Research Journal of Applied Sciences 11 (12): 1445-1449, 2016
ISSN: 1815-932X


# Determining Maximum Value and Optimizing a Bivariate Function Using Genetic Algorithm

[1]Taherh Shokuhi, [2]Majid Pazand and [3]Nasrin Hamidi
[1]Department of Applied Mathematics,
[2]Department of Agricultural Economics,
Higher Educational Complex of Saravan, Saravan, Iran
[3]Ministry of Education of Zahedan, Zahedan, Iran

**Abstract:** Now a days, obtaining the extermum points of functions are important in optimization. One of the used methods for optimization and finding extermum points of functions is genetic algorithm. This study has introduced genetic algorithm briefly and used this method to examine maximum points of function: $F = t^2+\beta\times E^2$ within interval of variables $t<63$ and $E<63$. This process is done based on two assumption of $\beta = 0$ and $\beta = 1$. The obtained results are used to find maximum points. To assurance of output results of genetic algorithm, differentiation method was used to find maximum point of function in considered function. The obtained results indicate that output of this method is equal to output of genetic algorithm method. In the following part of study, the effect of mutation is examined on process of obtaining results of genetic algorithm. The next step includes using a set of schema to speeding up the convergence. The obtained results indicate that this action would considerably speeds up the convergence.

**Key words:** Reproduction, bivariate function, single-variable function, schema, mutation coefficient

## INTRODUCTION

Genetic algorithm is a searching technique in computer science to find an approximate solution for optimization and search issue. Genetic algorithm is a specific type of Evolutionary Algorithms that use recursive biological techniques such as heredity and mutation.

In fact, genetic algorithms use Darwin's natural selection principle to find optimal formula to pattern matching or forecasting. Genetic algorithms are usually good option for forecasting techniques based on regression. On the other hand, the ability to find maximum and minimum on an extensive range of functions are applied to optimize engineering issues. It is briefly stated that genetic algorithm is a programming technique that uses genetic evolution as a problem-solving model. The issue that should be solved is input and solutions are coded based a pattern that fitness function or cost function evaluates each considered solution while majority of them are randomly chosen. Generally, these algorithms consist of following parts:

- Fitness function
- display
- Selection
- Change

Genetic algorithm motor creates a new population from formula. Every person is tested in front of a set of data and the most appropriate of them (maybe 100% of the most appropriates) are remained and others are removed. The most appropriate persons are paired (displacement of DNA factors) and changed (random change in DNA factors). It is observed that genetic algorithm tend to create accurate formulas passing through numbers of generations while neural networks are non-linear and non-parametric. The high-level attractiveness of genetic algorithm is that the results of it are considerable. The final formula will be observable for human user and to represent confidence level of results it is possible to apply common statistical techniques on these formulas (Samarasinghe, 2006).

The following definitions are important within process of genetic algorithm method: initial population, reproduction, cross over, mutation genetic algorithm has four features as follows:

- Working with objective function instead of working with its derivatives
- Using a group of points instead of one point to start optimization algorithm
- Using binary code system of points instead of using points
- The ruling principles are probabilistic

**Corresponding Author:** Taherh Shokuhi, Department of Applied Mathematics, Higher Educational Complex of Saravan, Saravan, Iran

The conditions for encoding of genetic algorithms are as follows:

- Reaching to a fix number of generations
- The allocated time is ended
- There is a person (generated person) to meet minimum (the least) of criterion
- The highest rate of fitness is obtained for individuals or there is not any better result
- Manual inspection
- High combinations

If there is enough number of generation productions, the final number is the response of important issue. At the best state, all final identical chromosomes are equal to maximum value of objective function (Haykin, 1998; Dayhoff, 1990).

## MATERIALS AND METHODS

**Use of genetic algorithm to find maximum value of considered function**

**State $\beta$ = 1:** At this part, the maximum level of function: $F = t^2 + \beta. E^2$ would be examined separately for two value of $\beta$ zero and one within an interval of $t<63$ and $E<63$ using genetic algorithm.

As we know, we do not work directly with variables within genetic algorithm but work with their codes. Hence, the put values of decimal variables of E and t in accordance with their interval as binary method to six genes chromosomes.

According to bivariate objective function, chromosomes are considered similar to Fig. 1 that each chromosome includes 12 genes in accordance with interval of t and E so that the first 6 genes are related to t and second 6 genes are related to E. Figure 1 depicts the chromosome of bivariate function with two variables t and E within state $\beta$ = 1. Table 1 and 2 indicate first generation of chromosomes for randomly chosen state. Table 3 indicates values t, E and F for first generation of Chromosomes. Now the function tends to its maximum after thousand times generation producing. Table 4-6 indicate values of chromosomes and function within thousandth generation at state $\beta$ = 1.

Figure 2 shows average value of F within different generations at state $\beta$ = 1 without using schemas. As it is observed in Fig. 2, mutation operator would lead to some sudden instability within converged trend that is not always beneficial. Of course, operator has acted regarding convergence (for instance around generation 200); however, this method has obtained maximum point. This process can be implemented several times in succession to prevent from error making sure about well working of method.

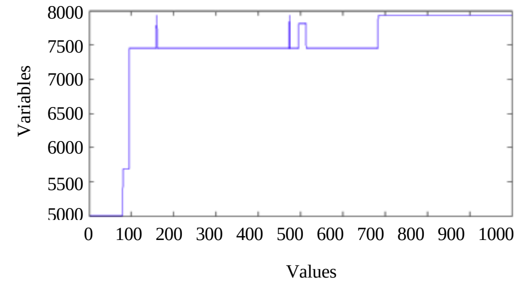


Fig. 1: Average value of F within different generations at state $\beta$ = 1

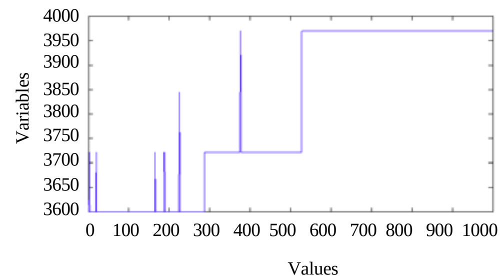


Fig. 2: Average value of F within different generations at state $\beta$ = 0

Table 1: First generation of chromosomes of E at state $\beta$ = 1

| Chromosome | E | Relevant chromosome to variable E | | | | | |
|---|---|---|---|---|---|---|---|
| 1 | 48 | 1 | 1 | 0 | 0 | 0 | 0 |
| 2 | 58 | 1 | 1 | 1 | 0 | 1 | 0 |
| 3 | 59 | 1 | 1 | 1 | 0 | 1 | 1 |
| 4 | 22 | 0 | 1 | 0 | 1 | 1 | 0 |
| 5 | 31 | 0 | 1 | 1 | 1 | 1 | 1 |
| 6 | 58 | 1 | 1 | 1 | 0 | 1 | 0 |
| 7 | 10 | 0 | 0 | 1 | 0 | 1 | 0 |
| 8 | 54 | 1 | 1 | 0 | 1 | 1 | 0 |

Table 2: First generation to chromosomes of t at state $\beta$ = 1

| Chromosome | t-values | Relevant chromosome to variable t | | | | | |
|---|---|---|---|---|---|---|---|
| 1 | 39 | 1 | 0 | 0 | 1 | 1 | 1 |
| 2 | 39 | 1 | 0 | 0 | 1 | 1 | 1 |
| 3 | 24 | 0 | 1 | 1 | 0 | 0 | 0 |
| 4 | 33 | 1 | 0 | 0 | 0 | 0 | 1 |
| 5 | 36 | 1 | 0 | 0 | 1 | 0 | 0 |
| 6 | 31 | 0 | 1 | 1 | 1 | 1 | 1 |
| 7 | 59 | 1 | 1 | 1 | 0 | 1 | 1 |
| 8 | 47 | 1 | 0 | 1 | 1 | 1 | 1 |

Table 3: Relevant values to first generation at state $\beta$ = 1

| Chromosome | t-value | E | F-value |
|---|---|---|---|
| 1 | 39 | 48 | 3825 |
| 2 | 39 | 58 | 4885 |
| 3 | 24 | 59 | 4057 |
| 4 | 33 | 22 | 1573 |
| 5 | 36 | 31 | 2257 |
| 6 | 31 | 58 | 4325 |
| 7 | 59 | 10 | 3581 |
| 8 | 47 | 54 | 5125 |

Table 4: Thousandth generation of chromosomes of E at state $\beta = 1$

| Chromosome | E | Relevant chromosome to variable E | | | | | |
|---|---|---|---|---|---|---|---|
| 1 | 63 | 1 | 1 | 1 | 1 | 1 | 1 |
| 2 | 63 | 1 | 1 | 1 | 1 | 1 | 1 |
| 3 | 63 | 1 | 1 | 1 | 1 | 1 | 1 |
| 4 | 63 | 1 | 1 | 1 | 1 | 1 | 1 |
| 5 | 63 | 1 | 1 | 1 | 1 | 1 | 1 |
| 6 | 61 | 1 | 1 | 1 | 0 | 0 | 1 |
| 7 | 63 | 1 | 1 | 1 | 1 | 1 | 1 |
| 8 | 63 | 1 | 1 | 1 | 1 | 1 | 1 |

Table 5: Thousandth generation of chromosomes of t at state $\beta = 1$

| Chromosome | t-values | Relevant chromosome to variable t | | | | | |
|---|---|---|---|---|---|---|---|
| 1 | 57 | 1 | 1 | 1 | 0 | 0 | 1 |
| 2 | 63 | 1 | 1 | 1 | 1 | 1 | 1 |
| 3 | 63 | 1 | 1 | 1 | 1 | 1 | 1 |
| 4 | 63 | 1 | 1 | 1 | 1 | 1 | 1 |
| 5 | 63 | 1 | 1 | 1 | 1 | 1 | 1 |
| 6 | 63 | 1 | 1 | 1 | 1 | 1 | 1 |
| 7 | 63 | 1 | 1 | 1 | 1 | 1 | 1 |
| 8 | 63 | 1 | 1 | 1 | 1 | 1 | 1 |

Table 6: Relevant values to thousandth generation at state $\beta = 1$

| Chromosome | t-value | E | F-value |
|---|---|---|---|
| 1 | 57 | 63 | 7218 |
| 2 | 63 | 63 | 7938 |
| 3 | 63 | 63 | 7938 |
| 4 | 63 | 63 | 7938 |
| 5 | 63 | 63 | 7938 |
| 6 | 63 | 61 | 7690 |
| 7 | 63 | 63 | 7938 |
| 8 | 63 | 63 | 7938 |

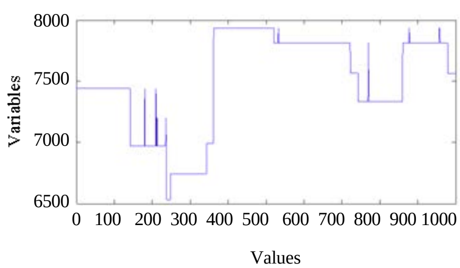


Fig. 3: Average value of F within different generations at state $\beta = 1$ with Pm = 0/01

**State $\beta$ = 0:** Now, we solve the issue with the same numbers of initial chromosomes for $\beta = 0$. Of course, the process is similar to previous state using three operators of reproduction, cross over and mutation in order to produce different generations. Figure 3 depicts function F with one variable of t for considered interval. Table 7 includes first generation of chromosomes have randomly chosen. Table 8 indicates values t, E and F for first generation of chromosomes. Now, the function tends to its maximum after thousand times generation producing. Table 4-6 indicate values of chromosomes and function within thousandth generation at state $\beta = 1$ (Table 9, 10). Figure 3 depicts average value of F within different generations at state $\beta = 0$ without using schemas.

Table 7: First generation of chromosomes at state $\beta = 0$

| Chromosome | t-values | Relevant chromosome to variable t | | | | | |
|---|---|---|---|---|---|---|---|
| 1 | 51 | 1 | 1 | 0 | 0 | 1 | 1 |
| 2 | 26 | 0 | 1 | 1 | 0 | 1 | 0 |
| 3 | 43 | 1 | 0 | 1 | 0 | 1 | 1 |
| 4 | 60 | 1 | 1 | 1 | 1 | 0 | 0 |
| 5 | 15 | 0 | 0 | 1 | 1 | 1 | 1 |
| 6 | 53 | 1 | 1 | 0 | 1 | 0 | 1 |
| 7 | 2 | 0 | 0 | 0 | 0 | 1 | 0 |
| 8 | 60 | 1 | 1 | 1 | 1 | 0 | 0 |

Table 8: Relevant values to first generation at state $\beta = 0$

| Chromosome | t-values | F |
|---|---|---|
| 1 | 51 | 2601 |
| 2 | 26 | 676 |
| 3 | 43 | 1849 |
| 4 | 60 | 3600 |
| 5 | 15 | 225 |
| 6 | 53 | 2809 |
| 7 | 2 | 4 |
| 8 | 60 | 3600 |

Table 9: Thousandth generation of chromosomes at state $\beta = 0$

| Chromosome | t-values | Relevant chromosome to variable t | | | | | |
|---|---|---|---|---|---|---|---|
| 1 | 51 | 1 | 1 | 0 | 0 | 1 | 1 |
| 2 | 63 | 1 | 1 | 1 | 1 | 1 | 1 |
| 3 | 63 | 1 | 1 | 1 | 1 | 1 | 1 |
| 4 | 63 | 1 | 1 | 1 | 1 | 1 | 1 |
| 5 | 63 | 1 | 1 | 1 | 1 | 1 | 1 |
| 6 | 63 | 1 | 1 | 1 | 1 | 1 | 1 |
| 7 | 63 | 1 | 1 | 1 | 1 | 1 | 1 |
| 8 | 63 | 1 | 1 | 1 | 1 | 1 | 1 |

Table 10: Relevant values to thousandth generation at state $\beta = 0$

| Chromosome | t-values | F-values |
|---|---|---|
| 1 | 51 | 2601 |
| 2 | 63 | 3969 |
| 3 | 63 | 3969 |
| 4 | 63 | 3969 |
| 5 | 63 | 3969 |
| 6 | 63 | 3969 |
| 7 | 63 | 3969 |
| 8 | 63 | 3969 |

**Examining the mutation coefficient within genetic algorithm in this issue:** This part of study includes study of effect of mutation coefficient on accuracy and speed of convergence in genetic algorithm. For this purpose, the response is examined for $\beta = 1$ with $P_m$-0/01 at this time. As it is observed in Fig. 4, transience has been increased with increase probability value of performance of mutation operator. It can be also observed that this operator does not always direct the method to reach the final answer. For instance, the function value is reduced near the generation 150 also leading to sudden growth of value of function near the generation 150 but the system has been instable with many fluctuations in value of function within different generations because of consideration of probability of

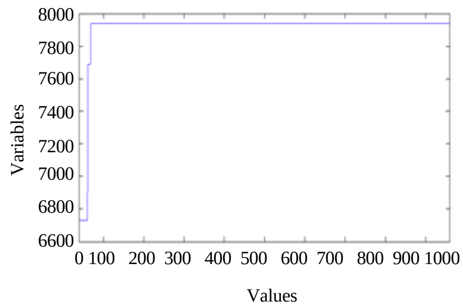


Fig. 4: The average value of F in different generations at state β = 1 using schema

high mutation so that the value does not reach to maximum value of function even after 1000 generations.

## RESULTS AND DISSCUSION

**Examining the effect of schemas on genetic algorithm in this issue**

**State β = 1:** At this stage, we first produce generations within 10 stages and then save the generated creator blocks during parallel process at these 10 stages. Now after the mentioned stage these creator blocks are used to produce new generations within other 10 stages so that the optimal solution of function will be obtained with low number of generations. The created values of variables t and F after 30 stages of population generation are presented in Table 11.

The generate values in generation 46 is presented in Table 12. As it is observed, the solution has been obtained after low number of generations compared to the state of without using schemas.

| | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 1 | 2 | 1 | 1 | 1 | 1 | 2 | 2 | 2 | 1 | 1 |

**The generated effective schemas at state β = 1:** The generated effective schemas for variables t and E are presented in Table 13. Figure 5 indicates the effect of schema for state β = 1. All of the conducted stages for part 4-1 are implemented for this stage too with the difference that the effects of schemas are applied after 10 stages. The generated values in generation 16 are presented in Table 14. As it is observed, the considered solution is obtained after passing the low number of generations compared to state without using schemas. Figure 5 indicates the effect of use of schema for state β = 0.

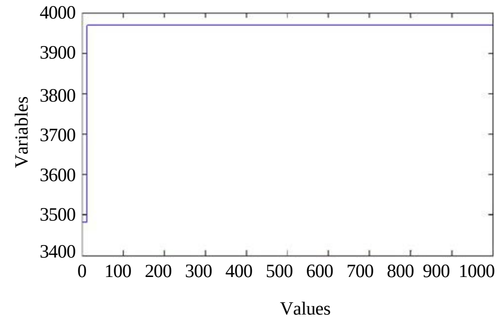


Fig. 5: Average value of F in different generations at state β = 0 using schema

Table 11: The Produced Values in generation 30 at state β = 1

| Chromosome | t-values | E | F |
|---|---|---|---|
| 1 | 57 | 59 | 6730 |
| 2 | 57 | 59 | 6730 |
| 3 | 57 | 59 | 6730 |
| 4 | 57 | 59 | 6730 |
| 5 | 57 | 59 | 6730 |
| 6 | 57 | 59 | 6730 |
| 7 | 57 | 59 | 6730 |
| 8 | 57 | 59 | 6730 |

Table 12: Generated values in generation 46 at state β = 1

| Chromosome | t-values | E | F |
|---|---|---|---|
| 1 | 63 | 63 | 7938 |
| 2 | 63 | 63 | 7938 |
| 3 | 63 | 63 | 7938 |
| 4 | 63 | 63 | 7938 |
| 5 | 63 | 63 | 7938 |
| 6 | 63 | 63 | 7938 |
| 7 | 63 | 63 | 7938 |
| 8 | 63 | 63 | 7938 |

Table 13: Generated values in generation 10 at state β = 0

| Chromosome | t-values | F |
|---|---|---|
| 1 | 59 | 3481 |
| 2 | 59 | 3481 |
| 3 | 59 | 3481 |
| 4 | 59 | 3481 |
| 5 | 59 | 3481 |
| 6 | 59 | 3481 |
| 7 | 59 | 3481 |
| 8 | 59 | 3481 |

Table 14: Generated values in generation 16 at state β = 0

| Chromosome | t-values | F |
|---|---|---|
| 1 | 63 | 3969 |
| 2 | 63 | 3969 |
| 3 | 63 | 3969 |
| 4 | 63 | 3969 |
| 5 | 63 | 3969 |
| 6 | 63 | 3969 |
| 7 | 63 | 3969 |
| 8 | 63 | 3969 |

## CONCLUSION

The results of optimization obtained from genetic method and mathematical optimization were matched at this study but the important point in genetic method is to reaching the solution with few calculations through smart choice of initial population using concept of schema. However, there is not always a relationship between function and existing variables or this relation is related to many variables while it is hard working with derivatives of these variables; hence, its analytical solution is not simple. Sometimes, there is not any function between inputs and outputs but a black box makes relation between inputs and outputs. In such cases, genetic algorithm indicates its priority to analytical solution as well as numerical solution so that maximum points of function are calculated through a simple process.

Genetic algorithm is an oriented optimization method because of its selection stages and on the other hand acts randomly because of using operators. One of the most important features of genetic algorithm is its unlimited objective function in terms of differentiation ability and continuity of function and the only condition for considered objective function is determination of function at different points.

The number of stages for population generation to obtain the solution for issue is related to the initial choice. In other words, different initial choices would lead to difference between generated numbers of populations in order to reach to the optimal solution. If the effective schemas are used in genetic algorithm, the considered solution will be obtained with fewer numbers of generations compared to the state of without using schemas.

If the mutation rate is very low, many of beneficial genes will lose possibility to be tested and if this rate is very high, the set of algorithm is excessively stochastic tending toward blind search. The high rate of crossover would prepare the possibility for more inspection of answer's space and reduced chance of reaching to an optimal wrong (local) solution. On the other hand, the very high rate of crossover would lead to spending time for search in spaces there is not any expectation in them.

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