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Numerical Solution of Non-Linear Filtration Issues for High Viscous Fluids at the Presence of Wells

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Abstract: The numerical simulation of the steady filtration process for highly viscous incompressible fluid following the nonlinear law of filtration in the presence of wells. It is assumed that the function determining the filtration law (dependence of filtration rate on the pressure gradient) is equal to zero if the module of the pressure gradient does not exceed a predetermined value the maximum gradient (the set of such points in the field of filtration where the flow does not occur is a stagnation zone) is continuous one, not decreasing and has exponential growth at infinity. The generalized statement of this problem is formulated in the form of an operator equation with a monotone operator in Banach space. The theorem of solvability is proved. In order to solve the operator equation a Two-Layer Iterative Method is proposed. Every step of the iterative process is reduced to the solution of the boundary issue for the Laplace equation. The convergence of the iteration process is investigated. This method was implemented numerically. The dependence of the boundaries on dead zones was investigated (sets in the field of filtration where the pressure gradient unit is less than the maximum one, the flow is absent) of the well rate. The results of numerical experiments performed for model issues the field of filtration, the number of wells, their location and rate varied the effectiveness of the proposed Iterative Method was confirmed. The proposed method can be used for the solution of problems concerning rational mining of highly viscous hydrocarbons.

Key words: Mathematical modeling, developed filtration, monotone operator, Iterative Method, numerical experiment

INTRODUCTION

The established processes of underground filtration for incompressible fluids are studied, the following non-linear filtration laws with limiting gradient. It is assumed that the function determining the filtration law (dependence of filtration rate on the pressure gradient) is equal to zero if the pressure gradient module does not exceed a predetermined value the maximum gradient (the set of such points in the field of filtration where the flow does not occur is a stagnation zone). Besides, it is believed that this function is continuous one not diminishing and is located in the area where the gradient pressure module is greater than the limiting gradient (flow area), the gradual order growth makes p-1>0. The generalized statement of the problem is formulated as an operator equation in a Banach space Sobolev space of functions with a generalized derivative, integrable with the degree p.

To solve the equation a two-layer iterative method is proposed, each step of which is actually reduced to the solution of the boundary issue for the Laplace operator.

A set of programs was developed in MatLab medium. The numerical experiments for model problems were performed.

Problem statement: The problem of the stationary pressure fields p and filtration rate $w = (w_1, w_2)$ is considered for highly viscous liquid in the area, $\Omega \subset R^2$, satisfying the equation of continuity and the effective nonlinear filtration law with the limiting gradient and the appropriate boundary conditions:

$$\operatorname{div} w = f_*, \; w = \frac{g(|\nabla p|)}{|\nabla p|}\nabla p, \; x \in \Omega \tag{1}$$

$$p(x) = 0, \; x \in \Gamma_1, \quad (w, n) = 0, \; x \in \Gamma_2 \tag{2}$$

Where:
$\Gamma_1 \cup \Gamma_2 = \Gamma$
Γ, Ω = Area limit, mesΓ_1>0
n = A single vector of external normal to Γ_2

Let's assume that the function g satisfies the following conditions g are continuous without decrease:

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$$g(\xi) = 0 \text{ at } \xi \le \beta \qquad (3)$$

where, $\beta \ge 0$ is the peak gradient; there are such $c_0>0$, $c_1>0$s $c_2>0$, $p\ge 2$ that:

$$c_0 \xi^{p-1} - c_1 \le g(\xi) \le c_2 \xi^{p-1} \text{ at } \xi \ge \beta \qquad (4)$$

Let's determine the operator G: $R^2 \to R^2$ according to the formula $G(y) = g(|y|)\, y/|y|$, $y \ne 0$, $G(0) = 0$. Let $V = \{\eta \in W^{(1)}_p(\Omega)\text{: } \eta(x) = 0,\ x \in \Gamma_1\}$, f_* generates a continuous linear functional on V. Due to the Eq. 3 and 4, the form $a(u, \eta) = \int_\Omega (G(\nabla u)(\nabla u, \nabla \eta)\,dx$ generates the following operator, $A: V \to V^* = W^{(-1)}_q(\Omega)$, $q = p/(p-1)$, according to the formula $\langle Au, \eta\rangle = a(u, \eta)$ (Lapin, 1998a, b; Badriev and Nechaeva, 2013) where $\langle ., .\rangle$ is the duality ratio between V and V^*. The filtration issue Eq. 1 and 2 is represented by the function $u \in V$, satisfying the following equation:

$$Au = f \qquad (5)$$

Where the element $f \in V^*$ is determined according to the following formula $\langle f, \eta\rangle = \int_\Omega (f\eta\, dx$. Operator A is a continuous, monotone and coercive one, therefore, the Eq. 5 has at least one solution (Lions, 1969; Gajewskii *et al.*, 1974; Ekeland and Temam, 1976).

Let's note that the filtration issue is studied in the articles (Badriev and Zadvornov, 2005; Badriev, 2013) with multi-valued filtration law in the case of p = 2. At that the task of filtering was formulated as a variation inequality. The duality method was used in (Badriev and Pankratova, 1992; Badriev, 1989; Badriev and Karchevskii, 1989) to study the filtration issues.

MATERIALS AND METHODS

Let's use a two-layer iterative process of the type to solve the operator Eq. 5 (Badriev and Karchevskii, 1994; Badriev, 1989a, b; Badriev and Zadvornov, 2003):

$$-\Delta(u^{k+1} - u^k) = \tau_k (Au^k - f), \quad k = 0, 1, 2, \ldots, \qquad (6)$$

where, $\tau_k>0$ is an iterative parameter, u^0 is the set element. If the function g, along with Eq. 3 and 4 also satisfies the following term $(g(\xi)-g(\zeta))/(\xi-\zeta) \le c_3 (1+\xi+\zeta)^{p-2}$, $c_3>0$ where $\tau_k = \min\{1, 1/(\alpha+\mu_k)\}$, $\mu_k = \mu(\|u^k\|+\|Au^k-f\|^{-1})$, $\mu(\xi) = c_4 (1+2\xi)^{p-2}$, $c_4>0$, a an arbitrary positive number, then the iterative sequence $\{u^k\}$, developed according to Eq. 6 is limited, any weak limit point of the sequence $\{u^k\}$ is the solution of Eq. 5 (Badriev and Karchevskii, 1994).

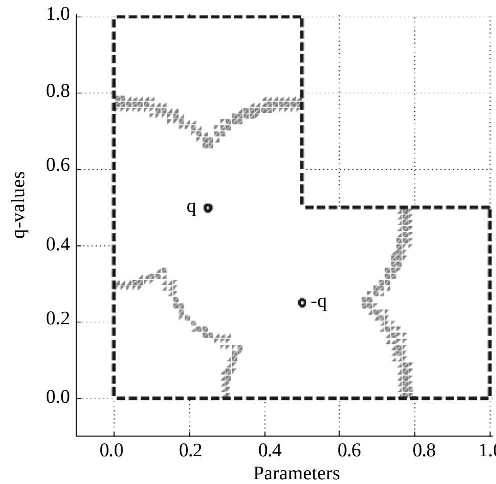

Fig. 1: L-shaped area, q = 2. Stagnant zone boundaries

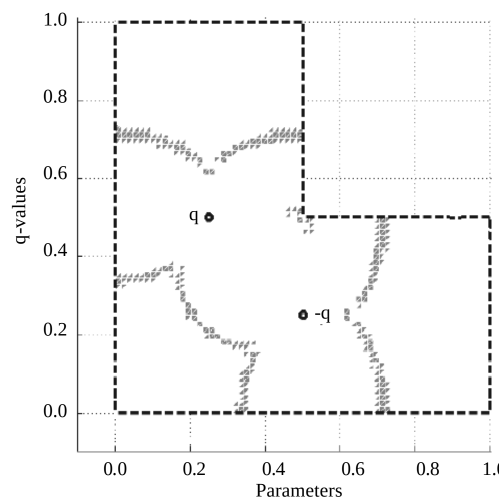

Fig. 2: L-shaped area, q = 1.3. Stagnant zone boundaries

In MatLab medium a set of programs was developed. The numerical experiments were performed for model filtration problems. The dependence of the of dead zone boundaries (the sets in the field of filtration where the pressure gradient unit is less than the limit one, the flow is absent) of the well rate.

The results of numerical experiments are presented on Fig. 1-5 which show the boundaries of stagnant zones. The filtration areas is an L-shaped and cruciform one. Figure 1-3 show two wells: the mining one (q) and injection one (-q), Fig. 4 and 5 demonstrate 4 production wells. $\beta = 1$, $g(\xi) = (\xi-\beta)^{p-1}$ were chosen during calculations at r $\xi \ge \beta$. As can be seen, the stagnant zones are reduced with the flow rate q increase. The obtained results correspond to the expected flow pattern.

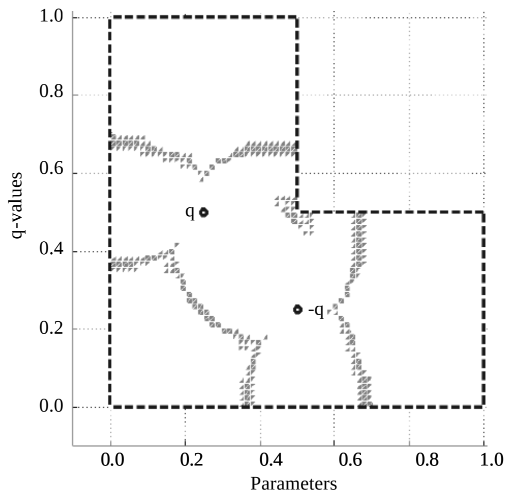

Fig. 3: L- shaped area, q = 1. Stagnant zone boundaries

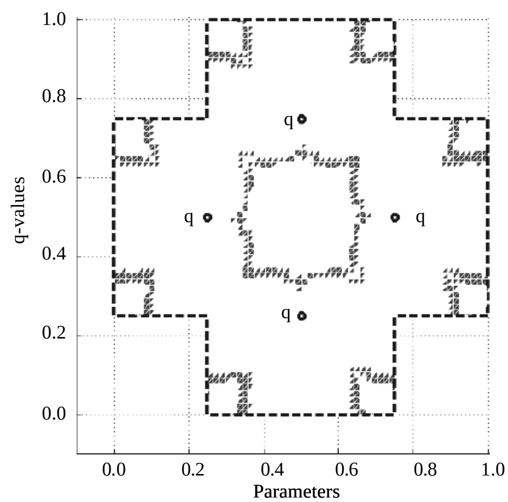

Fig. 4: Cruciform area, q = 1. Stagnant zone boundaries

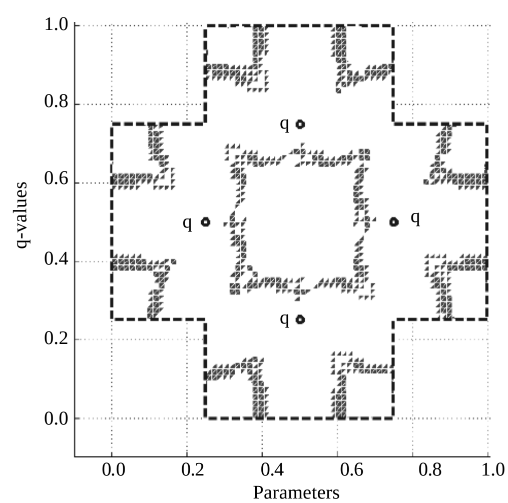

Fig. 5: Cruciform area, q = 0.7. Stagnant zone boundaries

It should be noted that for p = 2 (Hilbert space case), the results correspond to the results obtained by splitting method (Badriev *et al*., 2004; Badriyev *et al*., 2009; Badriev and Fanyuk, 2012).

Let's note also that the methods considered in this paper methods may be used to solve the problems of soft mesh membrane issues (Badriev and Banderov, 2014a, b; Badriev *et al*., 2013).

RESULTS AND DISCUSSION

Thus, the proposed iterative process allows to perform the solution of important problems for the rational mining of highly viscous hydrocarbons. The obtained results experimentally confirm, the theoretical conclusions about the convergence of iterative methods.

CONCLUSION

According to the stated above one may conclude that the proposed iterative process allows to calculate the basic characteristics of highly viscous hydrocarbon deposits in particular to define the boundaries of stagnant zones.

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