^{c}\frac{e^{-np}(np)^i}{i!}\right]^g \geq 1-\alpha \quad (10)$$

$$L(p) = \left[\sum_{i=0}^{c}\frac{e^{-np}(np)^{i-1}}{(i-1)!}\right]^g \geq 1-\alpha \quad (11)$$

The minimum termination time are determined for the Poisson and weighted Poisson distribution and placed in Table 1-8. These produce the minimum termination time of the proposed plan for the shape parameter $\lambda = 2$, number of tester $r = 3(1)9$, number of groups $g = 2(1)8$, acceptance number $c = 1(1)8$ and for producer's risk $\alpha = 0.25$, 0.10, 0.05, 0.01. Table 1-8 describe that the minimum termination time increases as the acceptance number increases. Moreover, the minimum termination time in a life test

Table 1: Minimum test termination time for the Pareto distribution of the 2nd kind for $\lambda = 2$ using Poisson distribution ($\alpha = 0.25$)

		g						
c	r	3	4	5	6	7	8	9
1	2	0.1265	0.0904	0.0704	0.0576	0.0487	0.0423	0.0373
2	3	0.2432	0.1661	0.1263	0.1019	0.0854	0.0735	0.0645
3	4	0.4282	0.2724	0.2003	0.1585	0.1311	0.1119	0.0976
4	5	0.7636	0.4269	0.2987	0.2302	0.1874	0.1581	0.1367
5	6	1.6390	0.6721	0.4342	0.3225	0.2569	0.2137	0.1830
6	7	*	1.1392	0.6334	0.4449	0.3442	0.2811	0.2378
7	8	*	2.6620	0.9624	0.6158	0.4568	0.3642	0.3033
8	9	*	*	1.6607	0.8754	0.6083	0.4692	0.3829

Table 2: Minimum test termination time for the Pareto distribution of the 2nd kind for $\lambda = 2$ using Poisson distribution ($\alpha = 0.10$)

		g						
c	r	3	4	5	6	7	8	9
1	2	0.0661	0.0483	0.0381	0.0314	0.0268	0.0233	0.0206
2	3	0.1428	0.1014	0.0786	0.0643	0.0543	0.0471	0.0415
3	4	0.2577	0.1751	0.1328	0.1070	0.0896	0.0770	0.0675
4	5	0.4367	0.2769	0.2032	0.1607	0.1329	0.1133	0.0989
5	6	0.7521	0.4224	0.2900	0.2283	0.1859	0.1569	0.1357
6	7	1.5170	0.6476	0.4218	0.3143	0.2509	0.2089	0.2508
7	8	*	1.0550	0.6023	0.4268	0.3317	0.2716	0.2302
8	9	*	2.1550	0.8885	0.5805	0.4345	0.3481	0.2908

Table 3: Minimum test termination time for the Pareto distribution of the 2nd kind for $\lambda = 2$ using Poisson distribution ($\alpha = 0.05$)

		g						
c	r	3	4	5	6	7	8	9
1	2	0.0432	0.0319	0.0253	0.0209	0.0178	0.0156	0.0138
2	3	0.1026	0.0740	0.0578	0.0475	0.0403	0.0350	0.0309
3	4	0.1911	0.1332	0.1022	0.0831	0.0699	0.0603	0.0531
4	5	0.3229	0.2140	0.1602	0.1281	0.1068	0.0915	0.0801
5	6	0.5330	0.3225	0.2352	0.1844	0.1518	0.1289	0.1121
6	7	0.9250	0.4870	0.3340	0.2550	0.2064	0.1735	0.1497
7	8	2.0800	0.7429	0.4690	0.3450	0.2734	0.2267	0.1936
8	9	*	1.2320	0.6654	0.4631	0.3567	0.2906	0.2454

Table 4: Minimum test termination time for the Pareto distribution of the 2nd kind for $\lambda = 2$ using Poisson distribution ($\alpha = 0.01$)

		g						
c	r	3	4	5	6	7	8	9
1	2	0.0176	0.0131	0.0105	0.0087	0.0074	0.0065	0.0058
2	3	0.0525	0.0386	0.0306	0.0253	0.2155	0.0188	0.0166
3	4	0.1073	0.0772	0.0603	0.0495	0.0420	0.0364	0.0322
4	5	0.1860	0.1300	0.1001	0.0812	0.0684	0.0591	0.0520
5	6	0.3010	0.2010	0.1511	0.1211	0.1011	0.0868	0.0760
6	7	0.4750	0.2967	0.2166	0.1703	0.1408	0.1199	0.1043
7	8	0.7700	0.4300	0.3005	0.2315	0.1885	0.1589	0.1375
8	9	1.4280	0.6280	0.4120	0.3081	0.2460	0.2052	0.1759

Table 5: Minimum test termination time for the Pareto distribution of the 2nd kind for $\lambda = 2$ using weighted Poisson distribution ($\alpha = 0.25$)

		g						
c	r	3	4	5	6	7	8	9
1	2	0.0248	0.01848	0.0147	0.0122	0.0104	0.0091	0.0080
2	3	0.0963	0.06960	0.0545	0.0448	0.0380	0.0331	0.0292
3	4	0.2064	0.14310	0.1095	0.0887	0.0746	0.0644	0.0566
4	5	0.3776	0.24490	0.1816	0.1444	0.1199	0.1025	0.0895
5	6	0.6767	0.39110	0.2768	0.2146	0.1753	0.1483	0.1285
6	7	1.3864	0.61870	0.4067	0.3043	0.2435	0.2031	0.1743
7	8	*	1.03620	0.5953	0.4226	0.3287	0.2694	0.2283
8	9	*	2.22600	0.9004	0.5863	0.4382	0.3508	0.2928

Table 6: Minimum test termination time for the Pareto distribution of the 2nd kind for $\lambda = 2$ using weighted Poisson distribution ($\alpha = 0.10$)

		g						
c	r	3	4	5	6	7	8	9
1	2	0.0089	0.0066	0.0053	0.0044	0.0037	0.0031	0.0029
2	3	0.0519	0.0382	0.0302	0.0249	0.0213	0.0185	0.0160
3	4	0.1246	0.0892	0.0694	0.0568	0.0481	0.0417	0.0368
4	5	0.2339	0.1604	0.1222	0.0987	0.0828	0.0713	0.0626
5	6	0.4024	0.2585	0.1909	0.1514	0.1255	0.1072	0.0936
6	7	0.6930	0.3980	0.2811	0.2177	0.1777	0.1502	0.1301
7	8	1.3550	0.6111	0.4028	0.3017	0.2416	0.2016	0.1730
8	9	14.4000	0.9876	0.5762	0.4113	0.3208	0.2633	0.2235

Table 7: Minimum test termination time for the Pareto distribution of the 2nd kind for $\lambda = 2$ using weighted Poisson distribution ($\alpha = 0.05$)

		g						
c	r	3	4	5	6	7	8	9
1	2	0.0043	0.0032	0.0025	0.0021	0.0018	0.0016	0.0014
2	3	0.0344	0.0255	0.0202	0.0167	0.0143	0.0125	0.0111
3	4	0.0905	0.0656	0.0514	0.0432	0.0359	0.0312	0.0276
4	5	0.1753	0.1230	0.0948	0.0771	0.0650	0.0561	0.0494
5	6	0.3014	0.2014	0.1514	0.1214	0.1013	0.0869	0.0761
6	7	0.5000	0.3092	0.2248	0.1767	0.1456	0.1239	0.1078
7	8	0.8620	0.4644	0.3210	0.2460	0.1995	0.1679	0.1450
8	9	1.8420	0.7080	0.4520	0.3340	0.2654	0.2204	0.1885

Table 8: Minimum test termination time for the Pareto distribution of the 2nd kind for $\lambda = 2$ using weighted Poisson distribution ($\alpha = 0.01$)

		g						
c	r	3	4	5	6	7	8	9
1	2	0.0008	0.0006	0.0005	0.0004	0.0003	0.0003	0.0002
2	3	0.0143	0.0107	0.0085	0.0071	0.0060	0.0053	0.0047
3	4	0.0470	0.0347	0.0275	0.0227	0.0194	0.0169	0.0149
4	5	0.0996	0.0720	0.0563	0.0463	0.0393	0.0341	0.0302
5	6	0.1762	0.1236	0.0952	0.0776	0.0654	0.0564	0.0497
6	7	0.2870	0.1930	0.1453	0.1167	0.0976	0.0838	0.0734
7	8	0.4542	0.2860	0.2094	0.1654	0.1366	0.1164	0.1016
8	9	0.7360	0.4160	0.2920	0.2254	0.1840	0.1552	0.1344

decreases as the number of testers' increases. The minimum termination time required for testing under weighted Poisson distribution is less than the Poisson distribution. For an example, if r = 3, c = 1, g = 2 and $\lambda = 2$, the minimum termination time from Table 5 is 0.0248 and from Table 1, it is 0.1265. The similar tables can be developed for different values of the shape parameter for Pareto distribution of the 2nd kind, computer software is provide from the researchers upon demand. It is important to note that the tables developed in this research article considering the known values of shape parameter of the Pareto distribution of the 2nd kind. If the shape parameter is unknown, it can be evaluated using maximum likelihood method technique.

DESCRIPTION OF TABLES

A research analyst would like to know whether the average life of their product is higher than the specified average life, $\mu_0 = 10,000$ h. The analyst wants to run a life test in 10,000 h by using testers accommodate with three products each. Suppose that the lifetimes of product follow the Pareto distribution of the second kind with $\lambda = 2$ (using Poisson distribution) when the producers risk is 0.25. From Table 1, the design parameters of proposed plan are (g, r, c, a) = (2, 3, 1, 0.1265). So, the analyst needs to select a sample of size 6 products from the lot and put three products to two groups on the life experiment. The lot is not acceptable if more than one failed products are found in 1265(10000×0.1265) h in every group, otherwise acceptable.

COMPARATIVE STUDY

In Table 9-15, the upper entries are showing the proposed plan and lowest entries describing the existing plan developed by Aslam *et al*. (2011). In these tables, the minimum termination time obtaining by a proposed plan is very minimal when compared to an existing plan. The comparison of two acceptance sampling plans, we again consider the above example. From Table 9, the design parameters of proposed and exiting plan are (g, r, c, a) = (2, 3, 1, 0.1265) and (g, r, c, a) = (2, 3, 1, 2.0000), respectively. So, the proposed plan require 1,265 h and exiting plan required 20,000 h, respectively to reach a same conclusion about the submitted lot.

Table 9: Comparisons of Minimum test termination time for the Pareto distribution of the 2nd kind for $\lambda = 2$ using Poisson distribution ($\alpha = 0.25$)

		g						
c	r	3	4	5	6	7	8	9
1	2	0.1265						
		2.0000						
2	3		0.1661					
			0.7000					

Table 10: Comparisons of Minimum test termination time for the Pareto distribution of the 2nd kind for $\lambda = 2$ using Poisson distribution ($\alpha = 0.10$)

		g						
c	r	3	4	5	6	7	8	9
1	2	0.0661						
		1.2000						
2	3		0.1014					
			1.0000					
3	4			0.1328				
				1.0000				
4	5				0.1607			
					0.8000			
5	6					0.1859		
						0.8000		

Table 11: Comparisons of Minimum test termination time for the Pareto distribution of the 2nd kind for $\lambda = 2$ using Poisson distribution ($\alpha = 0.05$)

		g						
c	r	3	4	5	6	7	8	9
1	2							
2	3	0.0740						
		1.5000						
3	4		0.1022					
			1.2000					
4	5			0.1281				
				1.2000				
5	6				0.1518			
					1.0000			

Table 12: Comparisons of Minimum test termination time for the Pareto distribution of the 2nd kind for $\lambda = 2$ using Poisson distribution ($\alpha = 0.01$)

		g						
c	r	3	4	5	6	7	8	9
1	2							
2	3							
3	4							
4	5				0.0812			
					2.0000			
5	6					0.1011		
						2.0000		
6	7						0.1199	
							1.5000	

Table 13: Comparisons of Minimum test termination time for the Pareto distribution of the 2nd kind for $\lambda = 2$ using weighted Poisson distribution ($\alpha = 0.10$)

		g						
c	r	3	4	5	6	7	8	9
1	2	0.0089						
		1.0000						
2	3							

Table 14: Comparisons of Minimum test termination time for the Pareto distribution of the 2nd kind for $\lambda = 2$ using weighted Poisson distribution ($\alpha = 0.05$)

		g						
c	r	3	4	5	6	7	8	9
1	2	0.0043						
		2.0000						
2	3		0.0255					
			0.8000					

Table 15: Comparisons of Minimum test termination time for the Pareto distribution of the 2nd kind for $\lambda = 2$ using weighted Poisson distribution ($\alpha = 0.01$)

		g						
c	r	3	4	5	6	7	8	9
1	2	0.00084						
		2.00000						
2	3		0.0107					
			2.0000					
3	4			0.0275				
				1.2000				
4	5				0.0643			
					1.0000			

CONCLUSION

In this research article, minimum test termination time is found for the specified values of producer's risk using the Poisson and Weighted Poisson distributions when the lifetime of the product follows the Pareto distribution of 2nd kind. The test termination time obtained by proposed plan is very lesser than the exiting plan, so we can conclude that current research analysis is more economical in the sense of saving cost, time, energy and labor.

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