

## Finding the Best Possible Method of the Trajectory Shape Object Movement

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**Abstract:** An object tracking have multiple challenges and constraints in each application or field of research. The reason for this is because: finding the optimal trajectory shape of the video motion is not that predictable. Tracking the path of moving targets needs to describe the nature of the movement and behavior of properties for the moving target and determine the quality of movement (normal or abnormal). The aim of this study is to compare the three methods for moving object in order to determine the quality and shape of the movement (straight line, circle arc, ellipse, oscillating, S-shaped... etc.). In the first case, the work depends on the slope in mathematical model while in the second case the calculations depend on the fundamental of linear and non-linear regression specification which are followed by the computation of minimum mean square error, minimum absolute error and minimum mean error for each case. In the third case addressing a mathematical model using the description Fourier method dealing with shape of movements.

**Key words:** Video tracking, trajectory shape, tendency, gradient, object detection, slope, regression, fourier descriptor

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### INTRODUCTION

The fundamental problems facing us in the field to track the trajectory of movement and shape of objects is difficult to determine the appropriate to describe this track variables therefore it has been using many of the analysis methods for the form of object movement (Maggio and Cavallaro, 2011).

The movement of objects based on the change in its position with the three dimensions (x, y, z) of space but in digital images depends on the amount of change in the x and y axes which represents a reflection of these three axes, so we must take in to account the change in the two axes. Study the extent of this change and take advantage of it to see the quality and shape of the body to determine the movement of natural and non-natural movement with finding the equation or formula which represents the closest body movement when tracking moving objects.

To begin the implementation of the algorithm is to determine trajectory of object moving by identifying the center point of the object moving after applied any segmentation algorithm likeness region growing or the difference between the background and current image.

In first phase of this research has been the slope in the use of mathematical relationships between two points (these points is represented the center of object movement in sequence frames) and is characterized by the amount of increase or decrease in values as well as changes in the value of the gradient between the negative and

positive movement to achieve the above form and according to the conditions will be mentioned later property.

In the second phase, Fourier descriptors method is utilized with close shape or contour. We are using Fourier descriptors are strong features for the discrimination of two-dimensional connected shapes (Dalitz *et al.*, 2013). The Fourier descriptors are usually utilized to describe the shape of a presenting object in an image.

In the third phase, we use linear and non-linear regression to find a range of items, according to mathematical relationships for each movement likeness (straight line, circle, curved, etc.), then to find the equation that represents the shape of the movement and then is applied to multiple statistical metrics to calculate the error rate will be mentioned later to find the movement type of the body moving.

Some research associated with our research like this Israa Hadi and Mustafa Sabah, converting the trajectory points into approximation function using curve fitting function to smooth the data; improving the appearance of the trajectory, extracting important features such as slope and intersection point (Hadi and Sabah, 2014). How to detect the wandering trajectory based on angle and recognize human behavior of the moving target are key problems needed to be solved (Chen *et al.*, 2013). We propose a method to define Fourier descriptor even for broken shapes, the method is based on the convex hull of the shape and the distance to the closest actual contour point along the convex hull (Dalitz *et al.*, 2013).

**Literature review:** To detect trajectory shape we are using three ways to specified it and compare among these ways as (Slope module, Regression module and Fourier descriptor). The gradient or slope relation is single the most important features that used to set the shape of path object. It is calculated by finding the ratio of the “vertical change” to the “horizontal change” or  $\Delta y = (y_2 - y_1)$  to the  $\Delta x = (x_2 - x_1)$ . Slope or gradient of a line is a number that describes both the direction and the steepness of the line. Slope is often denoted by the letter m. The direction of a line is characterized as: A line is increasing (slope is positive) or decreasing (slope is negative) ora line is horizontal (slope is zero) or vertical (slope is undefined (infinity)). The slope is represented in mathematical by equation below:

$$m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1} \quad (1)$$

$$\theta = \text{atan} \left( \frac{y_2 - y_1}{x_2 - x_1} \right) \quad (2)$$

$$\text{Dis.} \left( F_{(x_1, y_1)}, F_{(x_2, y_2)} \right) = \sqrt{(X_1 - X_2)^2 + (y_1 - y_2)^2} \quad (3)$$

The Fourier descriptors are utilized to characterize the objects shape in terms of its spatial frequency content (Noor *et al.*, 2011). It is a method utilized in object recognition and image processing to represent the boundary shape of a segment in an image. The major idea is to describe a contour by a specific of numbers that represent the frequency content of a total shape. Based on frequency analysis we can select a small set of numbers (the Fourier coefficients) that characterized a shape. The regression analysis is a statistical operation for rating the relationships among variables or phenomena’s (Wang and Bovik, 2009). In a cause and effect relationship, the independent variables are represented cause and effect is represented by dependent variables.

**MATERIALS AND METHODS**

**Suggested method:** Suggested method consists of four major modules (Fig. 1):

- Input video
- Moving object detection
- Detection of Trajectory Shape
- Statistic Measurement

**Input video:** Transform video signal to sequence of frames according to properties as shown in the Table 1 below:

**Moving object detection:** The first stage in tracking targets is separating the targets from the background. Two popular methods are background subtraction and segmentation. Background subtraction method is used to detect the moving target from background (Lehmann and Casella, 1998). We used Eq. 4- 6 to find center of object movement, then we save these center points in vector and used them to test.

**Area of n object:** The area of a binary object is given by:

$$A = \sum_i \sum_j \text{object}[i, j] \quad (4)$$

$$X_c = \frac{\sum_i \sum_j [i \times \text{object}(i, j)]}{A} \quad (5)$$

$$\sum_i \sum_j \frac{[j \times \text{object}(i, j)]}{A} \quad (6)$$

Where:

- $X_c, Y_c$  = Center of axis X and Y
- A = Area of object

**Detecting shape of trajectory:** Trajectory shape is detecting by three ways and compare among these ways as shown below:

**Slope module:** To detection the behavior and shape of the trajectory (straight line, arc, ellipse, circle, or spiral) of each movable object and the slope has been utilized to determine the direction of motion trajectory and the angle in the following cases below:

- Slope and angle are nearest or equal values in total points; the trajectory of object movement is straight line
- Slope and angle are changing in equal magnitude, the trajectory of object movements is circle and radius calculate by equation 3 is equal in all cases
- Slope and angle are changing in equal magnitude, the trajectory of object movements is ellipse when the radius determine by equation 3 is variables and different values in all cases
- Slope and angle are oscillating (alternate) between negative and positive values, the trajectory of object movements is S-shaped or spiral
- Slope value and the angle have changed suddenly and the value is too large at a specific location for the movement of the object is called the arc shape of object movement

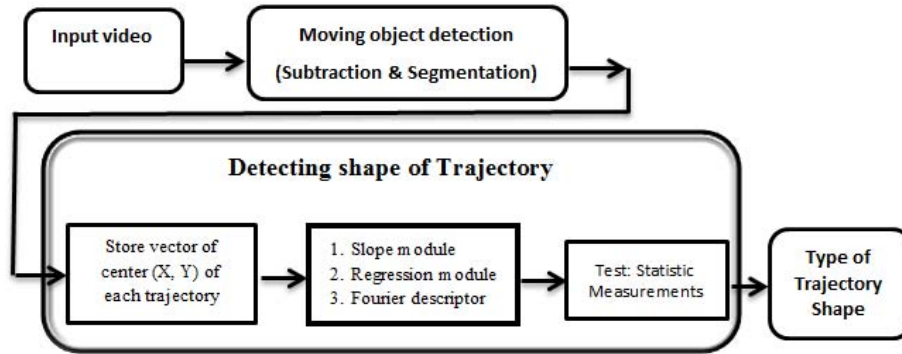


Fig. 1: Block diagram represents detection Trajectory object shape

Table 1: The movie properties

Movie name	Person	Movies time (sec)	13
Frame No.	324	Data rate (kb sec <sup>-1</sup> )	820
Frame size	160*120	Type of Files	.avi
Frame rate Fr/sec	25		

**Fourier descriptors:** Fourier descriptors are appropriated to closed form of object movement. It is allow us to bring the power of Fourier theory to shape description.

A contour provides a more accurate description of the target shape (Willmott and Matsuura, 2005). In general, regular and smooth curves are preferred to high-curvature contours. We applied equations as:

$$F(n) = F_R(n) + jF_I(n) \tag{7}$$

$$F_R(n) = \sum_{i=0}^{M-1} \frac{\Delta x(i)}{L_i} \cos\left(\frac{2\pi i n}{M}\right) \tag{8}$$

$$F_I(n) = \sum_{i=0}^{M-1} \frac{\Delta y(i)}{L_i} \sin\left(\frac{2\pi i n}{M}\right) \tag{9}$$

$$\Delta x(i) = x(i) - x(i+1), \tag{10}$$

$$\Delta y = y(i) - y(i+1)$$

$$L_i = \sqrt{(\Delta x(i))^2 + (\Delta y(i))^2} \tag{11}$$

Where M is no. of points, N is no. of features,  $F_R(n)$  is real part,  $F_I(n)$  is imaginary part and L is distance or length between two points.

**Regression module:** The aim of regression is to make predication or to estimate the value of variable

(phenomena) consequence to variant another. The regression is divided into two types as linear model and non-linear model.

**Regression linear model:** It can be divided into kinds as follow:

**A-Simple linear model (SLM):** relationship between two variables only. A formulate (SLM) model as:

$$\text{modelas : } y_i = \beta_0 + \beta_1 X_i + \epsilon_i \tag{12}$$

Where:

$Y_i$  = Response (dependent) variable which observed

$X_i$  = Independent variable which observed too

$\beta_0$  = Parameter call by intersect coefficient,

$\beta_1$  = Parameter call by slope and

$\epsilon_i$  = Error term

**B-general Linear Model (GLM):** Relationship among many variables. A formulate (GLM) model as:

$$\text{modelas : } y_i = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \dots + \beta_p X_p + \epsilon_i \tag{13}$$

**The least square line:** Least Square Line regression is a technique for finding a line that summarizes the relationship between the two variables, at least within the domain of the explanatory variable x. The method of least square is likely the most regular procedure to fit a singular curve through the specific data points. In this type we applied Eq. 14-17 to find parameters  $\beta_0, \beta_1$ :

$$\sum y = n\beta_0 + \beta_1 \sum x \tag{14}$$

$$\sum xy = \beta_0 \sum x + \beta_1 \sum x^2 \tag{15}$$

$$\beta_0 = \frac{(\sum y)(\sum x^2) - (\sum X)(\sum XY)}{N \sum X^2 - (\sum X)^2} \quad (16)$$

$$\text{Slop} = \beta_1 = \frac{N \sum xy - (\sum X)(\sum y)}{N \sum X^2 - (\sum x)^2} \quad (17)$$

**Regression non linear model:** It takes many different forms likeness as shown below:

Parabola or Quadratic curve. (18)  

$$Y = \beta_0 + \beta_1 X + \beta_2 X^2$$

Cubic curve  $Y = \beta_0 + \beta_1 X + \beta_2 X^2 + \beta_3 X_3$  (19)

nth degree curve.  $Y = \beta_0 + \beta_1 X + \beta_2 X^2 + \dots + \beta_n X^n$  (20)

Exponential.  $= ab^x$  (21)  
 or  $\text{Log } Y = \log a + (\log b) X$

$Y = aX^b$  or  $\text{Log } Y = \log a + b \log X$  (22)

Geometric curve (Power). We apply a set of linear and non-linear regression methods mentioned above and the use statistical methods and standards similar to root mean square error, mean absolute error and mean error to detection behavior and shape of the trajectory of object movement (Shatha, 2013). Below are some ways that have been adopted in the research to detect trajectory of object movement.

**Parabola or quadratic curve:** A quadratic function is a second-degree polynomial function of the form, where a, b and c are real numbers and. Every quadratic function has a “u-shaped” graph called a parabola. A parabola either opens up or opens down depending on the leading coefficient.

- If  $a > 1$ , as in Fig. 2a, the parabola will “open up
- If  $a < 1$ , as in Fig. 3b, the parabola will “open down
- If  $|a| > 1$ , the graph will be narrower than the graph of  $y = x^2$
- If  $0 < |a| < 1$ , the graph will be wider than the graph of  $y = x^2$

Let a parabola:

$$y = a + bx + cx^2 \quad (23)$$

which is fitted to a given data  $(x_1, y_1), (x_2, y_2), (x_3, y_3), \dots, (x_n, y_n)$ . Let  $y_e$  be the theoretical value for  $x_1$  then:

$$e_1 = y_1 - y_2 \quad (24)$$

$$e_1 = y_1 (a + bx_1 + cx_1^2)$$

$$e_1^2 = (y_1 - a - bx_1 - cx_1^2)^2$$

Now we have:

$$S = \sum_{i=1}^n e_i^2 = \sum_{i=1}^n (y_i - \alpha - bx_i - cx_i^2)^2$$

By the principle of least squares, the value of S is a minimum therefore:

$$\frac{\partial S}{\partial S} = 0, \frac{\partial S}{\partial b} = 0, \frac{\partial S}{\partial c} = 0 \quad (25)$$

Solving Eq. 25 and dropping suffix, we have:

$$\sum y = na + b \sum x + c \sum x^2 \quad (26)$$

$$\sum xy = a \sum x + b \sum x^2 + c \sum x^3 \quad (27)$$

$$\sum x^2 y = a \sum x^2 + b \sum x^3 + c \sum x^4 \quad (28)$$

Equations 26-28 are known as normal equations. On solving these equations, we get the values of a, b and c. Putting the values of a, b and c in Eq. 25, we get the equation of the parabola of best fit.

**Exponential curve:** The slope of the tangent to the graph at each point is equal to its y coordinate at that point. The exponential function has the characteristic that for each element increase in X the value of Y also increases by a fixed percentage. The inverse function of function is the natural logarithm  $\ln(x)$ . Assume an exponential curve of the formula below (Wang and Bovik, 2009):

$$Y = ae^{bx} \quad (29)$$

Taking logarithm on both the two sides, we obtain

$$\text{Log}_{10} y = \log_{10} a + bx \log_{10} e \quad (30)$$

$$\text{i.e., } Y = A + Bx \quad (31)$$

where,  $Y = \text{Log}_{10} y$ ,  $A = \log_{10} a$  and  $B = b \log_{10} e$ . The normal Eq. 31 are:

$$\sum y = nA + b \sum x \quad \text{and}$$

$$\sum xy = A \sum x + B \sum x^2$$

On solving the above two equation, we obtain A and B:

$$a = \text{anti log } A \tag{32}$$

$$b = \frac{B}{\log_{10} a} \tag{33}$$

**The power fit ( $y = ax^m$ ):** Assume we ask  $ax^m$  as an approximation to a function  $y$  where  $m$  is recognized constant. We must get the value of  $a$  such that the equation:

$$Y = ax^m \tag{34}$$

is satisfied as nearly as possibly by each of the  $n$  pairs of observed values  $(x_1, y_1), (x_2, y_2), \dots, (x_n, y_n)$ . Using least square technique, we should minimize the error function:

$$e(a) = \sum_{i=1}^n (ax_i^m - y_i) \tag{35}$$

For this purpose, partial derivative of Eq. 35 with respect to  $a$  must vanish. So, we have:

$$0 = 2 \sum_{i=1}^n (ax_i^m - y_i)(x_i^m) \tag{36}$$

and so:

$$0 = \alpha \sum_{i=1}^n (ax_i^{2m} - \sum_{i=1}^n x_i^m y_i) \tag{37}$$

which yields:

$$\alpha = \frac{\sum_{i=1}^n x_i^m y_i}{\sum_{i=1}^n x_i^{2m}} \tag{38}$$

To know best curve fitting, we calculate the  $e_{rms}$  (corresponding errors). For the first power fit, we have the best curve is represented by minimum  $e_{rms}$ .

## RESULTS AND DISCUSSION

**Fidelity measurement to detect the kinds of object movement:** The aim of a signal fidelity measurement is to compare two signals (original and distorted) that describe the degree of similarity/fidelity and the level of error/distortion between them (Willmott and Matsuura, 2005).

In the previous paragraph has been applied a number of linear and non-linear regression ways to detect the form of object movement that is tracked in order to know which of these methods to the closest form of object movement, we apply a number of statistical standards as shown.

**Mean Square Error (MSE):** (MSE) is term referring to Mean Squared Error of an estimator measurement the average of the squares of the “errors”, that is, the difference between the original and distorted signals (Willmott and Matsuura, 2005). The error between two functions is given by:

$$e_t = x_t - \bar{x}_t \tag{39}$$

The total mean square error is:

$$MSE = \frac{1}{N} \sum_{t=1}^N e_t^2 = \frac{1}{N} \sum_{t=1}^N (x_t - \bar{x}_t)^2 \tag{40}$$

**MSE:** Positive value and best when it is less value: To calculate MSE with regression module using equation as:

$$MSE = \frac{\sum x_i y_i - \beta_0 \sum y_i - \beta_1 \sum x_i y_i}{n - k - 1} \tag{41}$$

The  $\beta_0$  and  $\beta_1$  are parameters calculate by equations 18 and 19,  $x_t$  and  $y_t$  are points values to any object trajectory,  $N$  is no. of points and  $K$  is no. of dependent variable.

**MSE:** Positive value and best when it is less value. MSE has numerous attractive properties as: It is easy, a clear physical meaning, the MSE is a perfect metric in the status of optimization and it is a desirable measurement in the statistics and estimation domain (Lehmann and Casella, 1998).

**Mean Absolute Error (MAE):** MAE is a quantity utilized to measure how close predictions or forecasts are to the final outcomes. MAE is a more natural measure of average error:

$$MAE = \frac{1}{N} \sum_{t=1}^N |e_t| = \frac{1}{N} \sum_{t=1}^N |x_t - \bar{x}_t| \tag{42}$$

Table 2: Illustrated the result for state that explains the value of slope and angle to adjacent points in trajectory of object

Point	X <sub>cen</sub>	Y <sub>cen</sub>	Adjacent point		References and other point		Movement type
			Angel	Slop	Angle	Slop	
1	6	64	-14.036	-0.250	-14.036	-0.250	Approximate straight line2
	26	59	3.180	0.056	-6.009	-0.105	
3	44	60	0.000	0.000	-3.945	-0.069	
4	64	60	-2.862	-0.050	-3.668	-0.064	
5	84	59	3.180	0.056	-2.386	-0.042	
6	1.02	60	-2.862	-0.050	-2.468	-0.043	
7	122	59	3.013	0.053	-1.697	-0.030	
8	141	60	-8.746	-0.154	-2.322	-0.041	
9	154	58	-70.017	-2.750	-6.382	-0.112	
10	158	47					

Av. Change of angle at adj. Po.=8.93; Av. Change of angle at ref. Po. = -4.291; Av. Change of slop at adi. Po. = -0.309; Change of angle at ref. Po. = -0.076

Table 3: Results of power fit best of movement object

Point	X	Y	X <sup>2</sup>	X <sup>3</sup>	X <sup>4</sup>	X <sup>6</sup>	YX <sup>2</sup>	YX <sup>3</sup>
1	2.0	5.1	4	8	16	64	20.4	40.8
2	2.3	7.5	5.29	12.167	27.984	148.035	39.675	91.252
3	2.6	10.6	6.76	17.576	45.698	308.918	71.656	186.306
4	2.9	14.4	8.41	24.389	70.729	594.831	121.104	351.202
5	3.2	19.0	10.24	32.768	104.858	1073.746	194.56	622.592
Sum	13	56.6	34.7	77.324	265.269	2189.53	447.395	1292.152

Table 4: Sample data of motion person is represented approximate straight line

No. Point	1	2	3	4	5	6	7	8	9	10	11
No. Frame	6	7	8	9	10	11	12	13	14	15	16
Xcen	157	156	156	154	152	150	146	141	137	134	130
Ycen	58	61	60	61	54	51	53	52	52	51	50
No. Point	12	13	14	15	16	17	18	19	20	21	22
No. Frame	17	18	19	20	21	22	23	24	25	26	27
Xcen	124	119	114	108	103	97	92	86	80	75	69
Ycen	51	52	52	54	56	56	55	55	54	54	55
No. Point	23	24	25	26	27	28	29	30	31	32	33
No. Frame	28	29	30	31	32	33	34	35	36	37	38
Xcen	32	57	51	45	39	33	27	21	15	11	06
Ycen	55	57	57	56	56	55	55	56	58	59	60

**Mean Error (ME):** An imprecise term that sometimes refers to mean deviation. It refers to measuring the accuracy of a prediction or model. It is average error of a number of observations found by taking the mean value of the positive and negative errors without regard to sign:

$$ME = \frac{1}{N} \sum_{t=1}^N e_t = \frac{1}{N} \sum_{t=1}^N (y_t - \hat{y}_t) \tag{43}$$

**Experimental result:** In the first step, all data points (x, y) are normalized by equations below:

$$X_n = \frac{X - X_{min}}{D} \tag{44}$$

$$Y_n = \frac{Y - Y_{min}}{D} \tag{45}$$

$$D = \max (X_{max} - X_{min}, Y_{max} - Y_{min}). \tag{46}$$

To check the performance and efficiency of the all models, we have executed the models with some data sequential.

**Slope module:** Table 2 the result for state that explains the value of slope and angle to adjacent points in trajectory of object (Table 2-5 and Fig 4-6).

**Regression module:** The Power Fit ( $y = ax^m$ ) By execute equation 40 to power from 1-5. Parameter when  $m = 2$  then  $a = 1.6866$  and  $m = 3$  then  $a = 0.5902$ . To know which of these is best fit, we calculate the corresponding errors. For the first power fit, we have:

$$e_{ms} = \left[ \frac{1}{5} \left\{ (ax_1^2 - y_1)^2 + (ax_2^2 - y_2)^2 + (ax_3^2 - y_3)^2 + (ax_4^2 - y_4)^2 + (ax_5^2 - y_5)^2 \right\} \right]^{1/2}$$

$e_{ms} = 1.3$  at  $m = 2$ ,  $e_{ms} = 0.29$  at  $m = 3$  then  $y = 0.5902 X^3$  is the best.

Table 5: Illustrated the result for regression linear and nonlinear model that explains the value of MSE, MAE, and ME to approximate straight line of trajectory object to database of Table 3

Motion types	MSE	MAE	ME	Equation
Straight line	0.0678	6.421	0.0	$y = 0.5946X - 0.2224$
Exponential	0.1597	8.834	0.248	$Y = 0.612e^{0.699x}$
Power one	0.3951	-	-	Minimum when power equal one
Power two	0.4247			
Power three	0.4361			
Parabola or quadratic curve	0.679	20.922	0.586	$y = 0.819 - 1.676x - 1.393x^2$

Result; straight line; MSE; MAE; and ME are Less

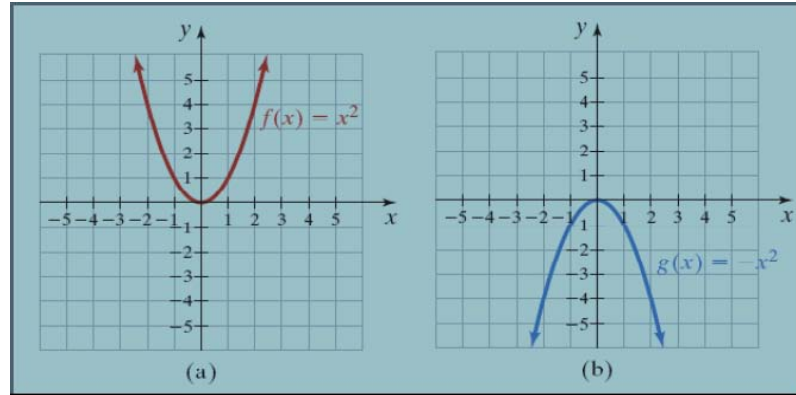


Fig. 2: Graph of (a)  $f(x) = X^2$ , (b)  $f(x) = -X^2$

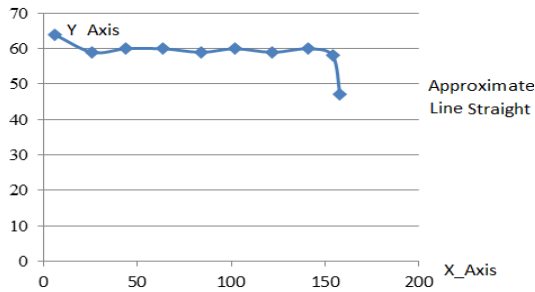


Fig. 3: Graph is representing object moving approximate straight line-shape respect to reference point

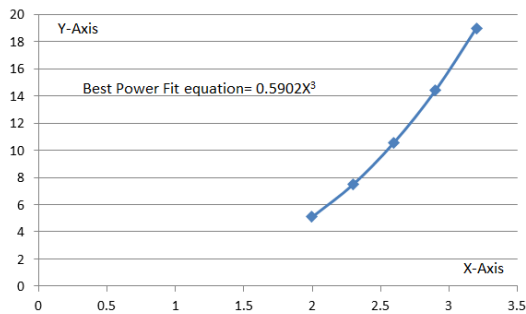


Fig. 4: Graph is representing object moving power fit of curve shape



Fig. 5: The samples data set (Frames 10, 20, 30 and 37) of approximate straight line movement of person

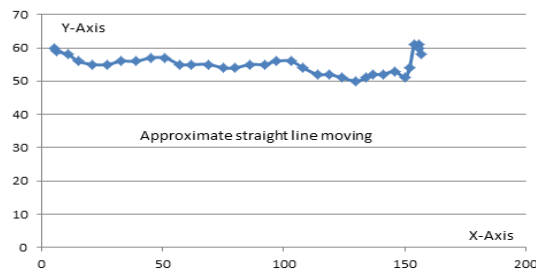


Fig. 6: Graph is representing object moving approximate straight line-shape

### CONCLUSION

In this research three mathematical models were used to determine the form of objects tracking movement in video applications this was accomplished through numerous experiments which concluded that in first model the change of slope and angle which resulted from

movement of object tracking provided us with the nature of object movement and form. In the second model Fourier descriptors methods were used to deal with closed loop object movement and in this case the mathematical model and trigonometric functions were intensively used compared to the first model. In the third model linear and nonlinear regression algorithm were used by applying variation of mathematical models followed by the application of number of statistic fidelity measurements to indicate the form of the movement which unfortunately led to long execution time.

As we concluded the slope method was better than the other two methods in terms of time, due to the (simple) mathematical model and less of trigonometric functions used application (computationally inexpensive).

#### REFERENCES

- Chen, Q., R. Wu, Y. Ni, R. Huan and Z. Wang, 2013. Research on human abnormal behavior detection and recognition in intelligent video surveillance. *J. Comput. Inf. Syst.*, 9: 289-296.
- Dalitz, C., C. Brandt, S. Goebbels and D. Kolanus., 2013. Fourier descriptors for broken shapes. *EURASIP. J. Ad. Signal Proc.*, 2013: 1-11.
- Hadi, I. and M. Sabah, 2014. Behavior formula extraction for object trajectory using curve fitting method. *Intl. J. Comput. Appl.*, Vol.104,
- Lehmann, E.L. and G. Casella, 1998. *Theory of Point Estimation*. 2nd Edn., Springer-Verlag, New York, ISBN: 0-387-98502-6.
- Maggio, E. and A. Cavallaro, 2011. *Video Tracking Theory and Practice*. John Wiley and Sons, Hoboken, New Jersey, USA.,
- Noor, S.M., I.A. Mohammed and L.E. George, 2011. Handwritten arabic (indian) numerals recognition using Fourier descriptor and structure base classifier. *J. Nahrain Univ.*, 14: 215-224.
- Shatha, M.N., 2013. Isolated multifont arabic character recognition using fourier descriptors. *J. Nahrain Univ.*, 16: 176-182.
- Wang, Z. and A.C. Bovik, 2009. Mean squared error: Love it or leave it? A new look at signal fidelity measures. *Signal Process. Mag.*, 26: 98-117.
- Willmott, C.J. and K. Matsuura, 2005. Advantages of the mean absolute error (MAE) over the root mean square error (RMSE) in assessing average model performance. *Climate Res.*, 30: 79-82.