

Particle Swarm Optimization Technique to Determine Class Location in Illiteracy Problem

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Abstract: Illiteracy problem is a problem facing the developing countries. The problem is the determination of the suitable locations to build literate classes at a city to educate illiterates with considering the calculation of cost in terms of time, distance and money. The problem under consideration will be represented as a multi-objective optimization model. Particle Swarm Optimization (PSO) will be used to solve this problem with a lot of conflict objectives and constraints and it will be applied this heuristic technique for a case study with these conditions.

Key words: Particle Swarm Optimization (PSO), multi-objective, optimization technique, geographic information system, money

INTRODUCTION

Illiteracy is one of the important problems that must be addressed and trying to make people to face it by saving the mechanisms to eliminate or decrease the illiterates. The class location Illiteracy Problem (IP) means that to build classes for illiterate people to learn how to read and write to be an attempt to encourage residents to go to these classes by some attracting factors. These factors including the choice of the classes' locations (must be close to the homes of residents). The transportation cost for these classes must be reasonable to make people to go and attend the lesson. The individual living is the important factor should be taking into account to success to make people to learn reading and write. There are some factors affect the building of these classes such that the building cost must be minimized to aid and encourage establishing the class. Each class contains building cost and running cost (teaching cost it could be reduced by selecting teachers from the local residents as much as possible).

Also the transportation time for the residents to the location of the class could be considered one of the factors that should be included to the problem restrictions. The transportation cost should be reduced to encourage illiterates to attend illiteracy classes. There are some restrictions must be take into account, like the number of building classes (must not exceed the planned

number of classes) and the number of illiterates individuals in each class (should not exceeds the total planned illiterates for each class).

There are critical factors affecting the construction process, like the cost to (build or transport) to a class must not exceed the available total planned building classes cost or transportation cost for these classes. Restrictions like the budget that covers the teachers' expenses and the material costs should not exceed the planned one for all classes. The distance of students to reach for each class also should be within (less than or equal) the planned total values. The distance parameters from Geographic Information System (GIS) tool to compute the shortest distance. The class location IP aims basically to determine the location of the classes in different best locations to be able to educate illiterate persons and assign the number of students for each class.

In this study, it is assumed that the assigned illiterates in each class will be at most n students but the decision variable that specifies the usefulness of the final decision for establishing a class in a certain area will appear as a binary variable in the proposed model (if the decision is YES and 0 if the decision is NO) in Amin a mathematical model was introduced and the problem under investigation was formulated as a multi-objective optimization problem with 5 different objectives and more than 30 constrains covering all the above mentioned restrictions.

In this study, researcher suggest the one of the heuristic algorithms called Particle Swarm Optimization (PSO) approach to solve and to determine the suitable sites for the classes to learn illiterate students according to all the above mentioned factors and circumstances.

PROBLEM DEFINITION AND OBJECTIVES

Illiteracy is not a problem that only poor countries face in their struggle to improve the quality of life of their people. The problem under consideration is a certain city that includes a lot of centers, each center includes a lot of local units and each local unit contains a lot of villages. The proposed problem is to build classes in several locations to solve location IP to learn a lot of students how to read and write.

The main goal of IP is to select the best sites to establish illiteracy classes for illiteracy persons and determine the number of registered students in each class. To be useful for building classrooms and play their role in the education of illiterate students must keep in mind some restrictions on different three levels such that village, a local unit and center level (Mazen *et al.*, 2013).

The three types of restrictions including in each level are cost, time and distance. These types of restrictions affect directly in locating building educational classes. In order the illiterate persons to attend literacy classrooms, we should keep in our track the rationalization of these three types of restrictions. Starting with the restrictions belongs to the village, the number of allocated classrooms in each village and the registered students of all allocated classrooms mustn't exceed the maximum number of allocated classes in each village and also the maximum number of all illiterate persons in a village respectively (Mazen *et al.*, 2013).

The total cost of all allocated classrooms includes Building cost (establishing the classroom at a location with blackboard and chalks) and running cost (teacher cost). The building and running cost must not exceed the available building and running budget in each village. The transportation budget is the important factor for the residents. The student cost includes transportation expenses, transportation time and transportation distance.

For the village, the transportation cost of all illiterate persons enrolled in all allocated classrooms should not exceed the transportation cost planned for each village. However, the transportation time of all illiterate persons in all allocated classrooms should not exceed the transportation time allocated for the village. The transportation distance of all illiterate persons in all allocated classrooms should not exceed the available

transportation distance for the village. The transportation budget is important factor for the other two levels (local unit and center) as for village (Mazen *et al.*, 2013).

The transportation distance is considered the critical goal would be calculated for the whole city. In this research, it is in need the GIS tool (such as ARCGIS, ARCMAP, ..., etc.) to deal with maps related to maps for the city and to compute distances especially the shortest distance between the selected sites for the classrooms and the homes of the residents.

There are some restrictions belongs to the whole city with all centers. The restrictions for allocated classrooms must not exceed the planned number of classes in the whole city. The total cost for building classrooms in the whole city will be at most the available budget for establishing process. The transportation cost (time and distance) of all illiterate persons in all allocated classrooms in all villages in all local units in all centers for the whole city mustn't exceed the planned one for the whole governorate (Mazen *et al.*, 2013). The critical assumption of the IP is the maximum number of illiterate persons is 25 in each allocated classroom.

There are targets of the class location IP moreover the building of classrooms in the best locations according to all discussed circumstances for the whole city. The 1st goal relates the building classrooms should be maximized to build in the best location. The 2nd goal reflects the total cost for building classrooms in the whole governorate should be reduced. Finally, the goal will minimize the total transportation cost in terms of transportation expenses, transportation time and transportation distance for all students. The objective functions for the multi-objective IP as the following (Talaat, 2008):

$$\sum_{r=1}^R \sum_{m=1}^M \sum_{v=1}^V \sum_{f=0}^F L_{mvf} \tag{1}$$

$$\sum_{r=1}^R \sum_{m=1}^M \sum_{v=1}^V \sum_{f=0}^F Tcost_{mvf} L_{mvf} \tag{2}$$

$$\sum_{r=1}^R \sum_{m=1}^M \sum_{v=1}^V \sum_{f=0}^F TSCOST_{mvf} Cap_{mvf} L_{mvf} \tag{3}$$

$$\sum_{r=1}^R \sum_{m=1}^M \sum_{v=1}^V \sum_{f=0}^F TSTIME_{mvf} Cap_{mvf} L_{mvf} \tag{4}$$

$$\sum_{r=1}^R \sum_{m=1}^M \sum_{v=1}^V \sum_{f=0}^F TSDIST_{mvf} Cap_{mvf} L_{mvf} \tag{5}$$

Where:

- r = Represents the rth center, $r = 1, 2, \dots, R$
- m = Represents the mth local unit in the rth center, $m = 1, 2, \dots, M$
- v = Represents the vth village in the mth local unit in the rth center, $v = 1, 2, \dots, V$
- f = Represents the fth classroom in the vth village in the mth local unit in the rth center, $f = 0, 2, \dots, F$
- L_{mvf} = It is one of the model decision variables, it takes the value 1 if the lth class located in vth village in mth local unit in the rth center and takes 0 otherwise
- Ca_{pmvf} = It is another one of the model decision variables, it represents the number of registered students in the fth class in the vth village in the mth local unit in the rth center
- $Tcost_{mvf}$ = Represents the total cost of the fth class in the vth village in the mth local unit in the rth center total cost of each class equals the sum of the construction (building) cost of the class and the running (teaching) cost of the class
- $TSCOST_{mvf}$ = Represents the transportation cost to the fth class in the vth village in the mth local unit in the rth center
- $TSTIME_{mvf}$ = Represents the transportation time to the fth class in the vth village in the mth local unit in the rth center
- $TSDIST_{mvf}$ = Represents the transportation distance to the fth class in the vth village in the mth local unit in the rth center

A multi-objective optimization problem has a number of objective functions which are to be minimized or maximized. The IP problem has a number of constraints described in appendix which any feasible solution (including the optimal solution) must satisfy. Finally, the class location IP is a multi-objective problem with objectives and a lot of constraints according to the above discussion (Talaat, 2008). Four different sets of constraints are mentioned: the first set related to the total number of classes covering the three levels, village, local unit and center and also the aggregate provenance level, the second set of constraints related to the number of illiteracy inhabitants covering the three levels, the third and the fourth sets of constraints related to the facility costs, and the transportation costs respectively and covering the three spatial levels beside the provenance aggregation level.

OVERVIEW OF PSO

PSO is a stochastic, population-based optimization technique introduced by Kennedy and Eberhart. It belongs to the family of swarm intelligence computational techniques and is inspired by social interaction in human beings and animals (especially bird flocking and fish schooling) (Rezaee Jordehi and Jasni, 2013).

The PSO algorithm belongs to category of the Evolutionary Computation (EC) for solving global optimization problems. In a PSO system, multiple candidate solutions coexist and collaborate simultaneously. Each solution called a particle, flies in the problem search space looking for the optimal position to land. A particle, during the generations, adjusts its position according to its own experience as well as the experience of neighboring particles. PSO system combines local search method (through self-experience) with global search methods (through neighboring experience), attempting to balance exploration and exploitation. A particle status on the search space is characterized by two factors: its position (X_i) and velocity ($Veloc_i$) (Rezaee Jordehi and Jasni, 2013). The weighting factors $Const_1$ and $Const_2$ are used to controlling the balance in moving toward posbest position and gposbest position. The r_1 and r_2 are random numbers constructed from uniform distribution with 0 and 1 parameters. Each particle has a memory and hence it is capable of remembering the best position in the search space ever visited by it. The position corresponding to the best fitness is known as posbest and the overall best out of all the particles in the population is called gposbest. The modified velocity and position of each particle can be calculated using the current velocity and the distance from the posbest to gposbest as shown in the following formulas (El-Sawy *et al.*, 2011):

$$Veloc_i(t+1) = weth \times Veloc_i(t) + Const_1 \times r_1 \times (posbest_i(t) - X_i(t)) + Const_2 \times r_2 \times (gposbest_i(t) - X_i(t)) \quad (6)$$

$$X_i(t+1) = X_i(t) + Veloc_i(t+1) \quad (7)$$

Where:

- i = 1, 2, ..., n
- n = Number of particles
- t = Number of iterations (generations)
- $Veloc_i(t)$ = Velocity of particle i at iteration t
- $X_i(t)$ = Current position of i at iteration t
- weth = Inertia weight factor
- posbest_i = Posbest of particle i
- gposbest_i = Gposbest of the all particles
- $Const_1, Const_2$ = Cognitive and social acceleration factors
- r_1, r_2 = Random numbers uniformly distributed in the range (0, 1)

LOCATION PARTICLE SWARM OPTIMIZATION TECHNIQUE

In a PSO algorithm, multiple candidate solutions coexist and collaborate simultaneously. Each solution called a particle, flies in the problem search space looking for the optimal position to land. A particle, during the generations, adjusts its position according to its own performance as well as the performance of neighboring particles.

The class location IP is a multi-objective problem. It is founded to represent the IP as a multi-objective model with objectives and a lot of constrains related to targets and conditions (Mazen *et al.*, 2013). The position of the particle is L_{mvf} . Initialize the swarm L_{mvb} the position of particles are randomly initialized within the feasible space. In the proposed problem the particle contains two parts as in Fig. 1, the first part is L_{mvb} the position of particle are randomly initialized and takes binary values that takes value 1 if the f th classroom located in v th village in m th local unit in r th center, otherwise 0. The second part is Cap_{mvf} represents the number of registered students in each classroom f th in the v th village in the m th local unit in the r th center such that $Cap_{mvf} \leq Z$ for all r, m, v, f .

The proposed problem requires a particle to encode the decision variables L_{mvb} Cap_{mvf} . Assume number of particles (swarm size) is 50, suppose the iteration number is 1000 and particle's length is 30 multiplied by 2. Suppose L_{mvf} contains 2D matrix swarm size particle's length. PSO starts by initializing the swarm by L_{mvb} the position of particles randomly as the matrix (50x60). The matrix contains the number of rows is swarm size and the number of columns is particle's length multiplied by 2. A PSO begins its search with a random set of solutions (particles). Once a random set of solutions is created, each particle is evaluated. PSO evaluates the performance F for each particle. Fitness is assigned to each particle. The evaluation of a solution means calculating the objective function value and constraint violations (Talaat, 2008) (Fig. 2).

Thereafter, a metric must be defined by using the objective function value and constraint violations to assign a relative merit to the solution (called the fitness). So, the substitution is done by each solution of random population to calculate the 5 objective functions value and >33 constraints violations. This is hardly the case when it comes to solving real world optimization problems. Constraints divide the search space into two divisions feasible and infeasible regions. All Pareto-optimal solutions must also be feasible. Constraints can be of two types quality and inequality constraints. The popular approach to handle the optimization problems with constraints is Penalty function

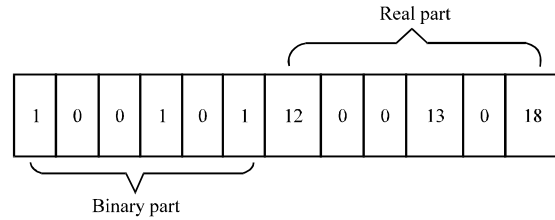


Fig. 1: Example of particle string

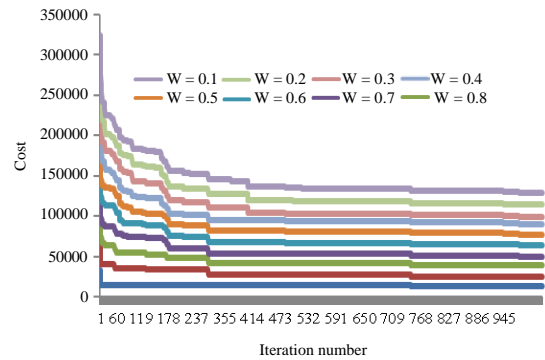


Fig. 2: Optimization cost at $Const_1 = Const_2 = 0.2$ with W in set $\{0, 1\}$

approach (Deb, 2001; Tallaat, 2008). For each solution $m^{(i)}$, the constraint violation for each constraint is calculated as follows:

$$cov_j(m^{(i)}) = \begin{cases} |y_j(m^{(i)})|, & \text{if } y_j(m^{(i)}) < 0 \\ 0, & \text{otherwise} \end{cases} \quad (8)$$

Thereafter, all constraint violations are added together to get the overall constraint violation:

$$scov(m^{(i)}) = \sum_{j=1}^J cov_j(m^{(i)}) \quad (9)$$

This constraint violation is then multiplied with a penalty parameter R_m and the product is added to each of the objective function values:

$$Fn_m(m^{(i)}) = fn_m(m^{(i)}) + R_m \Omega(m^{(i)}) \quad (10)$$

The functions Fn_m takes into account the constraint Ω violations. For a feasible solution, the corresponding term is zero and F_m becomes equal to the original objective function f_m . However, for an infeasible solution, $Fn_m > f_m$, thereby adding a penalty corresponding to total constraint violation. The penalty parameter R_m is used to make both of the terms on the right side of the above equation to have the same order of magnitude. Since, the

Table 1: Optimization cost at (Const₁ = Const₂) with weth in set {0,1}

Const ₁ = Const ₂	Weth								
	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
0.1	10078	10048	12069	13469	12063	9033	12042	10448	12036
0.2	12472	11869	16093	16093	12454	130363	11030	503	17469
0.3	14075	15078	14048	13063	13075	13081	8042	14036	13075
0.4	9075	15096	15063	11466	13081	14454	13066	14493	16078
0.5	15096	18472	15496	15463	11445	1236	15081	15472	15496
0.6	15069	16060	16084	14054	14090	16478	16099	16481	15445
0.7	15460	17057	15487	15102	16090	17520	15072	11066	14060
0.8	15072	15493	14493	10063	15096	14457	15072	15502	13051
0.9	14060	18087	15475	15475	15066	15084	17451	15809	13409

original objective functions could be of different magnitudes, the penalty parameter must also vary from one objective function to another. Once the penalized function is formed, any of the unconstrained multi objective optimization methods can be used with Fn_m (Deb, 2001).

A particle status on the search space is characterized by two factors: its position (X_i) according to Eq. 7 and velocity (Veloc_i) according to Eq. 6. It is noticed the posbest is equal to fitness matrix in the first generation but in each generation, PSO will compare the performance Fn_m of each individual solution to its best performance in posbest matrix. The next step, we will compare of the particles between each other in posbest matrix to get the best fitness gposbest (least objective). The result will be a one particle. It is calculated the best experience of the whole swarm whose position and objective value are called X_i and gposbest. Particles' velocities and positions are updated according to Eq. 6, 7. The parameter Veloc_i (t) (predisposition for each particle) is calculated by Eq. 6 and it will function as a probability threshold to make one of the two decisions (0 or 1). Such a threshold needs to stay in the range of {0, 1}. The sigmoidal function shown in Eq. 11, 12, maps (Jensen *et al.*, 2014; Rezaee Jordehi and Jasni, 2013):

$$\text{Sig}_{(x)} = \frac{1}{1 + \exp(-x)} \tag{11}$$

Final binary decision making is based on the following rule:

$$\left\{ \begin{array}{l} 0 \text{ if } R \text{ and } () \leq \text{sig. } x_i(t+1) \\ 1 \text{ otherwise} \end{array} \right\} \tag{12}$$

It is recommended to change the velocity of the particle according to Eq. 6 and move each particle to a new position using Eq. 7. The result shown of X_i(t+1) by applying (Eq. 11, 12) (Jensen *et al.*, 2014; Rezaee Jordehi and Jasni, 2013).

PSO algorithm will repeat until stopping criterion (usually a prespecified number of iterations or a quality

threshold for objective value) is reached. In the proposed experiment, PSO will be repeated to produce gposbest, optimal solution for the particles by changing its position during 1000 generations and 100 runs to obtain the optimal cost with best fitness for the IP.

EXPERIMENTS AND DISCUSSION

IP is an optimization problem to locate the best locations for classrooms according to the discussed objectives and constraints. For solving the proposed problem, it is recommended to solve by heuristic methods. Some popular global heuristic optimization algorithms include Genetic Algorithms (GA), Particle Swarm Optimization (PSO) may be used to solve the illiteracy mathematical model.

This study discussed IP and suggested to solve this problem by PSO. Initially, the PSO would have the swarm size equal 50. When running PSO system at 1000 generations, the best parameters for the algorithm are the value of (Const₁ and Const₂) equal 0.2 in the second and third part for Eq. 6 and the value of weight weth (the inertia weight weth was important in improving the performance in a number of applications) equal 0.8. It expressed the 100 trials in Table 1 for the algorithm. It is observed the average optimization cost is 5033. It is notably, the average execution time for PSO system is 3.762 sec. We would tried to run PSO system during 100 times with gradually changing for weth (the inertia weight) by values ranging from 0 to 1 at each 10 runs. The minimum cost will be 7439 for the IP problem with weth equal 0.4 and equal constants (Const₁, Const₂ are 0.2) The program will run at 1000 generations, the Fig. 2 will describe the result of costs to build the classrooms in the best locations with 0.2 for the (Const₁ and Const₂) and weth equal values between (0 and 1) for each generation.

CONCLUSION

PSO has been used to solve the illiteracy problem that defined in IP problem which contained three levels of

divisions; center r , local unit m and village v . We try to find the best location of literacy classroom respect to cost constraints. We determine the best parameters for PSO respect to our problem which is $w_{eth} = 0.8$ and $Const_1 = Const_2 = 0.2$. We run 100 times and we found the average minimum cost is 5033 that select the best location of illiteracy classrooms.

The number of publications reporting PSO applications has grown nearly exponential for the last few years. Clearly, the algorithm shines for its simplicity and for the ease with which it can be adapted to different application domains and hybridized with other techniques. The next decade will no doubt see further refinement of the approach and integration with other techniques (Jensen *et al.*, 2014).

APPENDIX

The proposed mathematical model for the problem under investigation has the following sets of constraints:

Constraints related the Number of Classrooms (NOC):

1.
$$\sum_{f=0}^F L_{mvf} \leq MNC_{mv}$$
2.
$$\sum_{v=1}^V \sum_{f=0}^F L_{mvf} \leq MNC_{rm}$$
3.
$$\sum_{m=1}^M \sum_{v=1}^V \sum_{f=0}^F L_{mvf} \leq MNC_r$$
4.
$$\sum_{r=1}^R \sum_{m=1}^M \sum_{v=1}^V \sum_{f=0}^F L_{mvf} \leq MNC$$

Where:

- MNC_{mv} = Represents the maximum NOC could be located in the v th village in the m th local unit in the r th center
- MNC_{rm} = Represents the maximum NOC could be located in the m th local unit in the r th center
- MNC_r = Represents the maximum NOC could be located in the r th center
- MNC = Represents the maximum NOC could be located in the whole governorate

5.
$$5 \leq Cap_{mvf} \leq 25$$

Constraints related the Number of Illiteracy Inhabitants (NII):

6.
$$\sum_{f=0}^F Cap_{mvf} L_{mvf} \leq ITv_{mv}$$
7.
$$\sum_{v=1}^V \sum_{f=0}^F Cap_{mvf} L_{mvf} \leq ITI_{rm}$$
8.
$$\sum_{m=1}^M \sum_{v=1}^V \sum_{f=0}^F Cap_{mvf} L_{mvf} \leq ITc_r$$

9.
$$\sum_{r=1}^R \sum_{m=1}^M \sum_{v=1}^V \sum_{f=0}^F Cap_{mvf} L_{mvf} \leq ITG$$

Where:

- ITv_{mv} = Represents the NII in the v th village in the m th local unit in the r th center
- ITI_{rm} = Represents the NII in the m th local unit in the r th center
- ITc_r = Represents the NII in the r th center
- ITG = Represents NII in the governorate

Constraints related the facility costs:

10.
$$\sum_{f=0}^F Tcost_{mvf} L_{mvf} \leq TVC_{mv}$$
11.
$$\sum_{f=0}^F Scost_{mvf} L_{mvf} \leq SVC_{mv}$$
12.
$$\sum_{f=0}^F Rcost_{mvf} L_{mvf} \leq RVC_{mv}$$
13.
$$\sum_{v=1}^V \sum_{f=0}^F Tcost_{mvf} L_{mvf} \leq TLC_{rm}$$
14.
$$\sum_{v=1}^V \sum_{f=0}^F Scost_{mvf} L_{mvf} \leq RLC_{rm}$$
15.
$$\sum_{v=1}^V \sum_{f=0}^F Rcost_{mvf} L_{mvf} \leq RLC_{rm}$$
16.
$$\sum_{m=1}^M \sum_{v=1}^V \sum_{f=0}^F Tcost_{mvf} L_{mvf} \leq TCC_r$$
17.
$$\sum_{m=1}^M \sum_{v=1}^V \sum_{f=0}^F Scost_{mvf} L_{mvf} \leq SCC_r$$
18.
$$\sum_{m=1}^M \sum_{v=1}^V \sum_{f=0}^F Rcost_{mvf} L_{mvf} \leq RCC_r$$
19.
$$\sum_{r=1}^R \sum_{m=1}^M \sum_{v=1}^V \sum_{f=0}^F Tcost_{mvf} L_{mvf} \leq TCG$$
20.
$$\sum_{r=1}^R \sum_{m=1}^M \sum_{v=1}^V \sum_{f=0}^F Scost_{mvf} L_{mvf} \leq SCG$$
21.
$$\sum_{r=1}^R \sum_{m=1}^M \sum_{v=1}^V \sum_{f=0}^F Rcost_{mvf} L_{mvf} \leq RCG$$

Where:

- $Scost_{mvf}$ = Represents the setup cost of the f th classroom in the v th village in the m th local unit in the r th center
- $Rcost_{mvf}$ = Represents the running cost of the f th classroom in the v th village in the m th local unit in the r th center
- TVC_{mv} = (Total village costs) represents the available total costs of the located classes in the v th village in the m th local unit in the r th center

- SVC_{mv} = (Setup village costs) represents the available setup costs of the located classrooms in the vth village in the mth local unit in the rth center
- RVC_{mv} = (Running village costs) represents the available running costs of the located classes in the vth village in the mth local unit in the rth center
- SLC_m = (Setup local unit costs) represents the available setup costs of the located classes in the mth local unit in the rth center
- RLC_m = (Running local unit costs) represents the available running costs of the located classes in the mth local unit in the rth center
- TLC_m = (Total local unit costs) represents the available total costs of the located classes in the mth local unit in the rth center
- SCC_{mvf} = (Setup center costs) represents the setup cost of the fth classroom in the vth village in the mth local unit in the rth center
- RCC_{mvf} = Running center costs) represents the running cost of the fth classroom in the vth village in the mth local unit in the rth center
- TCC_{mvf} = (Total center costs) represents the total cost of the fth classroom in the vth village in the mth local unit in the rth center
- SCG = Setup cost for governorate) represents the available setup cost of the located classrooms in the governorate
- RCG = Running cost for governorate) represents the available running cost of the located classrooms in the governorate
- TCG = Total cost for governorate) represents the available total costs of the located classrooms in the governorate

Constraints related the transportation costs:

- 22.
$$\sum_{f=0}^F TSCOST_{mvf} Cap_{mvf} L_{mvf} \leq TSVC_{mv}$$
- 23.
$$\sum_{v=1}^V \sum_{f=0}^F TSCOST_{mvf} Cap_{mvf} L_{mvf} \leq TSLC_{rm}$$
- 24.
$$\sum_{m=1}^M \sum_{v=1}^V \sum_{f=0}^F TSCOST_{mvf} Cap_{mvf} L_{mvf} \leq TSCC_r$$
- 25.
$$\sum_{r=1}^R \sum_{m=1}^M \sum_{v=1}^V \sum_{f=0}^F TSCOST_{mvf} Cap_{mvf} L_{mvf} \leq TSCG$$
- 26.
$$\sum_{f=0}^F TSTIME_{mvf} Cap_{mvf} L_{mvf} \leq TSVTIME_{mv}$$
- 27.
$$\sum_{v=1}^V \sum_{f=0}^F TSTIME_{mvf} Cap_{mvf} L_{mvf} \leq TSLTIME_{m}$$
- 28.
$$\sum_{m=1}^M \sum_{v=1}^V \sum_{f=0}^F TSTIME_{mvf} Cap_{mvf} L_{mvf} \leq MTIME_C$$
- 29.
$$\sum_{r=1}^R \sum_{m=1}^M \sum_{v=1}^V \sum_{f=0}^F TSTIME_{mvf} Cap_{mvf} L_{mvf} \leq TSTIMEG$$
- 30.
$$\sum_{f=0}^F TSDIST_{mvf} Cap_{mvf} L_{mvf} \leq TSVDIST_{mv}$$
- 31.
$$\sum_{v=1}^V \sum_{f=0}^F TSDIST_{mvf} Cap_{mvf} L_{mvf} \leq TSLDIST_m$$

- 32.
$$\sum_{m=1}^M \sum_{v=1}^V \sum_{f=0}^F TSDIST_{mvf} Cap_{mvf} L_{mvf} \leq TSCDIST_r$$
- 33.
$$\sum_{r=1}^R \sum_{m=1}^M \sum_{v=1}^V \sum_{f=0}^F TSDIST_{mvf} Cap_{mvf} L_{mvf} \leq TSDISTG$$

- Where:
- TSVC_{mv} = Represents the maximum transportation cost for all students to reach the allocated class in the vth village in the mth local unit in the rth center
 - TSLC_m = Represents the maximum transportation cost for all students to reach the allocated class in the mth local unit in the rth center
 - TSCC_r = Represents the maximum transportation cost for all students to reach the allocated class in the rth center
 - TSCG = Represents the maximum transportation cost for all students to reach the allocated class in the governorate
 - TSVTIME_{mv} = Represents the Maximum transportation time for all students to reach the allocated class in the vth village in the mth local unit in the rth center
 - TSLTIME_m = Represents the maximum transportation time for all students to reach the allocated class in the mth local unit in the rth center
 - TSCTIME_r = Represents the maximum transportation time for all students to reach the allocated class in the rth center
 - TSTIMEG = Represents the maximum transportation time for all students to reach the allocated class in governorate
 - TSVDIST_{mv} = Represents the maximum transportation distance for all students to reach the allocated class in the vth village in the mth local unit in the rth center
 - TSLDIST_m = Represents the maximum transportation distance for all students to reach the allocated class in the mth local unit in the rth center
 - TSCDIST_r = Represents the maximum transportation distance for all students to reach the allocated class in the rth center
 - TSDISTG = Represents the maximum transportation distance for all students to reach the allocated class in the governorate

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