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# Numerical Solution of the Issue about Geometrically Nonlinear Behavior of Sandwich Plate with Transversal Soft Filler

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**Abstract:** One-dimensional geometrically nonlinear problem of stability loss mixed forms for outer layers of a sandwich plate composed of two carrier layers and disposed there between transversely with a soft filler related to carrier layers with adhesive bonding at face axial compression of one outer layer among other outer ones. We assume that the edges of the plate carrier layers are hinged. The problem is described by the system of nonlinear differential equations. Using the method of summation identities the finite-difference problem approximations were developed. In order to solve the difference scheme a two-layer iterative process was used with the lowering of non-linearity to the lower layer. The central place is occupied by the determination of critical bifurcation points and respective critical loads. The bifurcation points are determined as the points of branching for the problem solution. These points may be found by the linearization of nonlinear equations in some area of solution. At that the need to address a non-linear (quadratic) eigenvalue problem on eigen values appears. The set of programs was developed in Matlab for the numerical realization of the proposed iterative method. The numerical experiments were performed for the model problem. An optimal iteration parameter (by the number of iterations) is selected empirically. In order to solve polynomial (quadratic) issue on eigenvalues the Matlab medium was used. As the result of numerical experiments, the dependence of end load on the deflection at the central point of the carrier layer was developed. The behavior of the plate near the critical point which is the bifurcation point is studied. The critical loads are determined. It is established that the result of a geometrically nonlinear problem solution by tabulating according to kinematic loading parameter as well as the linearized problem in the area of a nonlinear problem solution have almost identical values of the critical load.

**Key words:** Sandwich plate, geometric nonlinearity, transversal-soft filler, iterative method, numerical experiment

## INTRODUCTION

In this study we consider the problem of the stress-strain state determination for a sandwich plate with transversal-soft filler (Fig. 1). The kinematic relations for the filler are produced by successive integration over the transverse coordinate of initial three-dimensional equations of elasticity theory (Paimushin and Bobrov, 2000, Paimushin, 1987, 1999) previously simplified by the introduction of the assumption about the equality to zero of the tangential stress components (Paimushin, 2007; Berezhnoi and Paimushin, 2011; Badriev *et al*., 2015). It is believed that the edges of the second plate carrier layers are pivotally secured and the end load makes an impact on the first layer.

The finite-difference approximation of the problem is developed for the approximate solution. The numerical solution is carried out using a two-layer iteration method (Badriev and Banderov, 2014a, b) with a preconditioner which is a linear part of the developed difference scheme operator.

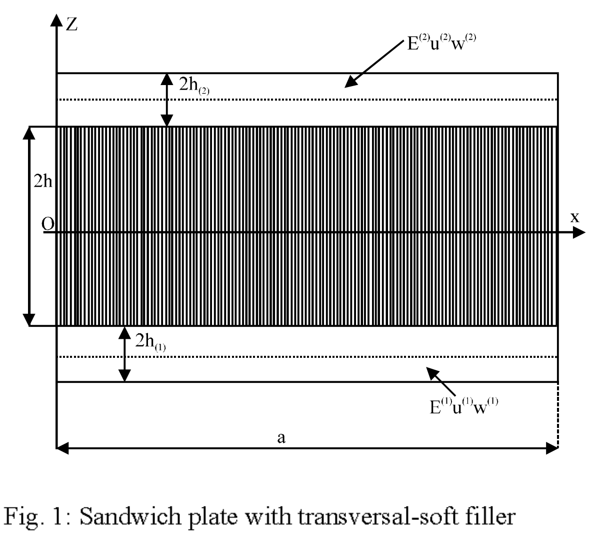


Fig. 1: Sandwich plate with transversal-soft filler

On the basis of the developed set of programs in Matlab the numerical experiments for the model problem are performed. By the tabulation according to the parameter

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of the kinematic load the load behavior and the deflection in the central part of the first carrying layer was monitored. The critical load is determined after the passage of which the deflections at the center point continued to expand at the decreasing values of the external load. Let's note that various geometric (Sagdatullin and Berezhnoi, 2014; Berezhnoi *et al*., 2014; Sultanov and Davydov, 2014) and physical (Sultanov, 2014; Badriev *et al*., 2014) setting of non-linear problems in the theory of shells were considered, including the cases of final deformations (Badriev, 2013; Badriev and Shagidullin, 1995; Badriev *et al*., 2001) as well as the approximate methods of their solution (Badriev and Karchevskii, 1994; Badriev and Zadvornov, 2003; Badriev *et al*., 2013). The nonlinear issues on eigen values were considered by Zheltukhin *et al*. (2014).

## MATERIALS AND METHODS

**Problem statement:** The problem of stress-strain state determination for a sandwich plate is considered with the transversal-soft filler. The problem of a sandwich plate equilibrium with transversal-soft filler in one-dimensional geometrically nonlinear formulation is described by the following system of differential equations:

$$\begin{cases} dT^{11}_{(1)}/dx + X^1_{(1)} + q^1 = 0, \quad dT^{11}_{(2)}/dx + \\ X^1_{(2)} - q^1 = 0,\ 0 < x < a, \\ dS^1_{(1)}/dx + c_3(w^{(2)} - w^{(1)}) + X^3_{(1)} = 0,\ dS^1_{(2)}/dx - \\ c_3(w^{(2)} - w^{(1)}) + X^3_{(2)} = 0,\ 0 < x < a, \\ u^{(1)} - u^{(2)} - H_{(1)}\dfrac{dw^{(1)}}{dx} - H_{(2)}\dfrac{dw^{(2)}}{dx} + \\ \dfrac{q^1}{G_{13}}2h - \dfrac{2h^3}{3E_3}\dfrac{d^2q^1}{dx^2} = 0,\ 0 < x < a \end{cases} \qquad (1)$$

Where:

| | | |
|---|---|---|
| a | = | The plate length |
| 2h and $2h_{(k)}$ | = | The filler and base layer thickness |
| $T^{11}_{(k)}$ | = | Membrane efforts |
| $X^1_{(k)}$ and $X^3_{(k)}$ | = | The surface load components, brought to the median surface |
| $w^{(k)}$ and $u^{(k)}$ | = | Deflections and axial displacements of the middle surface points |
| $S^1_{(k)}$ | = | Generalized cutting forces |
| $G_{13}$ and $E_3$ | = | The modules of transverse shear and a filler compression |
| $q^1$ | = | Stress tangents in the filler |

Where (let's assume that k = 1, 2, (k) index denotes values, related to k layer) $H_{(k)} = h_{(k)} + h$.

$$S^1_{(k)} = dM^{11}_{(k)}/dx + T^{11}_{(k)}\omega^{(k)} + M^1_{(k)} + H_{(k)}q^1,$$
$$T^{11}_{(k)} = B_{(k)}(du^{(k)}/dx + \frac{1}{2}(\omega^{(k)})^2)$$

Where:

| | | |
|---|---|---|
| $\omega^{(k)} = dw^{(k)}/dx$ | = | The angles of normal rotation to the middle surface $c_3 = E_3/(2h)$ |
| $M^{11}_{(k)} = -D_{(k)}$ $d^2w^{(k)}/dx^2$ | = | The internal bending moments |
| $M^1_{(k)}$ | = | Surface moment of the external forces brought to the middle surface |
| $B_{(k)} = 2h_{(k)}E^{(k)}/(1-v^{(k)}_{12}v^{(k)}_{21})$ | = | Stiffness in tension and compression |
| $E^{(k)}$ | = | The elasticity module of the first kind |
| $D_{(k)} = B_{(k)}h^2_{(k)}/3$ | = | The flexural stiffness |
| $v^{(k)}_{12}$, $v^{(k)}_{21}$ | = | Poisson's ratios of the material |

We assume that the edges of the plate carrying layers are hinged, so Eq. 1 are supplemented by boundary conditions (at x = 0, x = a, k = 1, 2):

$$B_{(1)}\left(\frac{du^{(1)}}{dx} + \frac{1}{2}(\omega^{(1)})^2\right) = p,$$
$$B_{(2)}\left(\frac{du^{(2)}}{dx} + \frac{1}{2}(\omega^{(2)})^2\right) = 0, \qquad (2)$$
$$w^{(k)} = \frac{d^2w^{(k)}}{dx^2} = 0,\ \frac{dq^1}{dx} = 0$$

where, p is the end load. It should be noted that the boundary conditions formulated for $q^1$ (Eq. 2) correspond to the presence on the edges x = 0, x = a diaphragms.

**Difference scheme:** In order to solve the problem (Eq. 1, 2) the dual layer iterative process proposed by Badriev *et al.* (2014) is used with the lowering of nonlinearity to the lower layer. However, this analysis provides information only about the possible stable equilibrium positions. One should find bifurcation points which lead to the need of finding the approximate, simplified ways of the system behavior study (Paimushin and Bobrov, 2000) under the action of the load applied thereto in order to search the unstable equilibrium positions. For the preliminary approximate solution of the task by the method of summation identities (Karchevsky and Lyashko, 1976; Samarsky and Andreev, 1976; Weinberg, 1972), its finite difference approximations were developed. To this end, we introduce uniform grids on the interval [0, a] with the step:

$$h_e = a/N: \overset{\circ}{\omega} = \{x_i, i = 1, 2, \ldots, N-1\},$$
$$\omega = \{x_i, i = 0, 1, 2, \ldots, N\},$$
$$\bar{\omega} = \{x_i, i = -1, 0, \ldots, N+1\}$$

Let's note that:

$$v_{x,i} = (v_{i+1} - v_i)/h_e,\ v_{\bar{x},i} = (v_i - v_{i-1})/h_e,\ v_{\mathring{x},i} = (v_{i+1} - v_{i-1})/(2h_e)$$

Then, the difference scheme is written as follows (for difference functions we leave the same notations as for the differential ones $w^{(k)}$ are determined on $\bar{\omega}$, the functions $u^{(k)}$, $q^1$ are determined on $\omega$):

$$-D_{(k)}w^{(k)}_{\bar{x}x\bar{x}x,i} + B_{(k)}((u^{(k)}_{\bar{x}} + \tfrac{1}{2}(w^{(k)}_{\bar{x}})^2)w^{(k)}_{\bar{x}})_{x,i} + H_{(k)}q^1_{\mathring{x},i} + (3-2k)$$
$$c_3(w^{(2)}_i - w^{(1)}_i) + M^{(1)}_{(k)\mathring{x},i} + X^{(3)}_{k,i} = 0,\quad i = 1,2,\dots,N-1,\quad k = 1,2,$$

$$B_{(k)}((u^{(k)}_{\bar{x}} + \tfrac{1}{2}(w^{(k)}_{\bar{x}})^2)w^{(k)}_{\bar{x}})_{x,i} + (3-2k)q^1_i + X^{(1)}_{k,i} = 0, i = 1,2,\dots,N-1, k = 1,2,$$
$$(u^{(2)}_i - u^{(1)}_i) - H_{(1)}w^{(1)}_{\mathring{x},i} - H_{(2)}w^{(2)}_{\mathring{x},i} + \frac{2h}{G_{13}}q^1_i - \frac{2h^3}{3E_3}q^1_{\bar{x}x,i} = 0,\quad i = 1,2,\dots,N-1$$

The boundary conditions (Eq. 2) are approximated in the following way:

$$w^{(k)}_0 = 0,\ w^{(k)}_{\bar{x}x,0} = 0,\ w^{(k)}_N = 0,\ w^{(k)}_{\bar{x}x,N} = 0,\ B_{(1)}((u^{(1)}_{\bar{x},1} + \tfrac{1}{2}(w^{(1)}_{\bar{x},1})^2 = -p,\ B_{(2)}((u^{(2)}_{\bar{x},N} + \tfrac{1}{2}(w^{(2)}_{\bar{x},N})^2 = 0,\ q^1_{x,0} = 0,\ q^1_{\bar{x},N} = 0$$

Let's denote:

$U$ = $(w^{(1)}, w^{(2)}, u^{(1)}, u^{(2)}, q^1)$

$V_{2h}$ = The set of difference functions z, determined on $\bar{\omega}$, such that $z_0 = 0$, $z_{\bar{x}x,1} = 0$, $z_N = 0$, $z_{\bar{x}x,N} = 0$, $w_{x,N} = 0$

$V_{1h}$ = The set of difference functions $\eta$, determined on $\omega$, $V_h = V_{1h}\times V_{1h}\times V_{2h}\times V_{2h}\times V_{1h}$

Let's consider the difference operators $A^{(k)}_{ju}: V_h \to V_{1h}$, $A^{(k)}_{jw}: V_h \to V_{2h}$, $k = 1,2$, $A_{jq}: V_h \to V_{1h}$, $j = 1,2$ according to the formulae:

$$A^{(k)}_{1u}U(x) = B_{(k)}u^{(k)}_{\bar{x}x} + (3-2k)q^1,\ k = 1,2,\ x \in \mathring{\omega}$$

$$A^{(k)}_{1w}U(x) = -D_{(k)}w^{(k)}_{\bar{x}x\bar{x}x} + H_{(k)}q^1_{\mathring{x}} + (3-2k)c_3(w^{(2)} - w^{(1)}),\ k = 1,2,\ x \in \mathring{\omega},$$

$$A_{1q}U(x) = -(u^{(2)} - u^{(1)}) - H_{(1)}w^{(1)}_{\mathring{x}} - H_{(2)}w^{(2)}_{\mathring{x}} + \frac{2h}{G_{13}}q^1 - \frac{2h^3}{3E_3}q^1_{\bar{x}x},\ x \in \mathring{\omega},$$

$$A^{(k)}_{2u}U(x) = -B_{(k)}((u^{(k)}_{\bar{x}} + \tfrac{1}{2}(w^{(k)}_{\bar{x}})^2)\, w^{(k)}_{\bar{x}})_x,\ k = 1,2,\ x \in \mathring{\omega},$$

$$A^{(k)}_{2w}U(x) = -\tfrac{1}{2}B_{(k)}((w^{(k)}_{\bar{x}})^2)_x,\ k = 1,2,\ x \in \mathring{\omega},\ A_{2q}U(x) = 0,\quad x \in \mathring{\omega},$$

and the function:

$$F = (f^{(1)}_w, f^{(2)}_w, f^{(1)}_u, f^{(2)}_w, f_q) \in V_h,\ f^{(k)}_w = M^{(1)}_{(k)\mathring{x}} + X^{(3)}_k,\ f^{(k)}_u = -X^{(1)}_k,\ k = 1,2,\ f_q = 0 \text{ on } \mathring{\omega}, f^{(1)}_{u,0} = -p$$

Let's note that $A^{(k)}_{1w}$, $A^{(k)}_{1u}$, $A_{1q}$ are the linear operators, $A^{(k)}_{2w}$, $A^{(k)}_{1u}$ are the non-linear ones. Then the difference scheme may be put down as follows:

$$AU = (A_1 + A_2)U = F,\ A_j = (A^{(1)}_{jw}, A^{(2)}_{jw}, A^{(1)}_{ju}, A^{(2)}_{ju}, A_{jq}),\ j = 1,2 \qquad (3)$$

**Tabulating by kinematic load:** In order to solve the difference scheme Eq. 3, the following two-layer iterative process was used with the lowering of nonlinearity to the bottom layer (Badriev *et al*., 2014):

$$A_1(U^{(n+1)} - U^{(n)})/\tau + (A_1 + A_2)U^{(n)} = F \qquad (4)$$

Where:

$U^{(0)}$ = The set initial approximation

$\tau>0$ = An iteration parameter

It should be noted that during the tabulation along the transverse load p, set in the boundary condition Eq. 2, the iteration process stops to converge at some large value of the load. So, instead of this condition the following boundary conditions corresponding to the kinematic loading were introduced: $u^{(1)}(0) = U_0$, $u^{(1)}(a) = -U_0$.

**Setting of a problem on eigen values:** In this study, the central place is occupied by the determination of critical bifurcation points and the corresponding critical loads. The bifurcation points are defined as the points of the problem solution branching (Eq. 1, 2). These points may be found by the linearization of the nonlinear equations in some area of solutions.

The basic idea of bifurcation point determination via homogeneous linearized equations is as follows. Suppose that one some form of the system equilibrium is known (in fact, this form was considered by Badriev *et al.* (2015)) and it is necessary to find the bifurcation point for this form of equilibrium. To do this, it's enough without being interested in the behavior of the system away from the well-known form of equilibrium:

$$U = (\overset{0}{w}{}^{(1)}, \overset{0}{w}{}^{(2)}, \overset{0}{u}{}^{(1)}, \overset{0}{u}{}^{(2)}, \overset{0}{q}{}^{1})$$

and find the terms of another form existence U+ΔU, differing from the initial one but infinitely close to it: A(U+ΔU) = F, i.e., the point, the area of which has such a balance form will be a bifurcation point where $\Delta U = (w^{(1)}, w^{(2)}, u^{(1)}, u^{(2)}, q^1)$ is a small increment. Let B is the operator A Frechet derivative, i.e. (Weinberg, 1972) A(U+ΔU) = AU+B(U)ΔU+R(U+ΔU) where the remaining member R(U, ΔU)→0 at ΔU→0. Then AU + B(U)ΔU-F→0 at ΔU→0. Therefore for small ΔU we obtain that:

$$B(U)\Delta U \approx 0 \tag{5}$$

The system of Eq. 5 is the following one for the considered problem:

$$-D_{(k)}\frac{d^4w^{(k)}}{dx^4} - \frac{d\overset{0}{w}{}^{(k)}}{dx}q^1 - \overset{0}{q}{}^1\frac{dw^{(k)}}{dx} + \overset{0}{T}{}^{11}_{(k)}\frac{d^2w^{(k)}}{dx^2} + \frac{d^2\overset{0}{w}{}^{(k)}}{dx^2}B_{(k)}\left[\frac{du^{(k)}}{dx} + \frac{dw^{(k)}}{dx}\frac{d\overset{0}{w}{}^{(k)}}{dx}\right] + $$
$$+ H_{(k)}\frac{dq^1}{dx} + (3-2k)c_3(w^{(2)} - w^{(1)}) = 0, \quad k = 1, 2;$$

$$B_{(k)}\left[\frac{d^2u^{(k)}}{dx^2} + \frac{d^2\overset{0}{w}{}^{(k)}}{dx^2}\frac{dw^{(k)}}{dx} + \frac{d\overset{0}{w}{}^{(k)}}{dx}\frac{d^2w^{(k)}}{dx^2}\right] + (3-2k)q^1 = 0, \quad k = 1, 2;$$

$$(u^{(1)} - u^{(2)}) - H_{(1)}\frac{dw^{(1)}}{dx} - H_{(2)}\frac{dw^{(2)}}{dx} + \frac{q^1}{G_{13}}2h - \frac{2h^3}{3E_3}\frac{d^2q^1}{dx^2} = 0$$

Let's consider the following auxiliary problem $B(\lambda\tilde{U})\Delta U = 0$, $\tilde{U} = U/p$ or:

$$(L(\tilde{U}) + \lambda\Phi(\tilde{U}) + \lambda^2\Psi(\tilde{U}))\Delta U = 0 \tag{6}$$

which is the task for its own values. Here, the same designations are introduced for L, **Φ**, **Ψ** as for F at that:

$$L_w^{(k)}(\tilde{U})\Delta U = -D_{(k)}\frac{d^4w^{(k)}}{dx^4} + H_{(k)}\frac{dq^1}{dx} + (3-2k)c_3(w^{(2)} - w^{(1)}), \quad k = 1, 2$$
$$L_u^{(k)}\Delta U = B_{(k)}\frac{d^2u^{(k)}}{dx^2} + (3-2k)q^1, \quad k = 1, 2$$
$$L_u^{(q)}\Delta U = (u^{(1)} - u^{(2)}) - H_{(1)}\frac{dw^{(1)}}{dx} - H_{(2)}\frac{dw^{(2)}}{dx} + \frac{q^1}{G_{13}}2h - \frac{2h^3}{3E_3}\frac{d^2q^1}{dx^2}$$
$$\Phi_w^{(k)}(\tilde{U})\Delta U = \frac{d\tilde{w}^{(k)}}{dx}q^1 - (3-2k)\,\tilde{q}^1\frac{dw^{(k)}}{dx} + \tilde{T}^{11}_{(k)}\frac{d^2w^{(k)}}{dx^2} + \frac{d^2\tilde{w}^{(k)}}{dx^2}\frac{du^{(k)}}{dx}, \quad k = 1, 2, \tag{7}$$
$$\Phi_u^{(k)}(\tilde{U})\Delta U = \frac{d^2\tilde{w}^{(k)}}{dx^2}\frac{dw^{(k)}}{dx} + \frac{d\tilde{w}^{(k)}}{dx}\frac{d^2w^{(k)}}{dx^2}, \ \Phi_q^{(k)}(\tilde{U})\Delta U = 0, \quad k = 1, 2$$
$$\Psi_w^{(k)}(\tilde{U})\Delta U = B_{(k)}\frac{d^2\tilde{w}^{(k)}}{dx^2}\frac{d\tilde{w}^{(k)}}{dx}\frac{dw^{(k)}}{dx}, \Psi_u^{(k)}(\tilde{U})\Delta U,\ k = 1, 2, \Psi_q(\tilde{U})\Delta U = 0a$$

## RESULTS AND DISCUSSION

**Numerical solution of geometrically non-linear problem by tabulation via kinematic load:** A set of programs was developed in Matlab environment for the numerical implementation of the iterative method (Eq. 4). The numerical experiments were performed for the model problem. The iterative parameter was selected empirically. The calculations were performed for the following characteristics: $a = 1$ cm, $h_1 = h_2 = 0.005$ cm, $h = 0.05$ cm, $G_{13} = 15$ MPa, $E_3 = 25$ MPa, $X^3_{(1)} = 0.0319$ MPa, $X^3_{(2)} = 0$, $E^{(k)} = 7.10^4$ Mpa, $v^{(k)}_{12}$, $v^{(k)}_{21} = 0.3$, $X^1_{(k)} = 0$, $M^1_{(k)} = 0$, $k = 1, 2$. The number of grid points makes $N = 100$. The initial approximation $U^{(0)}$ was set as a zero one. The calculations according to Eq. 4 were performed till the standard of misalignment $\|F-(A_1 + A_2) U^{(n)}\|$ remained greater than the determined accuracy $\varepsilon = 5.10^{-8}$. The tabulation of kinematic load, starting with $U_0 = 10^{-6}$, monitored the load behavior $p = -B_{(1)} = (du^{(1)}/dx + (\omega^{(1)})^2/2)$ and the deflections in the central part of the first bearing layer. Figure 2 shows the dependence of the load p on deflection in the central part of the first layer $w^{(1)}$.

Figure 2 shows that the end load begins to decrease with the deflection increase, indicating the critical load, which corresponds to $p_a = 16.65$ MPa. Figure 3-6 show the behavior of the plate near the critical point which is the bifurcation point.

**Numerical solution of quadratic spectral problem:** To solve the polynomial (quadratic) eigenvalue problem Eq. 7, the Matlab medium was used (the integrated function polyieg). Due to Eq. 6 $B(\lambda\tilde{U})\Delta U = 0$, i.e., it is necessary to obtain such an end load value p* that the value of eigen number will be equal to the end load $\lambda = p^*$. In fact a critical end load applied to the first bearing layer and equal to $p = 16.6535$ MPa was

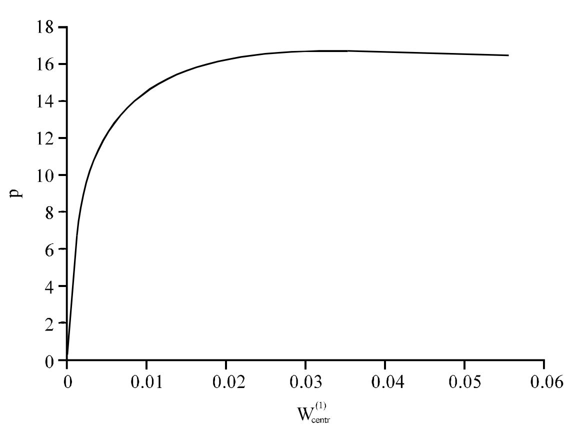


Fig. 2: The dependence of the end load p on deflection $w^{(1)}$ at the center point

determined. The results of numerical experiments are shown by the Fig. 7-10. Figure 7-9 demonstrate the graphs

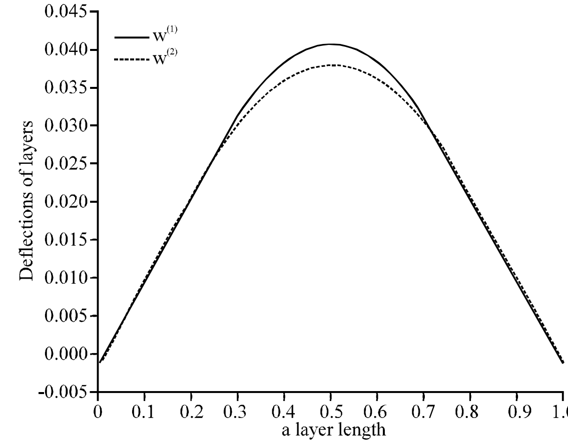


Fig. 3: Bearing layer deflections

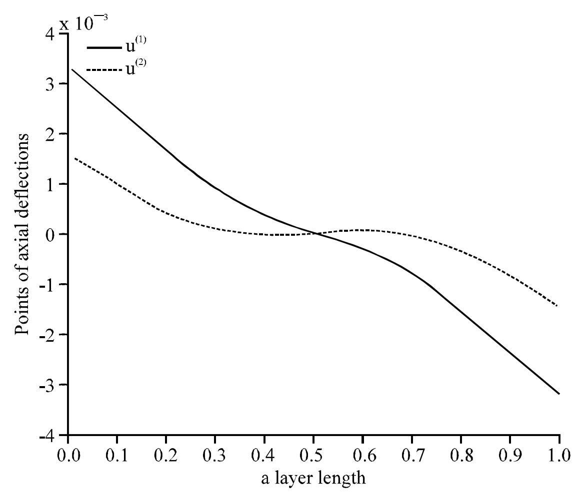


Fig. 4: Axial displacements of bearing layers

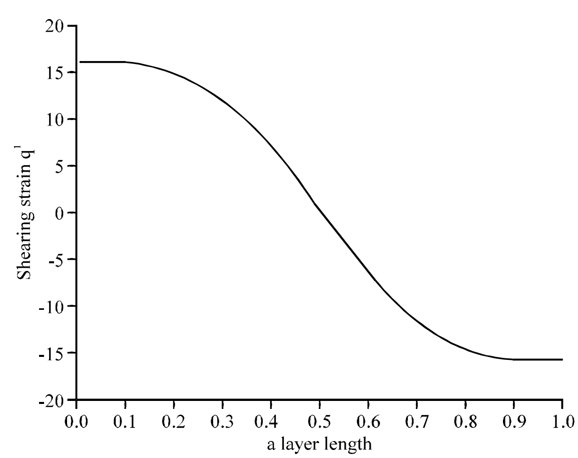


Fig. 5: Tangent stresses in the filler

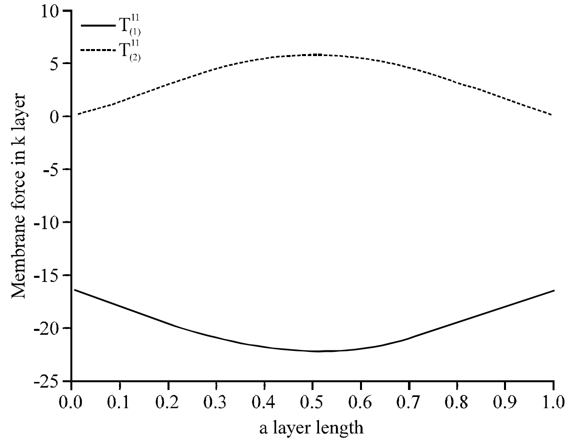


Fig. 6: Membrane efforts in bearing layers

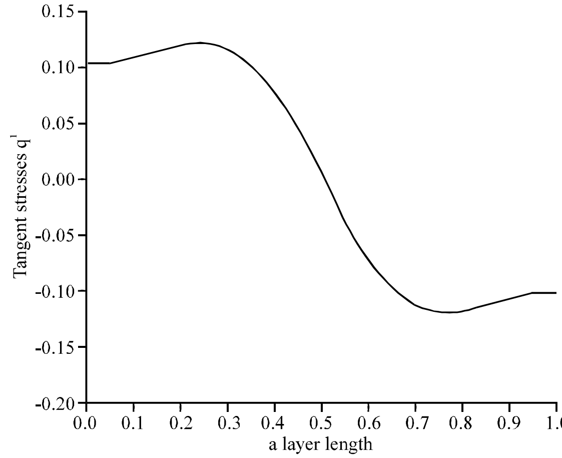


Fig. 9: Eigen function of tangent stresses

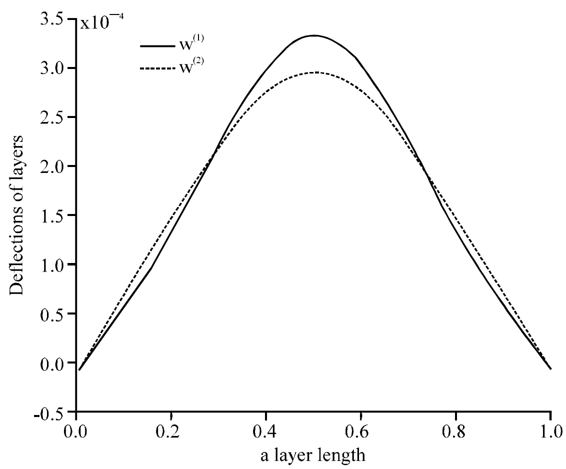


Fig. 7: Eigen function of deflections

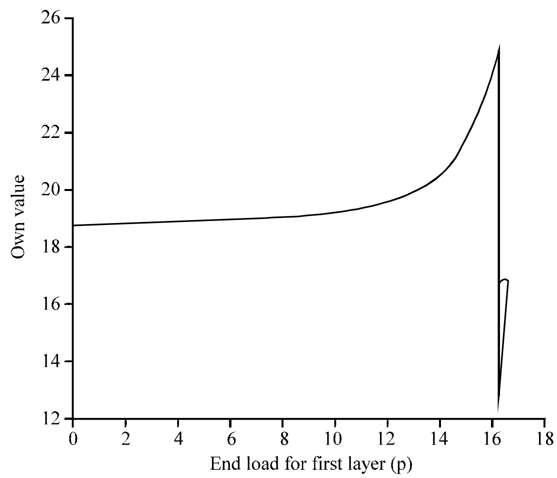


Fig. 10: Eigen function of axial displacements

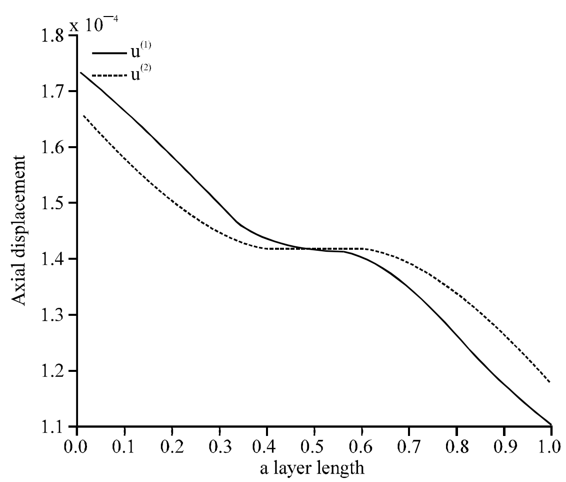


Fig. 8: Eigen function of axial displacements

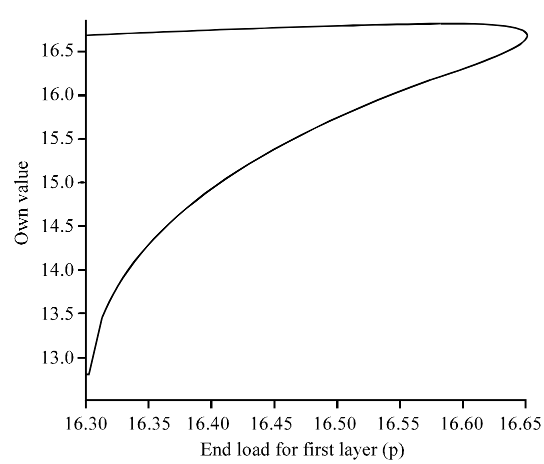


Fig. 11: Eigen function of end load for first layer

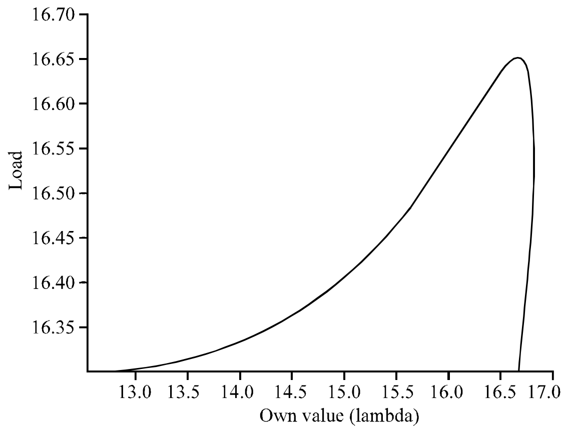


Fig. 12: Eigen function of own value (lambda)

of eigen functions for a quadratic eigenvalue problem Eq. 7. The behavior of their own values was also analyzed at different load values on Fig. 10. The dependence of the eigenvalue was demonstrated from the applied end load. A more detailed graph fragment after a sharp fall is depicted by Fig. 11. Figure 12 shows the inverse function to the function of Fig. 11. It is easily seen that at the load equal to $p_{bif} = 16.6535$, the eigenvalue makes $\lambda = 16.6535$. Let's note that solving a geometrically nonlinear problem (tabulating it by kinematic load) and a linearized problem in a non-linear area (the quadratic problem on eigenvalue), we obtained approximately the same load values at the bifurcation point ($p_a = 16.6535$ MPa, $p_{bif} = 16.6535$ MPa).

## SUMMARY

One of the main methods for three-layer structural elements production is a bonded joint of external carrier layers with filler which may often be accompanied by the appearance of technological defects on the mating surface layers in the form of disbonded places. The study of deformation processes for such elements is primarily dictated by the need to determine their suitability for further use. In this study, we consider the problem of determining the stress-strained state of a sandwich plate with a transversal-soft filler. Therefore, the methods proposed in this study are relevant both from a theoretical point of view and in terms of possible applications.

## CONCLUSION

The methods proposed in this paper concerning the determination of loss forms can be widely used in the design of structures made of laminated plates. The obtained results show the effectiveness of the proposed methods.

## ACKNOWLEDGEMENTS

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