Research Journal of Applied Sciences 10 (11): 728-733, 2015
ISSN: 1815-932X

Application of the Precise Isochrome Equation in the Monoaxial Crystals Optical Homogeneity Metrology

A.I. Kolesnikov, I.A. Caplunov, S.A. Tretyakov,
K.A. Morozova D.S. Kutuzov and N.V. Aydinyan
Tver State University, Zhelyabova St., 33, 170100 Tver, Russia

Abstract: On the basis of the precise equation isochrome analysis application in researches of monoaxial crystals by conoscopy method the analysis of the method application for monocrystals test evaluation is performed. On the basis of isochrome pictures calculations combination and applications of optical researches it is possible to receive the most informative monoaxial crystal conoscopic picture. By the pilot studies of paratellurite and niobate of lithium monocrystals with laser application technical capabilities of conoscopic research method of optical anomalies and tension in monocrystals are defined. Summing up the research result of the application opportunities analysis of the precise equation isochrome of monoaxial crystals by a conoscopy method it is possible to note that with its help all issues concerning definition of crystal surfaces crystallographic orientation are resolved. After predesigns by means of Eq. 1 it is possible, combining element parameters of the optical scheme to resolve issues concerning definition of crystal surfaces crystallographic orientation.

Key words: Isochrome, monoaxial crystals, conoscopic pictures, index of refraction, optical anomalies, paratellurite

INTRODUCTION

Recently published (Kolesnikov *et al*., 2015) precise without the approximations applied earlier-isochrome equation of monoaxial crystals allows to expand possibilities of conoscopy method in metrology of optical homogeneity of such crystals, among which many are important for optics and optoelectronics of materials.

Despite perfecting of cultivation technologies for the majority of monoaxial crystals increasing structural perfection and optical homogeneity is desirable. The dislocation density in the best crystals seldom turns $<10^3$ cm^2 and average is within (10^3-10^5) cm^2. In crystals such defects and anomalies as variations of refraction index, non-homogeneously entrapped impurity, gas bubbles (Punin and Shtukenberg, 2005; Foldvari *et al*., 1982) internal waviness, the abnormal biaxiality are often observed. They distort optical indicatrix of a crystal that is shown in distortions of the interference figures received by means of Conoscopy Method. The most sensitive to changes of refraction index in conoscopic pictures are isochromes. In order to find, correctly interpret and numerically estimate optical inhomogeneities of isochrome form distortions it is necessary to have a correct representation of theoretical form isochrome which has to be observed in case of ideal crystal of known substance with known principal values of refraction index of N_0 and N_e with ordinary and inordinary waves with the known thickness of h with known orientation of a crystal normal to optical axis determined by angle ψ with a known wavelength λ radiations and also with a known focal distance of f of the optical system projecting images on the observation plane.

Meanwhile, so far the monoaxial crystals isochrome equations were derived with application of a large number of approximations (Born and Wolf, 1985). According to these equations isochrome of monoaxial crystals can only be curves of second order circles, ellipses and hyperboles though in the pilot studies of crystals with high optical homogeneity conducted by conoscopy method on exemplars with an angle between normal and optical axis ψ, not equal to 00° and 90°, isochrome with the form which is not corresponding to curves of the second order appear (Kolesnikov *et al*., 2015, 2013). The approximate equations which are roughly distorting isochrome form at the qualitative level especially are unsuitable for the quantitative researches of isochrome form of the actual crystals always having optical inhomogeneities in volume.

Corresponding Author: A.I. Kolesnikov, Tver State University, Zhelyabova St., 33, 170100 Tver, Russia

The interest in conoscopy method which increased in recent years is bound not only for new appendices (Tretiakov *et al*., 2015; Saito *et al*., 2000; Sirat and Psaltis, 1985) but also for application as visible band source of laser radiation (Kolesnikov *et al*., 2013). The image from the screen is fixed by the digital camera and is researched by means of computer methods. Thus, the possibility of obtaining the conoscopic pictures of large-size monocrystals which is absent in case of application of microscopes is realized. Mathematical processing of the received images requires the precise theory isochrome monoaxial crystals.

The objective of the current work was determining the technical capabilities given by theprecise isochrome research theory of optical homogeneity of various sizes monoaxial crystals and with the various orientation of normal towards surface of optical axis.

THE TECHNICAL CAPABILITIES ANALYSIS OF THE CONOSCOPY METHOD BY MEANS OF ISOCHROME EQUATION

Received by Kolesnikov *et al.* (2015) precise isochrome equation of monoaxial crystal in definitions given above (N_o, N_e, h, ψ, λ, f) is the following:

$$(N_0^2-N_e^2)\left[\frac{Y\sin\psi}{m\lambda\sqrt{X^2+Y^2+f^2}/h+\sqrt{N_0^2(X^2+Y^2+f^2)-X^2-Y^2}}+\cos\psi\right]^2=$$
$$N_0^2\left[\frac{X^2+Y^2-N_e^2(X^2+Y^2+f^2)}{\left(m\lambda\sqrt{X^2+Y^2+f^2}/h+\sqrt{N_0^2(X^2+Y^2+f^2)-X^2-Y^2}\right)^2}+1\right] \qquad (1)$$

where, X and Y are coordinates of a point, belonging to isochrome (m of order maximum) lying in the observation plane. The coordinate of Y in this plane corresponds to projection of crystal optical axis. In case of the normal coinciding with an axis (ψ = 0) isochrome represents family of circles with the centers in crossing point of a normal and axis within the observation plane. Monoaxial crystal isochrome in case of coincidence of a normal to an optical axis are circles which radiuses, R_m calculated by means of Eq. 2 received from Eq. 1:

$$R_m = fN_e\{4h^2m\lambda N_0^2N_e^2-m^3\lambda^3N_e^2-h^2m\lambda N_0^2-h^2m\lambda N_e^2+$$
$$2\sqrt{h^4N_0^2(h^2N_0^4-2h^2N_0^2N_e^2+h^2N_e^4+m^2\lambda^2N_e^2)m\lambda}/m^4\lambda^4N_e^4-$$
$$4h^2m^2\lambda^2N_0^2N_e^4+2h^2m^2\lambda^2N_0^2N_e^2+2h^2m^2\lambda^2N_e^4+h^4N_0^2-$$
$$2h^4N_0^2N_e^2+h^4N_e^4\}^{1/2} \qquad (2)$$

The analysis of Eq. 1 leads to the following conclusions which are falling into form of curves isochromes and also to a symmetry of isochrome picture in general:

- The isochrome equation of monoaxial crystal in the most general case is the equation of the eighth degree (sum total of indeterminate)
- Only at coincidence of a normal to a crystal with optical axis (ψ = 0) isochromes are curves of the second order circle. There can be no other isochrome-curves of the second order hyperboles, ellipses and parabolas in a conoscopic picture of monoaxial crystal
- Only in case when the optical axis is orthogonal to normals towards crystal (ψ = 90°) isochromes are the quartic curves just superficially resembling hyperboles
- The isochrome picture is generally symmetric to axis Y a projection of an optical axis to the observation plane and is not symmetric to axis X

In Fig. 1 forms calculated according to the isochrome (Eq. 1) for monoaxial crystals of paratellurite with thickness h = 1 cm are presented, lit by conic light beam of the helium-neon laser with radiation wavelength λ = 0.6328 microns, thus, the conoscopic picture is projected on the observation plane by means of the optical system having a focal distance of f = 20 cm. Isochrome pictures are given for the crystals having corners between a normal and surfaces and an optical axis λ = 0°, 10°, 75° and 90°. For receiving an idea of isochromes of high orders, isochrome through 25 orders on are shown first (m = 1).

With increase in distance from the center of a picture $\rho = (x^2+y^2)^{1/2}$ both orders of maxima and angle of incidences of beams α increase by a crystal, defined by equation:

$$\alpha = \text{arctg}\left[\frac{\sqrt{x^2+y^2}}{f}\right] \qquad (3)$$

Sensitivity of isochrome form to inhomogeneities in a crystal, i.e., actually to variations of refraction index of ΔN_0 and ΔN_e increases with increase in an order of m. Researches of influence change of each of the parameters entering Eq. 1 on a picture monoaxial crystal isochrome at constancy of other parameters lead to the conclusions concerning technical aspects of Conoscopy Method and ways of optimization of observing conditions and filing of images:

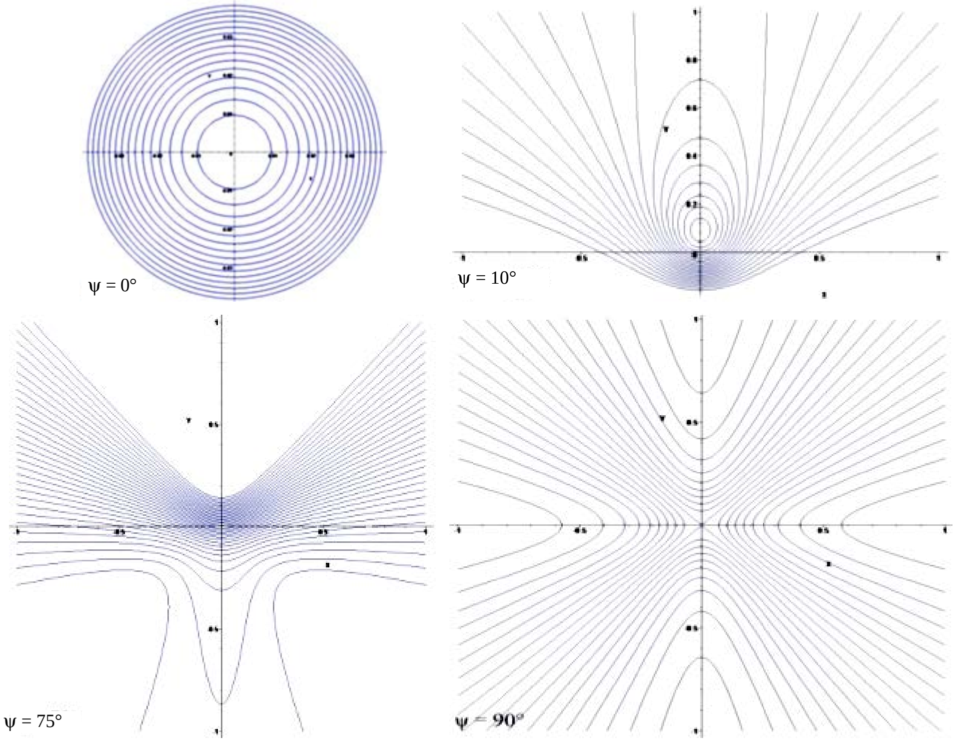

Fig. 1: Theoretical isochrome forms for paratellurite crystals of thickness h = 1 cm with corners between a normal and an axis ψ = 0, 10, 75 and 90°, lit by light with a length λ = 0.6328 mcm for which refraction index of ordinary and inordinary beams N_0= 2.2597, N_e = 2.4119. Focal distance of projective system f = 40 cm

- At increase in thickness of a crystal h the number observed isochrome increases and distances between the next isochromes decrease
- At increase in an angular aperture A (the maximal angle between beams sharing in formation of conoscopic picture) the number of observed isochromesincreases along with simultaneous increase in the area of conoscopic picture
- At increase in the module of difference in the main refraction index ΔN and respectively, the relative double refraction $\Delta N/N_0$, the isochrome number increases

Summing up the research result of the analysis application opportunities of the precise equation for monoaxial crystals isochrome by Conoscopy Method it is possible to note that with its help all issues concerning definition of crystal surfaces crystallographic orientation are resolved. After previous calculations by means of Eq. 1 it is possible, combining combinations of optical scheme elements parameters to receive the most informative conoscopic picture of a large-size monoaxial crystal.

IDENTIFICATION AND ANALYSIS OF OPTICAL ANOMALIES AND VIOLATIONS OF THE GEOMETRICAL FORM OF MONOAXIAL CRYSTALS BY MEANS OF CONOSCOPY METHOD

Experiment technique: In the current research, the large-size monoaxial crystals of paratellurite and niobate of lithium, presented in Fig. 2 are investigated.

For receiving the conoscopic pictures, the isochrome installation which is turning on the laser with a wavelength of the radiation of 533 nanometers, the collimator, polarizer, the lens transforming parallel light beam in conic, the analyzer, the projection lens and the semidiaphanous screen behind which there was a digital camera fixing images was used.

In observed conoscopic pictures use of projective system is not obligatory. In such cases in the analysis of isochrome form in Eq. 1 focal distance of f is replaced with l distance between an output surface of a crystal and the screen.

The matter is that the actual optical systems applied at illumination of crystals never give strictly concentric

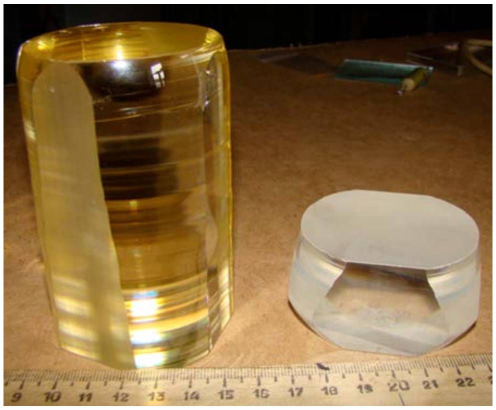

Fig. 2: Large-size monocrystals of lithium niobate (on the left) with a diameter of 70 mm and paratellurite (on the right) with a diameter of 65 mm, studied by Conoscopy Method are presented

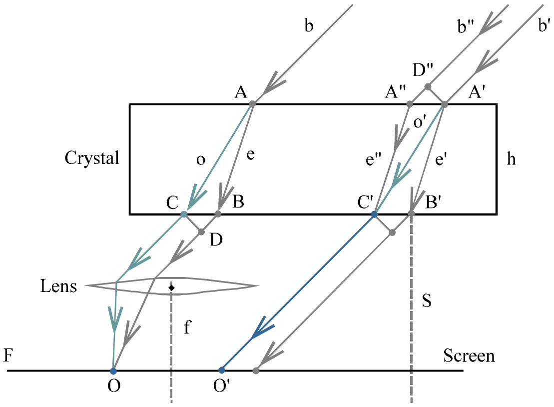

Fig. 3: The explanation of equality of the course difference between ordinary and inordinary beams when receiving a conoscopic picture with the projection lens (on the left) and without it (on the right)

beams of light and it is necessary to describe falling bunch as cone system of terminating thickness for each angle to the common axis of cones.

In the absence of a lens as shown in the right member of Fig. 3, ordinary and inordinary beams, coming from one beam b', certainly do not interfere. But at very small distance from a point of falling A' in some point A' there always will be beam b' parallel to the beam b', the inordinary beam from which e" in a point C' will interfere with an ordinary beam o' from beam b'. Thus, the course difference between beams o' and e" will be the same as course difference between beams o and e in case of lens application (on the left-hand part of drawing). Thus, conoscopic pictures with lens and without lens will be identical.

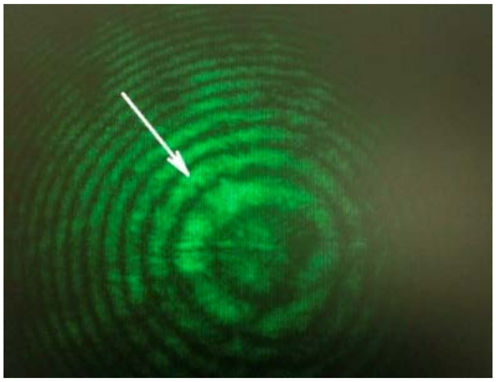

Fig. 4: Breaks on isochromes in the conoscopic picture of a monocrystal of paratellurite 40 mm thick received in the direction of an optical axis in a conic bunch of laser light with a wavelength of 533 nm. The arrow specified the largest break on isochrome of the 2nd order

On change of the isochrome form described by Eq. 1, it is possible to reveal optical inhomogeneity but it is impossible to specify its location in a crystal. It can be in any part of the volume occupied by a cone of beams. In the current research the express methods consisting in several movements and in various directions of a crystal towards cone of beams are developed for defining localization of the found in homogeneity in a large crystal. Computer subtraction of isochrome images allows to find position of optical anomaly with high precision.

Detection of optical anomalies: It is most convenient to investigate optical homogeneity of crystals in the exemplars which are cut out orthogonally to optical axis. In Fig. 4, the conoscopic picture of paratellurite monocrystal received by means of the laser with a wavelength $\lambda = 533$ nanometers and thickness $h = 40$ cm in the direction of an optical axis is presented [001].

Equation 1 gives the chance on changes of isochrome radiuses ΔR, measured in lengths of waves (distance between the next R_i-R_{i+1} isochromes of ordinary and inordinary waves corresponding to change of a course Δ difference on one wavelength λ) to calculate changes of refraction index ΔN_0 and ΔN_e. In Fig. 5, three-dimensional dependence of isochromesituation change ΔR of the second order in the conoscopic picture of a paratellurite monocrystal received in the direction [001] for a radiation wavelength $\lambda = 533$ nanometers from changes of refraction index ordinary ΔN_0 and inordinary ΔN_e beams is shown.

Observed size ΔR corresponds to change of a difference of the course $\Delta\lambda/\lambda(0.25\text{-}0.25)$ that gives an assessment for ΔN_0, ΔN_e 0.01-0.03. By means of a known approximate ratio for mechanical tension σ:

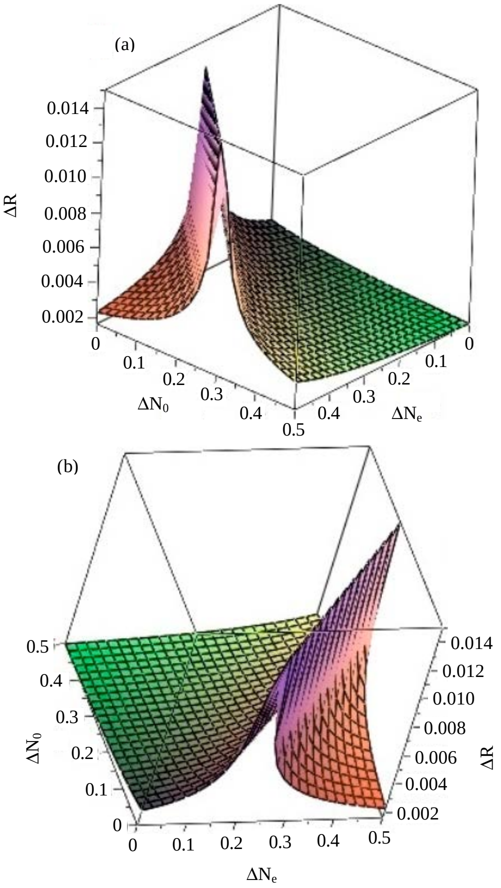

Fig. 5: a, b) Three-dimensional dependence of position change of an isochrome of the second order ($m = 2$) a paratellurite monocrystal in a conoscopic picture for the direction [001] when using light with a wavelength λ = 533 nm from changes of refraction index ordinary ΔN_0 and inordinary beams ΔN_e on Eq. 1

$$\sigma^2 (\Delta n/n)/n^2 ps \qquad (4)$$

where, p and s effective values of stress-optical constants and constants of resilient pliability. The mechanical tension responsible for observed break of an isochrome is estimated in limits σ (1-2). The 10^7 Pa that shows essential violations of ideal structure in some volume of a crystal.

In many crystals optical inhomogeneities an internal waviness are observed. The nature of an internal waviness in crystals is studied insufficiently. In research, it is shown that in paratellurite crystals they represent layers 40-60 μ thick between two flat, rather stretched surfaces. The dislocation density in volumes with an internal waviness of 1-2 orders exceeds average of a crystal. Internal waviness comes to light at illumination of a crystal by plane-parallel bunch of plane-polarized laser light and observation on the screen in a distant zone. On such picture, it is impossible to estimate numerically changes of refraction index in area of internal waviness.

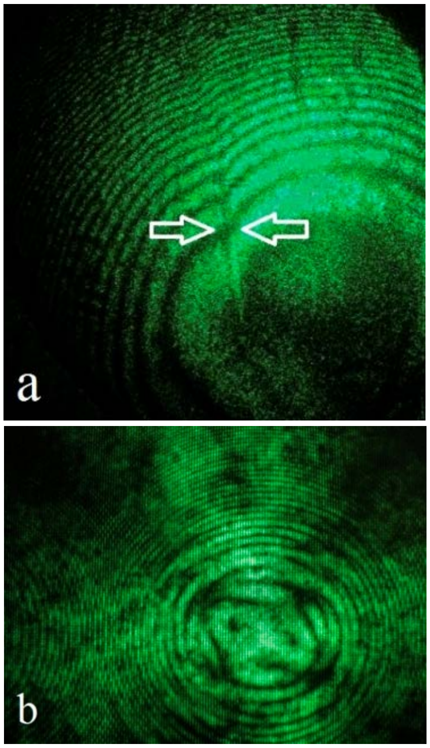

Fig. 6: a) a series of breaks on the next isochromes in area twisted in the conoscopic picture of a monocrystal of paratellurite received on the screen in the direction of an optical axis (Ψ = 0) and b) manifestation of the abnormal biaxiality in the conoscopic picture of a monocrystal of lithium niobate with a diameter of 70 mm and height along the optical axis of 120 mm is received in laser light with a wavelength λ = 533 nm

Most clearly an internal waviness is visible when studying crystals by a conoscopy method. In this case in area of internal waviness the breaks located on the next isochromes on one straight line along the internal waviness are observed according to Fig. 6a.

The assessment by means of Eq. 4 gives variations of refraction index of ΔN value~0.03 and the mechanical tension of value $\sigma = (3\text{-}4)\cdot 10^7$ Pa that points to the strong distortions of crystal structure in area of internal waviness and is the cause for growth technology optimization. Residual mechanical stresses in rather large volumes of monoaxial crystals lead to abnormal biaxiality. In Fig. 6b, the conoscopic picture of theoretically monoaxial large-size monocrystal of lithium niobate received by means of a laser method in the direction of an optical axis is shown.

The angle 2V between the induced axes can reach in the most intense volumes of crystals of paratellurite the maximal values of 2.0-2.5°. At the abnormal biaxiality <10'-15' exits of axes are not resolved on the screen, however on an eccentricity of ellipses which are observed instead of circles it is also possible to calculate small values of angles 2V.

For calculation of mechanical tension σ, responsible for the abnormal biaxiality, the following approximate formula is known:

$$\mathrm{tg}V = \frac{\sqrt[4]{[(\pi_{1\mu}-\pi_{2\mu})\sigma_\mu]^2+(2\pi_{6\mu}\sigma_\mu)^2}}{\sqrt{\left|N_0^{-2}-N_e^{-2}\right|}} \qquad (5)$$

Where:

$\pi_{j\mu}$ = Piezooptical coefficients

V = Half of an angle between the induced axes

The experimental picture presented in Fig. 6 corresponds to monoaxial pressure load of a crystal of paratellurite in the direction [100] (axis X) size $3\cdot10^6$ Pa.

CONCLUSION

Application of the precise monoaxial crystals isochrome equation in the computer analysis of isochrome pictures in the conoscopic pictures received by means of lasers allows to find, classify and define with a great accuracy the location of optical anomalies of various type in any even very large, exemplars. Variations of refraction index, internal waviness, the abnormal biaxiality belong to such anomalies. Thus, defining sizes of mechanical tensions and their signs is possible. Consequently, the combination of the created mathematical apparatus and laser application of radiation sources use increases informational content and sensitivity and opens new opportunities of Conoscopy Method in optical metrology of monoaxial crystals.

ACKNOWLEDGEMENTS

Research is performed in the Tver State University with assistance of the Ministry of Education and Science of the Russian Federation within implementation of the Federal target program “Researches and Development in the Priority Directions of Development of Scientific and Technological Complex of Russia for 2014-2020”, Agreement 14.574.21.0113 (RFMEFI57414X0113).

REFERENCES

Born, M. and E. Wolf, 1985. Principles of Optics. Pergamon Press, New York, pp: 640-648.

Foldvari, L., R. Voszka and A. Peter, 1982. Comments on gas-bubble entrapment in TeO_2 single crystal. J. Crystal Growth, 59: 651-653.

Kolesnikov, A.I., R.M. Grechishkin, S.A. Tretiakov, V.Y. Molchanov and A.I. Ivanova, E.I. Kaplunova and E.Y. Vorontsova, 2013. Laser conoscopy of large-sized optical crystals. IOP Conf. Seri.: Mater. Sci. Eng., Vol. 49. 10.1088/1757-899X/49/1/012037.

Kolesnikov, A.I., S.A. Tretiakov, R.M. Grechishkin, K.A. Morozova and K.B. Yushkov *et al*., 2015. A study of optical uniformity of lithium niobate and paratellurite crystals by the method of conoscopy. Acta Phys. Pol. A., 127: 84-86.

Punin, Y.O. and A.G. Shtukenberg, 2005. Anomalous crystal optics of heterogeneous crystals. Crystallogr. Rep., 50: 297-307.

Saito, K., S. Sugawara, J.Q. Guo and A.P. Tsai, 2000. Determination of quasi-crystallographic orientations of Al-Pd-Mn icosahedral phase by means of light figure method. Jpn. J. Applied Phys., 39: 5173-5176.

Sirat, G. and D. Psaltis, 1985. Conoscopic holography. Optics Lett., 10: 4-6.

Tretiakov, S., R. Grechishkin, A. Kolesnikov, I. Kaplunov and K. Yushkov *et al*., 2015. Characterization of temperature field distribution in lonqe-size paratellurite crystals applied in acousto-optic devices. Acta Phys. Pol. A., 1: 72-74.