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Algorithm for Calculation of Confidence Intervals of Low-Cycle Fatigue Curve

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Abstract: The study considers developed in kinetic theory of fatigue mathematical model for forecasting of stress-cycled machine parts lifetime with specified probability of non-destruction. The results processing of samples fatigue tests is carrying out on a base of this model. Two parameters of this model: tensile strength and number of cycles before upper inflection point of low-cycle fatigue curve, related with tensile strength by non-linear dependence are random values with unknown distribution laws. In that way left limits of confidence intervals of low-cycle fatigue curve, corresponding to specified probability (1-5%) of samples non-destruction used for forecasting of its lifetime are impossible to calculate. Researchers propose the original algorithm for model parameters determination of fatigue curve passing through any experimental point. Using this algorithm it is possible to calculate during computer modeling the set of fatigue curves. Obtained set of fatigue curves allows generating the random sample the number of cycles before destruction, for any fixed level of actual stress. This random sample is necessary for calculation of its quantile estimation at plotting of limits of confidence intervals. For regeneration of unknown density function of distribution this random value authors use mathematical apparatus of non-parametric statistics ensuring the task decision independently of complexity distribution law of researched random value. The results of developed algorithm realization are illustrated on example of limits of confidence intervals determination for results of low-cycle fatigue test of flexible pipes samples made of HS80 steel.

Key words: Tensile strength, cyclic deforming, low-cycle fatigue curve, probability of samples destruction, limits of confidence intervals, methods of non-parametric statistics

INTRODUCTION

The basis of task decision of lifetime forecasting and durability estimation of machine parts and units is fatigue curve (Wohler curve) (Susmel and Lazzarin, 2002), connecting number of cycles of deforming (N) and stress value in dangerous place (σ). The most perspective mathematical models for description of fatigue curve are semi-empirical models developed within the fatigue kinetic theory (Ye, 1982; Syzrantsev *et al*., 2015) taking into account process of accumulation of fatigue damages in material at cyclic deforming of machine parts. So, for fatigue curve in low-cyclic area $N \leq 10_5$ the mathematical model described by equation:

$$\sigma = \sigma_B + \vartheta \times \lg\left(\frac{N}{H} + 1\right) \quad (1)$$

Where:

- σ_B = The yield material value
- ϑ = The slope of a fatigue curve in system of coordinates lgN-σ
- H = Number of deformation cycles to the top point of the low-cyclic fatigue curve bend, calculated on dependence

$$H = \frac{Q}{\sigma_B}\ln\left\{1 + \left[\exp\left(\frac{\sigma_B - \sigma_r}{\sigma_r - \sigma_{rT}}\right) - 1\right]^{-1}\right\} \quad (2)$$

where, N is number of loading cycles, calculated on dependence:

$$N = \frac{Q}{\sigma}\ln\left\{1 + \left[\exp\left(\frac{\sigma - \sigma_r}{\sigma_r - \sigma_{rT}}\right) - 1\right]^{-1}\right\} \quad (3)$$

Where:

- N = The number of loading cycles
- σ = Maximum cycle tension
- Q = Fatigue ratio
- σ_r = The fatigue limit at coefficient of cycle asymmetry r
- σ_{rT} = Cyclic yield limit (below its level plastic deformation marks even after several million cycles of loading are missing)

Algorithm and calculation of parameters Q, σ_r, σ_{rT} of ϑ model (Eq. 1) on the basis of available data set for the samples destruction σ_i, N_i, $i = \overline{1, n}$ is considered in work (Syzrantsev *et al*., 2015). As a result of these algorithms realization the fatigue curve is constructed in a kind of

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regression dependence $N = N(\sigma)$. This curve corresponds to 50% probability of samples destruction. It is understood that there's no sense to use such curve. Thus, the most important practical application of processing of samples fatigue tests results is not fatigue curve but its left (lower) limits of confidence intervals, corresponding to 1 or 5% probability of samples destruction. The purpose of present study is development of calculation method of points of confidence intervals limits for low-cycle fatigue curve in kind (Eq. 1).

PROCEDURE AND ALGORITHM OF LOW-CYCLIC FATIGUE CURVE CONFIDENCE LIMITS CALCULATION

The fatigue curve confidence limits calculation requires knowledge of density function f(N) or f(lgN) with σ = const. Normally (Susmel and Lazzarin, 2002), it is accepted that these functions correspond to normal (lognormal) distribution laws of random values. However, as shown in the papers (Syzrantsev and Syzrantseva, 2008; Syzrantseva, 2009a), the functions f(N) and f(lgN) are significantly more complicated not described by the laws, researched in the theory of parametrical statistics.

In this study for the problem solution the idea of statistical modeling was implemented, first time proposed in the study to get a fatigue limit selection in order to restore its density function by non-parametric statistics methods (Syzrantseva, 2009b).

It follows from the dependency analysis (Eq. 1) that it contains two parameters which nature is casual, this is a strength limit (σ_B) and a number of cycles to the top point of low-cyclic fatigue curve bend point (H) which is relatively σ_B described by non-linear dependence (Eq. 2).

For the moment of the considered problem solution the parameters $\sigma_y = \sigma^*_y$, $\sigma_{yT} = \sigma^*_{yT}$, $Q = Q^*$, $\vartheta = \vartheta^*$, $Q_T = Q^*_T$, $D = D^*_0$ are known. We use the data of samples destruction by the pull test machine on the basis of which we will determine the statistical characteristics σ_B: average value $\overline{\sigma_B}$ and limits of its confidential interval, for example, for probability of 90% $\sigma^{0.90}_{Bmin}$ and $\sigma^{0.90}_{Bmax}$.

Substituting in the Eq. 2 $\sigma_B = \overline{\sigma_B}$, we will calculate a mathematical expectation $\overline{H}$ of cycles number H and its 99% confidence limits: $H^{0.99}_{min}$ and $H^{0.90}_{max}$.

Let's set the task to get a dependency for fatigue curve which passed through any experimental point σ_i = const, N_i = const $i = \overline{1, n}$ and was within the established confidence limits. We enter the dimensionless value χ by this we set the current values σ_B and H:

$$\begin{aligned} \sigma_B(\chi) &= \sigma^{0.99}_{Bmin} + \chi\times\left(\sigma^{0.99}_{Bmax} - \sigma^{0.99}_{Bmin}\right), \\ H(\chi) &= H^{0.99}_{min} + \chi\times\left(H^{0.99}_{max} - H^{0.99}_{min}\right) \end{aligned} \tag{4}$$

Entering the function $\sigma_B(\chi)$ and $H(\chi)$ in the Eq. 1, we have:

$$\begin{aligned} \sigma = {} & \sigma^{0.99}_{Bmin} + \chi\times\left(\sigma^{0.99}_{Bmax} - \sigma^{0.99}_{Bmin}\right) + \\ & \vartheta\times\lg\left(\frac{N}{H^{0.99}_{min} + \chi\times\left(H^{0.99}_{max} - H^{0.99}_{min}\right)} + 1\right) \end{aligned} \tag{5}$$

Entering the equation with $\sigma = \sigma_i$ = const and $N = N_i$ = const we have the transcendental equation relating to one variable χ:

$$\begin{aligned} \sigma_i = {} & \sigma^{0.99}_{Bmin} + \chi\times\left(\sigma^{0.99}_{Bmax} - \sigma^{0.99}_{Bmin}\right) + \\ & \vartheta\times\lg\left(\frac{N_i}{H^{0.99}_{min} + \chi\times\left(H^{0.99}_{max} - H^{0.99}_{min}\right)} + 1\right) \end{aligned} \tag{6}$$

Solving this equation by any numerical method for each couple of σ_i, N_i, $i = \overline{1, n}$ experimental data values, we will determine the selection χ_i, $i = \overline{1, n}$ which allows to receive a series of length n of low-cycle fatigue curves:

$$\begin{aligned} \sigma = {} & \sigma^{0.99}_{Bmin} + \chi_i\times\left(\sigma^{0.99}_{Bmax} - \sigma^{0.99}_{Bmin}\right) + \\ & \vartheta\times\lg\left(\frac{N}{H^{0.99}_{min} + \chi_i\times\left(H^{0.99}_{max} - H^{0.99}_{min}\right)} + 1\right) \end{aligned} \tag{7}$$

This set of fatigue curves allows to create as selection N^*_i, $i = \overline{1, n}$ with $\sigma = \sigma^*$ = const:

$$N^*_i = \left[10^{\frac{\sigma^* - \sigma^{0.99}_{Bmin} - \chi_i\times\left(\sigma^{0.99}_{Bmax} - \sigma^{0.99}_{Bmin}\right)}{\vartheta}} - 1\right]\times\left[H^{0.99}_{min} + \chi_i\times\left(H^{0.99}_{max} - H^{0.99}_{min}\right)\right] \tag{8}$$

It is required to restore the cycles number density function before destruction $f_N(N)$ with the fixed tension value $\sigma = \sigma^*$ = const.

Though a priori the distribution law of random value N is unknown, we will use the mathematical tool of non-parametric statistics which is successfully applied recently for the solution of similar tasks (Botev *et al*., 2010; Nerodenko and Syzrantseva, 2012; Syzrantsev and Syzrantseva, 2008; Syzrantseva, 2009a). Initial information for the function determination $f_N(N)$ is set of values N^*_i, $i = \overline{1, n}$ calculated on dependency (Eq. 8).

To restore the function $f_N(N)$, we will use Parzen-Rosenblatt Method (Syzrantsev *et al*., 2014; Syzrantseva, 2009a). Following this method, unknown function of density $f_N(N)$ is described in the equation:

$$f_N(N) = \frac{1}{n \times h_N} \sum_{i=1}^{n} K\left(-\frac{\left(N - N_i^*\right)}{h_N}\right) \quad (9)$$

Where:
K(N) = The kernel function (core)
h_N = The "blurring" parameter

Solving the problem it is necessary (Syzrantseva, 2009a) to a kernel function type and to determine the value of the "blurring" parameter. At the moment from a set of kernel functions, the function with a normal kernel is mostly used:

$$f_N(N) = \frac{1}{n \times h_N \times \sqrt{2 \times \pi}} \sum_{i=1}^{n} \exp\left(-\frac{\left(N - N_i^*\right)^2}{h_N}\right) \quad (10)$$

Optimal value $h_N = h_N^*$ is calculated in accordance with euqation:

$$h_N^* = D_N \times n^{-\frac{1}{5}} \quad (11)$$

Where:

$$D_N = \frac{1}{n-1} \sum_{i=1}^{n} \left(N_i^* - \overline{N_i^*}\right)^2, \ \overline{N_i^*} = \frac{1}{n} \sum_{i=1}^{n} N_i^*$$

Having the function $f_N(N)$, the quantiles value $N_{min}^{0.99}$, required as per terms of the fatigue tests data processing with $\sigma = \sigma^*$ = const are calculated to solve the equations by numerical method:

$$\int_0^{N_{min}^{0.99}} f_N(N)dN = 0.01 \quad (12)$$

The value $N_{min}^{0.99}$, established as a result of the stated algorithm implementation with the set tension value σ^* = const defines one point of confidence limit, corresponding to probability of samples destruction of 1%. To calculate the other limit points of this confidential interval it is enough to repeat the considered procedure for tension σ^* within the required range.

IMPLEMENTATION OF THE DEVELOPED ALGORITHMS ON THE EXAMPLE OF LOW-CYCLIC TESTS FOR SAMPLES FROM HS80 STEEL DATA PROCESSING

The results of fatigue tests of samples cut of flexible pipe HS80 are presented in research (Syzrantsev *et al*., 2015). We will carry out the calculation of the confidence limits. On the basis of data processing of stretching the samples from HS80 steel by the pull test machine the following was obtained: $\sigma_{Bmin}^{0.90}$ = 591.1 MPa; $\sigma_{Bmax}^{0.90}$ = 613.1 MPa; $\sigma_{Bmin}^{0.95}$ = 586.8 MPa; $B_{max}^{0.95}$ = 617.4 MPa; $\sigma_{Bmin}^{0.99}$ = 575.3 Mpa; $\sigma_{Bmax}^{0.99}$ = 628.9 MPa. As per the (Eq. 2), we calculate the values $H_{min}^{0.90}$ = 269.727; $H_{max}^{0.90}$ = 181.926; $H_{min}^{0.95}$ = 291.365; $H_{max}^{0.95}$ = 168.479; $H_{min}^{0.99}$ = 358.268; $H_{max}^{0.99}$ = 137.237.

Let's consider the Eq. 6 solving it by the numerical method n once for each couple of values σ_i, N_i, we determine the selection of dimensionless value χ_i, $i = \overline{1, n}$. Then for any fixed tension value $\sigma = \sigma^*$ = const according to Eq. 8, using the array χ_i, $i = \overline{1, n}$, we calculate the selection N^*_i (and $\lg N^*_i$), $i = \overline{1, n}$. Similarly, fixing value of cycles number $N = N^*$ = const on the basis of the array χ_i, $i = \overline{1, n}$ the selection N^*_i, $i = \overline{1, n}$ is determined by the Eq. 8. Having used the mathematical tool of non-parametric statistics on the basis of the selections N^*_i, $i = \overline{1, n}$, σ^*_i, $i = \overline{1, n}$ we restore the unknown function of distribution density $f_N(N_*)$, described by type dependences Eq. 10. As an example in Fig. 1 the distribution histogram N^*_i, $i = \overline{1, n}$ and the function $f_N(N^*)$ with $\sigma = \sigma^*$ = 250 MPa are shown.

To calculate the confidence limits, for example with probability of samples destruction 0.5, 2.5, 5 and 50% it's enough to implement the considered restoring procedure $f_N(N^*)$ for a number of fixed tension values and to calculate the corresponding quantile estimates of cycles numbers

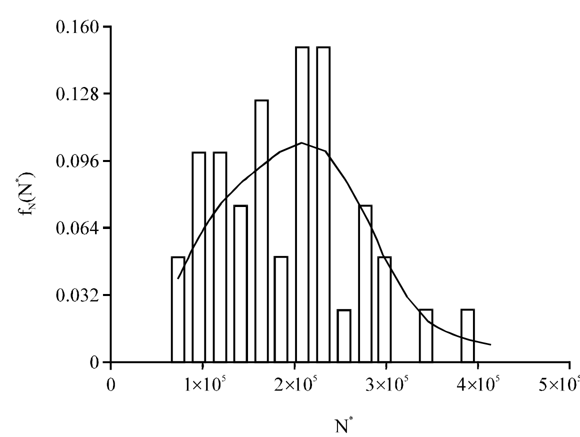

Fig. 1: Density function $f_N(N^*)$

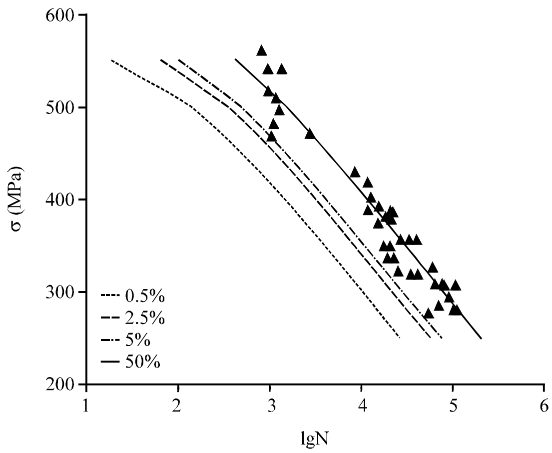

Fig. 2: Confidence limits in the system of coordinates lgN-σ, obtained for the real distribution law of random value N law

by solving the integrated Eq. 12. The results of such executed calculations are presented in Fig. 2 in system of coordinates lgN-σ.

CONCLUSION

- The original algorithm of confidence limits calculation for the low-cyclic fatigue curve, based on the use of the nonparametric statistics mathematical tool which allows while solving the task to consider the actual distribution laws of cycles number before destruction with fixed tension value was proposed
- The developed procedures and computing algorithms are illustrated on the example of data processing for low-cyclic fatigue tests of the samples from HS80 steel

REFERENCES

Botev, Z.I., J.F. Grotowski and D.P. Kroese, 2010. Kernel density estimation via diffusion. Ann. Stat., 38: 2916-2957.

Nerodenko, D.G., K.V. Syzrantseva, 2012. Calculation of reliability of elements of technological pipelines by methods of non-parametric statistics. Oil Ind., 6: 94-96.

Susmel, L. and P. Lazzarin, 2002. A bi-parametric wohler curve for high cycle multiaxial fatigue assessment. Fatigue Fract. Eng. Mater. Struct., 25: 63-78.

Syzrantsev, V.N., K.V. Syzrantseva and L.A. Chernaya, 2014. Calculation of the strength reliability of parts under random loading. Global J. Res. Eng., 14: 11-17.

Syzrantsev, V. and K. Syzrantseva, 2008. Non-parametric approach to the durability estimation task of details with complicated geometry. Monog. Mach. Des.

Syzrantsev, V.N., K.V. Syzrantseva, V.N. Ilinykh, 2015. Processing of low-cycle fatigue tests data on a base of kinetic fatigue theory. Int. J. Eng. Appl. Sci., 6: 7-10.

Syzrantseva, K.V., 2009a. Development of a method to calculate the strength reliability of tooth gears based on the fatigue resistance when the teeth bend. J. Mach. Manuf. Reliab., 38: 552-556.

Syzrantseva, K.V., 2009b. Predicting the failure of gear transmissions by non-parametric statistics. Russ. Eng. Res., 29: 1206-1208.

Ye, P.K., 1982. Accumulating of fatigue damages. Russ. Eng. Res., 1: 11-15.