

## Application of Tensor Methodologies for the Description of Non-Linear Processes in Electromagnetic Drive

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**Abstract:** The study describes the use of tensor methodology to describe non-linear processes occurring in the electromagnetic AC drives. Application of tensor approach provides increased value for models describing nonlinear processes in objects than, for example with conventional systems of differential equations. The advantage of the tensor approach is the possibility of a generalized description of the essence of the processes occurring in the drives that facilitates the search for solutions of the equations describing the processes of electromagnetic actuators.

**Key words:** Electromagnetic AC drive, magnetic drive model, tensor methodology, non-linear processes, describe

### INTRODUCTION

When you create a model of a non-linear electrical systems based on differential equations (Bakhvalov *et al.*, 2015; Gorbatenko *et al.*, 2015a, b; Lankin *et al.*, 2015; Shaykhutdinov *et al.*, 2013; Shaikhutdinov *et al.*, 2015) it is important to bear in mind that the usual differentiation does not give an adequate mathematical representation of the physical processes occurring non-linear. Even in the simplest case, a conductor with a current, building an adequate model is possible only if you apply the tensor approach (Kron, 1955). From this perspective, any electrical system is the subject of a tensor methodology (Krawczyk and Wiak, 2002). A typical example of a non-linear electrical system is a electromagnetic drive AC (Gorbatenko and Lankin, 1989; Gorbatenko *et al.*, 2004a, b, 2015a; Lankin *et al.*, 2015). The non-linear nature of the processes in such devices due to several reasons. The main reasons are displacement current in an electromagnetic actuator skein-AC and the saturation magnetic steel in the process of magnetization. Both phenomena allow tensor description which is confirmed by studies cited hereinafter.

### MATERIALS AND METHODS

To take into account the effect of the displacement current, each coil present as  $n$  thin wires. As a result, the effect of displacement current magnitude and phase of the current in different thin conductors will vary. In the thin conductors which are located closer to the surface of the base coil, the current will be greater in the thin conductors

located closer to the middle-less. The greater the number of the  $n$ , the less will be the effect of current displacement in each thin conductor. With this in mind, consider the electromagnetic actuator as the AC electrical system, comprising  $n$  interconnected electromagnetic circuits. Let impedance  $k$  branch  $Z_{kk}(s)$  ( $k = 1, 2, \dots, n$ ) such a system is defined as:

$$Z_{kk}(s) = R_{kk} + sL_{kk} + \frac{1}{sC_{kk}}$$

Where:

- $s$  = Complex variable
- $R_{kk}$  = Active resistance  $k$  branch
- $L_{kk}$  = Private inductance  $k$  branch
- $C_{kk}$  = Private capacity  $k$  branch

Between the private inductances of branches  $L_{kk}$  where  $k = 1, 2, \dots, n$  may be electromagnetic communication. These relationships are determined by mutual inductance  $L_{ik}(i \neq k)$ . Also, each branch has a voltage source. Designation voltage  $k$  branch through  $u_k$ . The system of differential equations of balance voltage transients describes is as follows:

$$\begin{cases} \frac{d\psi_1}{dt} + R_{11}i_1 + \frac{1}{C_{11}} \int i_1 dt = u_1 \\ \frac{d\psi_2}{dt} + R_{22}i_2 + \frac{1}{C_{22}} \int i_2 dt = u_2 \\ \dots\dots\dots \\ \frac{d\psi_n}{dt} + R_{nn}i_n + \frac{1}{C_{nn}} \int i_n dt = u_n \end{cases}$$

Where:

- $\Psi_1, \Psi_2, \dots, \Psi_n$  = Flux
- $i_1, i_2, \dots, i_n$  = Current branches
- $R'_{11}, R'_{22}, \dots, R'_{nn}$  = Coefficients that are numerically equal to the active resistance determined DC
- $C_{11}, C_{22}, \dots, C_{nn}$  = Capacitance branches

This system of equations can be fairly high order which depends on the number of circuits non-linear electromechanical system.

This system includes options  $R'_{11}, R'_{22}, \dots, R'_{nn}$  which can be made dependent on the operating mode thereby simulating for example, the effect of current displacement at high frequencies. But it should always be borne in mind that the active resistance  $R'_{11}, R'_{22}, \dots, R'_{nn}$ , determined only by direct current and are constants. Therefore, if the coefficients made dependent on any variables this system of equations can not be considered an Adequate Mathematical Model. The inadequacy of such a model is explained hereinafter. The considered system of equations can be written in a more compact form:

$$\frac{d\Psi_i}{dt} + R'_{ik}i^k + \frac{1}{C_{ii}} \int i^i dt = u_i$$

where, I changing their values from 1 to n. The values  $R'_{ik}$  in this equation can be represented in the following table:

$$R'_{ik} = \begin{pmatrix} R'_{11} & 0 & \dots & 0 \\ 0 & R'_{22} & \dots & 0 \\ \dots & \dots & \dots & \dots \\ 0 & 0 & \dots & R'_{nn} \end{pmatrix}$$

If  $i_k$  and  $\Psi_i$  vectors by definition then  $R'_{ik}$  is tensor (Krawczyk and Wiak, 2002) (the object, linearly converting elements I linear space to (i+1) linear space) rank 2 (according to the inverse tensor basis (Joshi, 1995; Itskov, 2015)). However, derivatives:

$$\frac{d\Psi_i}{dt} (i = 1, 2 \text{ and } n)$$

are not tensor components  $R'_{ik}$ . Therefore, the equation:

$$\frac{d\Psi_i}{dt} + R'_{ik}i^k + \frac{1}{C_{ii}} \int i^i dt = u_i \tag{1}$$

tensor is not. Therefore this equation adequately describes the processes in electromagnetic actuators AC, because in this system are not taken into account the processes associated with the displacement current. As a result, the Mathematical Model is linear and therefore approximate.

The system of equations communication flux linkage and current can also be represented in tensor form. Contact each flux loop with all current set by the equation:

$$\Psi = L_{11}i^1 + L_{12}i^2 + \dots + L_{1n}i^n$$

Depending on the mode of operation of the electromagnetic actuator inductance values can be varied within wide limits. The expression for the inductance is given by:

$$L = \frac{\mu_0 \mu N^2 S}{l}$$

Where:

- N = No. of windings
- S = Cross-sectional area of the magnetic circuit
- $\mu$  = Relative magnetic permeability
- $\mu_0$  = Magnetic constant
- l = Winding length

Each inductance is generally a function of all currents:

$$L_{ik} = L_{ik}(i^1, i^2, \dots, i^n)$$

where, i, k = 1, 2, ..., n. Thus, a non-linear in the parameters, the system of equations due flux linkage and currents:

$$\begin{cases} \Psi_1 = L_{11}i^1 + L_{12}i^2 + \dots + L_{1n}i^n \\ \Psi_2 = L_{21}i^1 + L_{22}i^2 + \dots + L_{2n}i^n \\ \dots \\ \Psi_n = L_{n1}i^1 + L_{n2}i^2 + \dots + L_{nn}i^n \end{cases}$$

which in tensor form is:

$$\Psi_i = L_{ik}i^k \equiv \sum_{k=1}^{k=n} L_{ik}i^k (i = 1, 2 \text{ and } n)$$

The last equation is indeed tensor (in accordance with the opposite sign of the tensor) as  $\Psi^i$  and  $i^k$ : absolute vector components by definition. Therefore:

$$L_{ik} = L_{ik}(i^1, i^2, \dots, i^n)$$

rank tensor 2. One can assume that the voltage balance equation in the AC electromagnetic actuator (1) can be represented in tensor form, i.e., in the form of invariant to the transformation of coordinates. This model is the equation:

$$\frac{d\Psi_i}{dt} + \Omega_{ij}^k \Psi_k \frac{dx^j}{dt} + R'_{ik}i^k + \frac{1}{C_{ii}} \int i^i dt = u_i$$

in which the term:

$$\Omega_{ij}^k \psi_k \frac{dx^j}{dt}$$

It takes into account the effect of the displacement current.

### RESULTS

As a result an adequate mathematical model may be the following system of equations of tensor:

$$\frac{d\psi_i}{dt} + \Omega_{ij}^k \psi_k \frac{dx^j}{dt} + R_{ik} i^k + \frac{1}{C_{ii}} \int i^i dt = u_i \quad (2)$$

$$\psi_i = L_{ik} i^k \equiv \sum_{k=1}^{k=n} L_{ik} i^k \quad (i = 1, 2 \text{ and } n)$$

### DISCUSSION

The resulting Tensor Model (2) adequately describe nonlinear processes occurring in the electromagnetic actuator AC by taking into account the effects of the displacement of the current in the coil of the electromagnetic AC drive and the saturation of the magnetic steel in the process of magnetization.

### CONCLUSION

Development tensor methodology annexed to the electromagnetic AC drive is a perspective direction of modern engineering which will improve the accuracy of measurements by taking into account the influence of the non-linear processes.

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