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# A Unified Procedure for Optimization of Stacking Sequence of Laminated Composite Shells Subjected to Linear Buckling

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**Abstract:** A unified procedure for optimizing of laminate stacking sequence of composite cylindrical shells subjected to linear buckling in order to bear maximum critical load is presented. This approach is based on the using of Binary Ant Colony Optimization (BACO) Algorithm and Finite Element Method (FEM). The optimization parameter is fiber angle which will be selected from a finite set of angels. On one hand, BACO is used for optimization process integrating and on the other hand, FEM is applied for calculating critical buckling load. For flexibility of algorithm, the pheromone release on nodes instead of edges is utilized to avoid modeling this problem in the format of Knapsack one. Knapsack problem which has the most similarities with composite stacking sequence is considered for algorithm evaluation. Results have shown that BACO in comparison with other optimization algorithms like genetic and particle swarm, has found the optimal solution in less function evaluations.

**Key words:** Optimization, composite shell, BACO, FEM, linear buckling

## INTRODUCTION

Composites classified as advanced materials are interesting for engineers due to their high strength and low density. In spite of their superior characteristic, they poses numerous parameters in their design like: fiber angel, layer thickness, layer position and fiber-matrix type. Tuning these design parameters makes optimization algorithm deployment inevitable. On one hand, due to inherent complexity of composite design problems, it is needed to pay significant attention for selecting proper range of analysis and optimization (Tsai, 1992). On the other hand, most of the optimization algorithms face with two major drawbacks: local convergence and discrete input parameters. As a result, finding an appropriate approach for fixing these drawbacks is a necessity. As seen due to the literature review, it seems that the best approach is simultaneous implementation of optimization algorithm codes and FEM commercial software.

Usually buckling means losing a stable state in the structure without fracture or rupture of material or at least before them. Most of shells stability problems like many addressed engineering issues could not be solved analytically which makes numerical approach a good substitution. An elastic buckling of a cylindrical shells based on the exact solution is studied (Flugge, 1973). Buckling analysis of an isotropic cylindrical shell which is clamped on one side is presented (Lin and Yeh, 1994). An analytical response of thin composite cylindrical shells under different boundary conditions and loadings is shown (Geier *et al*., 2002).

Optimizing the composite structures in order to increase their buckling capacity has been conducted in the recent decades. Some common design parameters in optimization process of these structures are: number of layers, fiber angles and stacking sequence. Optimizing Cylindrical Composite shells with stiffeners by Ant Colony Optimization (ACO) algorithm is studied (Wang *et al*., 2010) which illustrates the robustness of the algorithm. The optimal stacking sequence of composite shells in order to increase their natural frequency is represented (Koide and Luersen, 2013). In this study, ABAQUS and MATLAB Software have been utilized for finding system response. Literature review shows that Binary Ant Colony Optimization (BACO) algorithm has not been applied for stacking sequence optimization of composite shells subjected to buckling load. In addition, in most of the studies on ACO, the pheromone is resealed on the edges not the nodes. This approach models some problems like stacking sequence optimization in the format of Knapsack problem which increases the level of complexity of the problem.

This study focuses on combined approach based on BACO as the optimizer for stacking sequence of cylindrical composite shells subjected to linear buckling

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and FEM as the solver for calculating the critical buckling load. For flexibility of the approach, the pheromone release on nodes instead of edges, has been utilized (Pahlevanpour, 2013).

## MATERIALS AND METHODS

**Binary Ant Colony Optimization (BACO) algorithm:** This method is one of the Meta heuristic approaches for solving complex optimization problems. The algorithm randomly but purposefully looks for optimized solutions in the problem space. BACO algorithm is based on the indirect relationship between a set of artificial ants by synthetic pheromone as a means of communication. Pheromone duty is to transfer ants' experience to each other in an indirect way (Dorigo and Stutzle, 2000). In the other words, ants will be transformed from one state to another one by using probabilistic approaches on the problem graph. These approaches are based on the amount of pheromone, heuristic Information and problem restrictions. Ants will release pheromone on the edges or nodes. It guides the next ants for optimizing search. It should be mentioned that pheromone will be evaporated gradually and as a result it will enables new routes to search for next ants.

**General form of ant colony algorithm**

```
End ACO metaheuristic
Schedule Activity
Manage Ant Activity ()
        Evaporate Pheromone
        Daemon Action ( )        {Optional}
   End Schedule Activity
   Return S
End ACO metaheuristic
```

In 2000, a new method called BACO was proposed for problems defined in binary continuous or discrete space (Hiroyasu *et al*., 2000). In this method, two parallel strings of zeros and ones (generally from 0-M) are considered. Ants will pass through them and make their own unique path of zeroes and ones. In this space, a specific amount of pheromone will be released based on the rout expenses after finishing ants' trip (Fig. 1). In this method, probability of selecting position 1 for the ant k which is in location 0 at time t will be determined by:

$$P_o^k(t) = \frac{t_{01}}{t_{01} + t_{00}}$$

Where:

$\tau_{01}$ = Pheromone intensity on the connecting path from 0-1

$\tau_{00}$ = pheromone intensity on the connecting path from 0-1 The intensity of released pheromone by ant k on the tripped path is determined by:

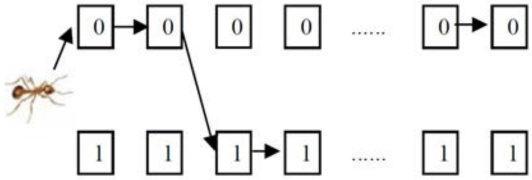

Fig. 1: Stacking sequence creation by each antFig

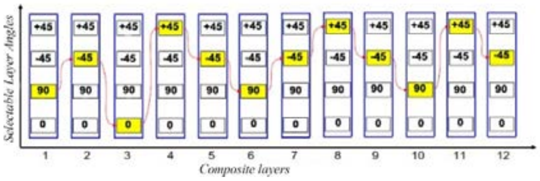

Fig. 2: Stacking sequence creation by each ant

$$\Delta\tau_{01}^k = \begin{cases} \frac{Q}{F_k} \text{if the ant k passes sub} - \text{path}(0 \rightarrow 1) \\ 0 \end{cases}$$

Where:

$F_k$: = The solution cost created by ant k

Q: = Constant determined respect to the upper and lower limits of the problem answer

After all ants choose their route, then Pheromone intensity will be updated by:

$$\Delta\tau_{01}(t, t+1) = \sum_{k=1}^{M} \Delta\tau_{01}^k(t, t+1)$$

Then, pheromone intensity will be updated by:

$$\tau(t, t+1) = p\tau_{01}(t) + \Delta\tau_{01}(t, t+1)$$

Where, p is pheromone evaporation coefficient ranging from 0-1.

**Optimization unified method via combining BACO and FEM:** Firstly, required input parameters for BACO and the number of layers will be determined by user. There are four different options available (0, +45, -45 and 90) as fiber angles for each layer which could be increased. Each ant will start his/her trip form an imaginary node and in the next layer will choose one the mentioned angels based on the level of pheromone on the node (Fig. 2). So, in each evaluation, a stacking sequence will be obtained by BACO via MATLAB code. Then a text file including fiber angles, layers thickness and mechanical property of the intended composite material is created. This text file will

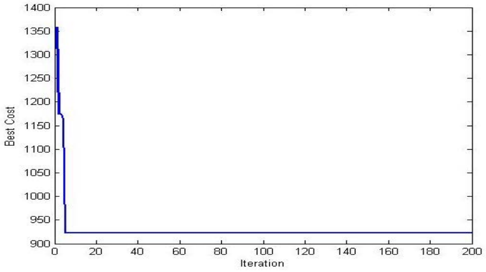

Fig 3: Performance of the BACO with ten variables

be updated during the optimization process. In addition to that text file, there are two other text files. One of them is responsible for creating the model and the other one is responsible for the model response. In each evaluation, the so-called APDL code will calculate the linear buckling load for each ant via ANSYS Software using FEM. Hereafter, optimizer code via MATLAB Software will read the result and by evaluating, it will release pheromone on the passed nodes by ants. This cycle will be continued until termination condition is met (Pahlevanpour, 2013) (Fig. 3).

**Baco verification through Knapsack problem response evaluation:** To compare the results of optimized staking sequence of this research, there was no similar case. As a result, MATLAB code verification is conducted by using Knapsack problem with most similarities to composite stacking sequence problem (Table 1).

In Knapsack problem, some objects with specific value ($v_i$) and specific weight ($w_i$) are available. They should be put in a backpack with overall weight capacity of (W). So, the target is to maximize the value of selected objects or minimize the value of unelected objects (Kellerer *et al*., 2003). This process can been stated in mathematical format as follows:

$$\max \sum_{i=1}^{n} v_i x_i$$

$$\sum\nolimits_{i=1}^{n} w_i x_i \leq W$$

$$x_i \in \{0,1\}$$

Now, transformed fitness function for Knapsack problem can be written by applying the accumulative penalty function:

$$Z = Z + \mu v = \sum_{i=1}^{n} v_i \left(1 - x_i\right)$$

$$\left(\frac{\sum\nolimits_{i=1}^{n} w_i x_i}{W} - 1.0\right)$$

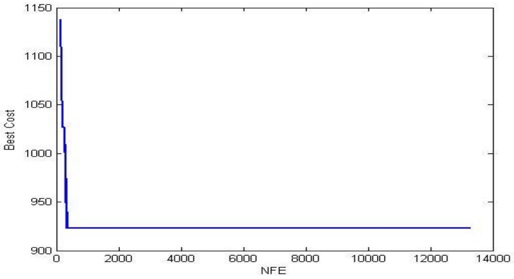

Fig. 4: Performance of the GA with ten variables

Table 1: Knapsack problem similarities with composite stacking sequence problem

| Knapsack problem | Composite Stacking sequence |
|---|---|
| Number of objects | Number of layers |
| Selection of each object | Selection of fiber angles |

Table 2: Comparing the results of BACO, GA and PSO algorithms

| Algorithm | $V_1$ | $V_0 = Z$ | $w_1$ | $w_0$ | NFE |
|---|---|---|---|---|---|
| BACO | 2390 | 923 | 300 | 195 | 100 |
| GA | 2390 | 923 | 300 | 195 | 258 |
| PSO | 2390 | 923 | 300 | 195 | 600 |

$v_1$ = Total value of taken; $v_0$ = Total value of not taken; $w_1$ = Total weight of taken; $w_0$ = Total weight of not taken; NFE = Number of Function Evaluation until final convergence (excellence criterion)

Where:

n = 10
W = 300

Where, $\zeta$ is constant factor which assumed as 10000. Number of objects are ten. Objects weight ranging from 200-500 and their values are ranging from 20-70. These parameters will be generated randomly by MATLAB code. The weight capacity of backpack is 300. (v, m) = [(366, 27), (227, 57), (412, 35), (330, 41), (334, 53), (467, 64), (230, 51), (366, 53), (136, 46), (445, 68)].

Results from BACO algorithm has been compared with genetic algorithm and particle swarm optimization based on the number of evaluations and same input parameters (Table 2).

As illustrated in Table 2, all three algorithms can find the optimized response. However, it can be concluded that BACO has better performance than GA and PSO by comparing Number of Function Evaluation (NFE). Performance of three algorithms has been shown (Fig. 4-6).

**Finite element model of composite shells:** Finding an exact equation to calculate the critical buckling load of composite cylindrical shells is hard and specific requirements for each equation must be established. As a result, approximate solutions for a variety of asymmetric or symmetric orthotropic buckling loads have been presented by different researchers, each of which has limitations and advantages.

For modeling composite material by ANSYS via this study, four nodes shell elements were used. Each node of this element has six degrees of freedom and it is capable of modeling composite layers considering their fiber angels. In order to validate the finite element modeling, study by Geier *et al.* (2002) has been considered. In this study, buckling load is calculated for shells with an internal radius of 250 mm and a length of 510 mm. Shells are made of 10 layers of unidirectional carbon/epoxy with 0.125 mm thickness and the following properties (Table 3). Multi-layer layups named as Z32 and Z33 are:

Z32: [0, 0, +19, -19, +37, -37, +45, -45, +51, -51]
Z33: [-51, +51, -45, +45, -37, +37, -19, +19, 0, 0]

According to the intended study, buckling loads and modes are: Error percentage are 2.6 and 2.8% for Z32 and Z31:

$$Z32: N_{x\,cr} = -62.54\frac{N}{mm}$$

$$Z33: N_{x\,cr} = -114.5\frac{N}{mmm}$$

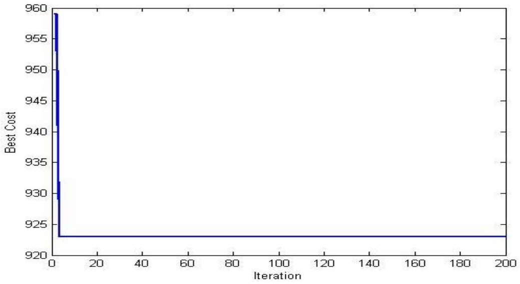

Fig 5: Performance of the PSO with ten variables

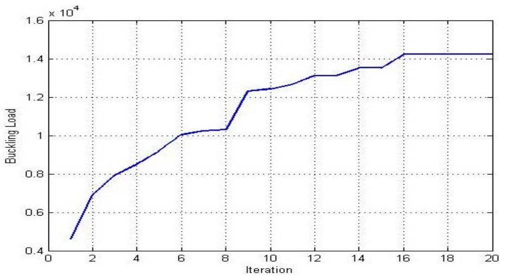

Fig 6: Performance of the algorithm for first case

Table 3: Mechanical properties of Carbon/Epoxy composite

| Index | Unit | Carbon/Epoxy |
|---|---|---|
| EL | MPa | 123550 |
| ET | MPa | 8708 |
| LT | - | 0.32 |
| G# | GPa | 5696 |

For FEA validation, the described shells with the same geometry and boundary conditions are modeled via ANSYS Software and the results are as follows:

$$Z32: N_{x\,cr} = -60.86\frac{N}{mm}$$

$$Z33: N_{x\,cr} = -111.32\frac{N}{mm}$$

Error percentage are 2.6 and 2.8% for Z32 and 31, respectively which indicates the validity of the finite element analysis

## RESULTS AND DISCUSSION

Results for composite shells subjected to linear buckling with different layers and geometries are provided. The geometry and boundary conditions are applied according to the following essential points:

- Boundary condition is clamped in all cases
- Stacking sequence is unsymmetrical
- Fiber Angles are selected from (0, +45, -45 and 90) set for the first three cases and from (+30, 0, -30, +45, -45, 90) set for the fourth case
- The intended number of repetitions for algorithm is termination condition
- The shells' geometry and material have been selected based on the industrial applications
- Number of ants and repetitions can be changed according to the problem type
- The Composite properties can be found in (Table 4)

According to conducted sensitivity analysis and values presented in different papers, the best set of parameters for composites layup are given (Table 5).

**First case:** The properties of six layers composite cylindrical shell geometry and its stacking sequence results are presented (Table 6 and 7): Optimal stacking

Table 4: Composite properties

| Index | Unit | Glass/Epoxy | Graphite/Polymer |
|---|---|---|---|
| EL | GPa | 39 | 155 |
| ET | GPa | 8.6 | 12.1 |
| ?LT | - | 0.28 | 0.248 |
| GLT | GPa | 3.8 | 4.4 |
| Density | Kg/m3 | 2060 | 1500 |
| SytL | MPa | 1080 | 1500 |
| SycL | MPa | -620 | -1500 |
| SytT | MPa | 39 | 35 |
| SycT | MPa | -128 | -230 |
| TLT | MPa | 89 | 65 |

Table 5: The set of parameters for composites layup

| Pheromone influence factor (a) | Initial pheromone | Pheromones evaporation rate |
|---|---|---|
| 0.6 | 0.9 | 0.4 |

Table 6: Six layers composite shell properties (first case)

| Length (m) | Inner radius (m) | Material type | Each layer thickness (m) | No. of layers |
|---|---|---|---|---|
| 6 | 0.6 | Glass/ Epoxy | 0.000125 | 6 |

Table 7: Six layers composite shell results (first case)

| Number of ants | No. repetitions | No. of function evaluation | Buckling load (N) |
|---|---|---|---|
| 12 | 20 | 183 | 14219.44 |

Table 8: Eight layers composite shell properties (second case)

| Length (m) | Inner radius (m) | Material type | Each layer thickness (m) | Number of layers |
|---|---|---|---|---|
| 7 | 0.3 | Graphite/Polymer | 0.000125 | 8 |

Table 9: Eight layers composite shell results (second case)

| Number of ants | Number of repetitions | Number of function evaluation | Buckling load (N) |
|---|---|---|---|
| 16 | 20 | 222 | 71129.48 |

Table 10: 15 layers composite shell properties (third case)

| Length (m) | Inner radius (m) | Material type | Each layer thickness (m) | Number of layers |
|---|---|---|---|---|
| 7 | 0.3 | Graphite/Polymer | 0.000125 | 15 |

Table 11: 15-layers composite shell results (third case).

| Number of ants | No. of repetitions | No of function evaluation | Buckling load (N) |
|---|---|---|---|
| 30 | 35 | 742 | 226803.92 |

Table 12: Six layers composite shell results (fourth case)

| Number of ants | No of repetitions | No of function evaluation | Buckling load (N) |
|---|---|---|---|
| 16 | 20 | 223 | 15524.83 |

sequence: [90, 0, -45, 0, +45, 90]. The performance of the algorithm is shown in Fig 7. The properties of eight layers composite cylindrical shell geometry and its stacking sequence results are presented (Tables 7 and 8): Optimal stacking sequence:

[90, 0, 90, 0, 90, 0, -45, +45]

The performance of the algorithm is shown in Fig. 8 The properties of 15 layers composite cylindrical shell geometry and its stacking sequence results are presented (Tables 9 and 10):

**Optimal stacking sequence:**

[90,90,0,45,0,90,90,-45,90,90,45,90,45,-45,-45]

The performance of the algorithm is shown in Fig 9.

**Fourth case:** The geometry is as ame as first case but the angles are selected form (+30, 0, -30, +45, -45, 90) set. Its stacking sequence results are presented in Table 11. Optimal stacking sequence: [0, 0, 0, +30, -30, 0] The performance of the algorithm is shown in Fig 10. Comparing the results from the first to fourth cases shows that critical buckling load has been increased about 9% by increasing the number of selectable angels.

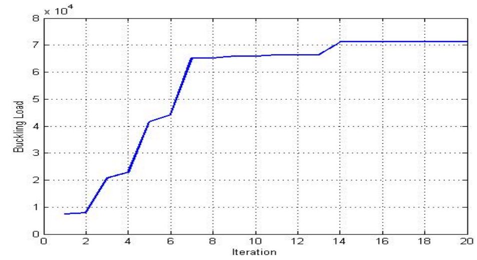

Fig. 7: Performance of the algorithm for second case

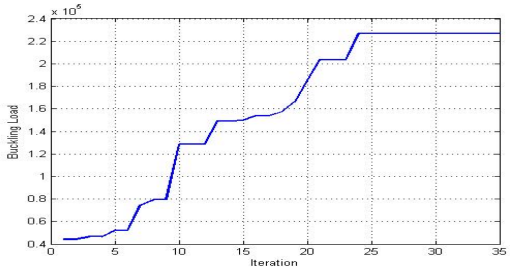

Fig. 8: Performance of the algorithm for third case

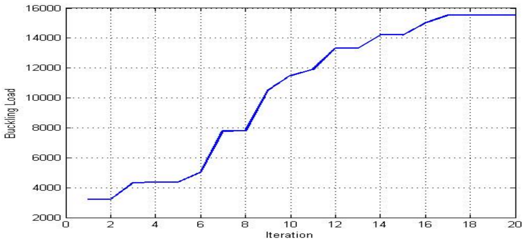

Fig. 9: Performance of the algorithm for fourth case

## CONCLUSION

The optimization of composite cylindrical shells stacking sequence subjected to linear buckling load was presented via a unified method. Fiber angles were taken into account as variables. Finite Element Method (FEM) and Binary Ant Colony Optimization (BACO) algorithm were used for buckling load calculation and optimization, respectively. One of the most important issues in the performance of optimization algorithms is the number of function evaluation to achieve optimal response. The BACO algorithm is performed very well for Knapsack problem and is found the optimal solution in less function evaluation in comparison with GA and PSO algorithms. The results show that the number of chosen layers and angles has a significant effect on increasing the bearable buckling load of shells. Buckling load increases about 68% when the number of layers increase from 8-15. Also, in two identical geometries, buckling load approximately increased 9% by increasing selectable angles from 4-6. Therefore, the optimization unified method can be employed as an efficient tool for composite layup design in a logical framework.

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