

## A Novel Scheme for Selecting Minimum Connected Dominating Set in AD HOC and WSNs

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**Abstract:** Wireless Sensor Networks (WSNs) are sets of energy constrained nodes. To achieve routing between nodes many research works have focused on selecting virtual backbone. This goal can be attained by constructing a Connected Dominating Set (CBS) in the network graph. The CBS can provide an efficient packet routing between nodes. Furthermore, selecting a minimum size CDS (MCDS) can decrease end to end delay, minimize consumed energy and maximize throughput. As a NP-hard problem, constructing MCDS was the aim of many research efforts but stills yet an open issue. On the other hand, although of the data mining tools power but they aren't well applied in WSNs. In this study, Formal Concept Analysis (FCA) is used in a novel proposed scheme. Indeed, FCA provides best dominated nodes selection. Through tests, the new technique seems to select always an optimal MCDS.

**Key words:** Connected dominating set, formal concept analysis, wireless sensor networks, AD HOC network, MCDS, consumed energy

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### INTRODUCTION

WSNs (Wireless Sensor Networks) are sets of sensor nodes controlled by a special node called sink. Last few years, WSNs have attracted the research community due to its low cost and its huge capabilities to collect data from physical world. Indeed, WSNs can be considered as the interface between physical and digital world. Furthermore, WSNs is a set of cooperative sensor nodes. Each node is a small device able to collect data, process and communicate results through wireless channel. On one hand, WSNs as well as AD HOC networks have no predefined infrastructure. On the other hand, sensors are constrained in term of their energy limitation, computing capacity and band width. Consequently, the task of collecting data from sensor nodes to the sink as well as routing between nodes are challenging problem. The selection of a virtual backbone in this kind of network can be considered as a solution to deal with this problem. The creation of a virtual backbone in WSNs can be achieved by constructing a Connected Dominating Set (CDS) in the network graph. Indeed, WSNs can be modelled using an undirected graph  $G(V, E)$  where  $V$  is the set of vertices representing nodes and  $E$  is the set of edges between vertices. An edge exists between two nodes if these nodes can communicate using wireless physical channel. CDS is a subset  $D(V', E')$  of  $G(V, E)$  where:

- All nodes in  $V'$  are connected
- For each node  $u \in V$ ,  $u$  is either  $\in V'$  or there exist a node  $v \in V'$  such that  $(u, v) \in E$

The minimization of CDS size can decrease end to end delay, minimizes consumed energy and maximizes throughput. As an NP-hard problem (Jeremy *et al.*, 2004), many research effort have been realized to deal with this problem. Most of heuristics will be discussed in the related research study.

Recently, the integration of data mining in WSNs tools seems to provide best and improved results (Moulaoui *et al.*, 2016). In this study, Formal Concept Analysis (FCA) is used as a data mining technique to construct MCDS. Indeed, nodes are classified in term of their neighbours numbers. FCA provide best sort of node to decide whatever it will be part of MCDS or not.

**Literature review:** Several research works have focused on selecting MCDS in a graph representing the AD HOC networks (Kaviya and Sankareeswari, 2014). These techniques are summarized in Fig. 1. In what follows, we outline and discuss most of these techniques by category. By the end of this study the goal of this study is shown.

**CDS according to the graph type:** The selection of CDC can be achieved into two types of graphs:

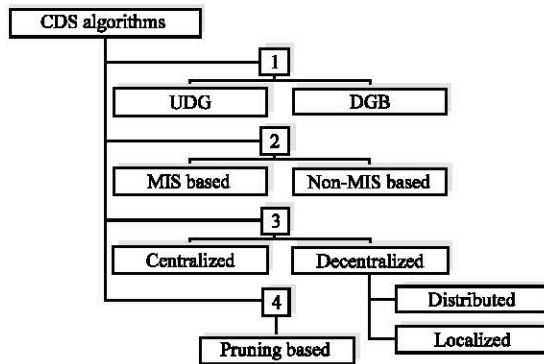


Fig. 1: CDS formation algorithm category

- Unit Disk Graph (UDG): where all nodes have the same transmission range
- Disk Graphs with Bidirectional (DGB): where node transmission range is not the same

Although, the difference between UDG and DGB if exists, the link between any couple of nodes is bidirectional. Whatever, in UDG or in DGB, MCDS is proved as a NP-hard problem (Torkestani and Meybodi, 2010; Liu *et al.*, 2010; Rai *et al.*, 2009; Thai *et al.*, 2007).

**MIS based and non mis based:** An Independent Set (IS) of a graph G can be defined as: a subset of G containing vertices where no two vertices are adjacent. The Maximal Independent Set (MIS) can be defined as: an IS which is not a subset of any other IS. The construction of MCDS based on MIS can be performed into two steps: selecting the set of MIS in the graph; connecting these sets. To be part of an MIS, the optimal nodes selection is based on multiple criterions such as: node remaining energy and node degree. Many research works have focused on selecting MCDS using this approach (Das and Bharghavan, 1997; Rai *et al.*, 2009; Wan *et al.*, 2002; Wu and Li, 1999).

**Centralized algorithms and decentralized algorithms:** Another, set of techniques aiming to construct MCDS (Alzoubi *et al.*, 2002; Das and Bharghavan, 1997; Guha and Khuller, 1998; Wan *et al.*, 2002; Wu and Li, 1999) can be classified into two categories.

**Centralized techniques:** Which are suitable for static AD HOC and WSNs. This kind of methods needs the knowledge of the whole network.

**Decentralized techniques:** Which are suitable for mobile and dynamic AD HOC and WSNs. Indeed, these techniques don't require the knowledge of the whole

networks. Decentralized techniques can be either distributed where the decision process is fully decentralized. The second type of decentralized techniques is called localized algorithms where the decision process requires communications rounds between nodes in addition to be distributed. Most of the distributed algorithm are performed into two steps: find the sets of MIS. Next, connecting them.

**Pruning-based algorithms:** The aim of using pruning rules is to minimize the size of backbone, i.e., the size of CDS (Torkestani and Meybodi, 2010; Azarderakhsh *et al.*, 2006; Butenko *et al.*, 2004; Cidon and Mokryn, 1998; Rai *et al.*, 2009; Wu and Li, 1999). In this kind of techniques, the construction process is started by considering all nodes in the CDS. Next, pruning rules are applied to eliminate redundant nodes in order to achieve the minimum size of the selected CDS.

**Summary and study aim:** Although, the considerable effort made by the research community to resolve constructing MCDS problem but stills open as a research issue. The proposed solutions seem to be similar and using the same point of view. Recently, there is a considerable orientation to apply data mining tools for trying to solve some open issues in AD HOC and WSNs. As example the use of FCA by Moulahi *et al.* (2016) has a significant effect on improving results and opening new horizon in WSNs. The aim of this study is to use FCA for selecting best MCDS.

## MATERIALS AND METHODS

### Preliminaries

**Formal concept analysis:** Formal Concept Analysis FCA attempts to study the concepts when they are formally described, i.e., the context and concepts are completely and accurately defined. It was introduced by Wille (1982) as an application of the lattice theory (Galois). It is based on the research by Barbut (1970) all of lattice theory and has also a solid philosophical base.

**Formal context:** A context is a triple  $(G, M, I)$  where G and M are sets and  $I \subseteq G \times M$ . The element of G are called objects and those of M attributes. The set I is the incidence of the context and it is seen as a binary relation. So,  $gIM$  is used as notation instead of  $(g, m) \in I$ . This notation is said: "The object g has the attribute m".

For  $A \subseteq G$ , we define:  $A' = \{m \in M \mid \forall g \in A: (g, m) \in I\}$  and dually for  $B \subseteq M$ :  $B' = \{g \in G \mid \forall m \in B: (g, m) \in I\}$ ,  $A'$  is the set of attributes shared by all objects of A and  $B'$  is the whole set of objects that have all attributes in B.

**Formal concept:** A formal concept  $(G, M, I)$  is a pair  $(A, B)$  where  $A \subseteq G$  and  $B \subseteq M$  which verifies  $A' = B$  and  $B' = A$ . For a concept  $(A, B)$ , we say that  $A$  is its extension and  $B$  is its intention.

**Concept lattice:** The concept lattice of  $(G, M, I)$  is a set with ordering relation representing the collection of all formal concepts denoted by  $L(G, M, I)$  of  $(G, M, I)$ , i.e.,  $L(G, M, I): \{(A, B) \in 2^G \times 2^M | A' = B, B' = A\}$ .

**Review on graph theory**

**Graph:** As mentioned in the introduction of this study, a network can be presented by a graph. Each object in a graph is denoted as a vertex or a node. The connections between nodes in a network are presented by edges (or links) in a graph. Each edge in a graph joins two distinct nodes. So that, a graph  $G$  can be defined formally as an ordered pair  $G: (V, E)$  where:

- $V$  is a set of vertices or nodes
- $E$  is a set of links or edges between nodes
- Each edge links two vertices, i.e., each element of  $E$  is a pair of elements of  $V$ . In Fig. 2 an graph sample is given

**Adjacency matrix:** For a graph  $G: (V, E)$  with  $n$  vertices ( $V = v_1, v_2, \dots, v_n$ ) the adjacency matrix is an  $n$  by  $n$  binary matrix  $M$  such that:

- If there is a vertex connecting  $v_i$  and  $v_j$  then  $M[i, j] = 1$
- $M[i, j] = 0$  otherwise

The adjacency matrix of the pervious graph is:

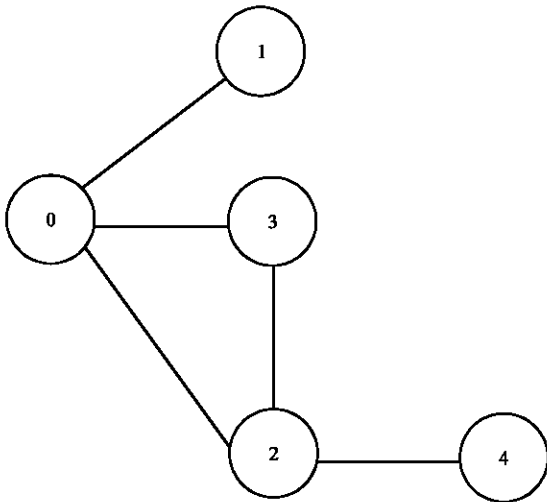


Fig. 2: Graph sample

$$M = \begin{pmatrix} 0 & 1 & 1 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 & 1 \\ 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \end{pmatrix}$$

**RESULTS AND DISCUSSION**

**MCDS through formal concept analysis:** In this study, used symbols are described in Table 1. Furthermore, we show the steps of our scheme by an example. Next, an example of execution is presented. Finally, the algorithm of our scheme is given.

**The proposed techniques by steps:** In Fig. 3, we give an example of a graph which will be used in the following steps. The aim of this subsection is to show by an example, how our proposed technique works.

**Step 1:** A list of concept  $T$  is generated from the adjacency list using FCA techniques through Lattice Minner software. The context is given in Table 2. The Lattice of the previous context is generated in Fig. 4. This

Table 1: Symbols and descriptions

Symbols	Descriptions
$G(V, E)$	The graph of AD HOC or WSNs where $V$ is the set of vertices representing nodes and $E$ is the set of edges between vertices
$M$	The adjacency matrix of $G(V, E)$
$T$	The set of concepts
$L_1$	List of concepts with only one element in their extension
$L_{imposed}$	The set of concepts containing imposed nodes
$L_{concur}$	$L_1/L_{imposed}$
$L_{other}$	$T/L_1$

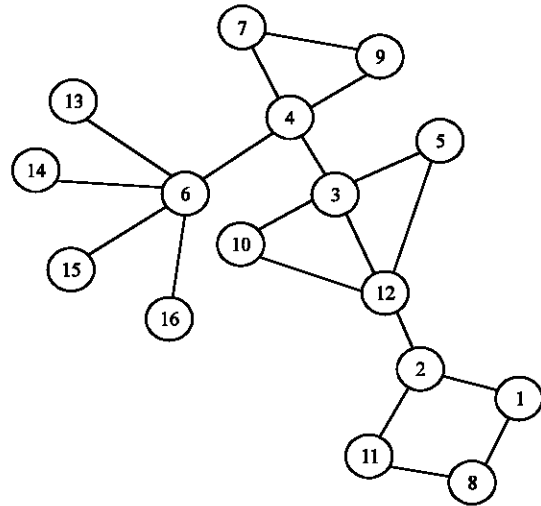


Fig. 3: Graph example

Table 2: Context-adjacency matrix of the graph example

Values	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
1	-	x	-	-	-	-	-	x	-	-	-	-	-	-	-	-
2	x	-	-	-	-	-	-	-	-	-	x	x	-	-	-	-
3	-	-	-	x	x	-	-	-	-	x	-	x	-	-	-	-
4	-	-	x	-	-	x	x	-	x	-	-	-	-	-	-	-
5	-	-	x	-	-	-	-	-	-	-	-	x	-	-	-	-
6	-	-	-	x	-	-	-	-	-	-	-	-	x	x	x	x
7	-	-	-	x	-	-	-	-	x	-	-	-	-	-	-	-
8	x	-	-	-	-	-	-	-	-	-	x	-	-	-	-	-
9	-	-	-	x	-	-	x	-	-	-	-	-	-	-	-	-
10	-	-	x	-	-	-	-	-	-	-	-	x	-	-	-	-
11	-	x	-	-	-	-	-	x	-	-	-	-	-	-	-	-
12	-	x	x	-	x	-	-	-	-	x	-	-	-	-	-	-
13	-	-	-	-	-	x	-	-	-	-	-	-	-	-	-	-
14	-	-	-	-	-	-	x	-	-	-	-	-	-	-	-	-
15	-	-	-	-	-	x	-	-	-	-	-	-	-	-	-	-
16	-	-	-	-	-	x	-	-	-	-	-	-	-	-	-	-

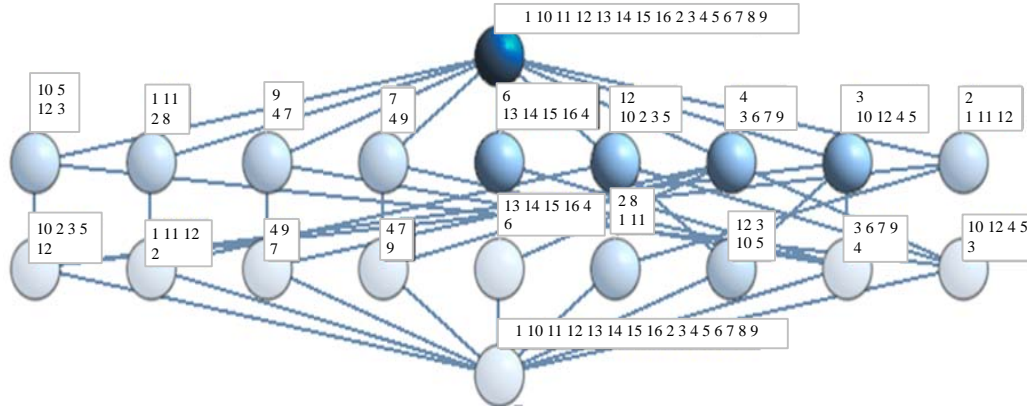


Fig. 4: Lattice

Lattice Minner is generated as a Galois Lattice but we just need the list of concept without relation order. In our example, the list of concepts is:

{1 10 11 12 13 14 15 16 2 3 4 5 6 7 8 9|∅}, {1 11 12|2}, {10 12 4 5|3}, {10 2 3 5|12}, {12 3|10 5}, {13 14 15 16 4|6}, {2 8|1 11}, {3 6 7 9|4} 1 {4 7|9} 1 {4 9|7} 1 {1 11|2 8}, {12|10 2 3 5}, {10 5|12 3}, {4|3 6 7 9}, {2|1 11 12}, {3|10 12 4 5}, {6|13 14 15 16 4}, {7|4 9}, {9|4 7}, {∅|1 10 11 12 13 14 15 16 2 3 4 5 6 7 8 9}

**Step 2:** Selecting  $L_1$  from  $T$ . In our example this list is: {12|10 2 3 5}, {4|3 6 7 9}, {2|1 11 12}, {3|10 12 4 5}, {6|13 14 15 16 4}, {7|4 9}, {9|4 7}.

**Step 3:** Selecting  $L_{imposed}$  from  $L_1$ . In our example, this list is: {12|10 2 3 5}, {2|1 11 12}, {3|10 12 4 5}, {6|13 14 15 16 4}.

We can deduce that the set of imposed node to be part of MCDS is {6, 12, 2}. As example to explain why the Node 12 is considered as imposed because as extension of the concept {12|10 2 3 5}, it exists nodes {10, 5} in the

intension of this same concept where they aren't extension in any concepts of  $L_1$ . However, the Node 7 in {7|4 9} is not considered as imposed because all nodes {4, 9} of this concept's intension are already extension in other concepts of  $L_1$  ({7|4 9} and {9|4 7}). The Node 3 is not taken into consideration because all of their neighbours are covered by the selected set {6, 12, 2}. So, it remains to cover the set {7, 9, 8}.

**Step 4:** Selecting  $L_{concur}$  from  $L_1$ . In our example, this list is: {4|3 6 7 9}, {7|4 9} and {9|4 7}.

The Node 4 is selected because it is imposed and covers Nodes 7 and 9. So, nodes in MCDS are now {6, 12, 2, 4}. So, it remains to cover the set {8}.

**Step 5:**  $L_{other} : TL_1$ . In our example, this list is: {1 10 11 12 13 14 15 16 2 3 4 5 6 7 8 9|∅}, {1 11 12|2}, {10 12 4 5|3}, {10 2 3 5|12}, {12 3|10 5}, {13 14 15 16 4|6}, {2 8|1 11}, {3 6 7 9|4}, {4 7|9}, {4 9|7}, {1 11|2 8}, {10 5|12 3}, {∅|1 10 11 12 13 14 15 16 2 3 4 5 6 7 8 9}.

Only the Node 8 is not covered yet. So, we have to select among the concept {1 11|2 8}. In that case either we

select the Node 1 or 11 because they have the same features. Therefore, MCDS will be {6, 12, 2, 1}. Finally, the Node 3 is added as relay node and the final MCDS is {6, 12, 2, 1, 4, 3}.

After explaining our technique steps by an example, the proposed algorithm is given in the next subsection.

**The proposed algorithm:** In the light of the previous analysis. There are three possibilities to consider a Node 11 as an element of MCDS: it has the maximum degree among unselected node and it is an imposed node. 11 is considered as an imposed node means that there is a node in the graph which is connected only to 11.

The 11 is a concurrent node, i.e., either in the case of circular graph or 11 has the same maximum degree and the same neighbours of another node. n 11 make the links 2 nodes in the MCDS.

According to these assumptions in what follow our Algorithm 1 is given as:

**Algorithm 1; MCDS selection:**

```

Input: The graph G (V; E)
Output: MCDS
M-Adjacency matrix of G (V; E)
T-FCA of M
L1-the list of concept Ci∈T where |ext(Ci)| = 1
Limposed-the list of concept Ci∈L1 such that for all Ci∈L1 and Ci≠Cj∃n∈Int
(Ci) then n∉Ext (Cj)
Sort Ci∈Limposed by decreasing order according to |int (Ci)|
foreach Ci∈Limposed do
if Ci has the maximum number of non covered neighbors then
Add Ci to the set MCDS
Update the list of uncovered node
Lconcur=L1 \ Limposed
Sort Ci∈Lconcur by decreasing order according to |int (Ci)|
//Imposed node from previous list but linked to concurrent nodes
foreach Ci∈Lconcur do
if Ci has no concurrence and has the maximum number of non covered
neighbors then
Add Ci to the set of MCDS
Update the list of uncovered node
Lother=T \ L1
foreach node ni non covered yet do
Find Ci in Lother where ni ∈ int (Ci)
Select a node nj∈ext (Ci), the priority for nodes which are neighbors of a
node in MCDS
Add nj to the set of MCDS
Add ni to the set of MCDS
Update the list pf uncovered node
// Optimization
Verify the connectivity of MCDS and add relay nodes
return MCDS
    
```

**Simulation study:** In the simulation study we use Matlab and Lattice Minner. In Fig. 5, the simulation environment is described. Our scheme execution is performed in 4 phases:

- Phase 1: using Matlab, generate a graph representing the network. This generation can be realized randomly or the user can input his own graph

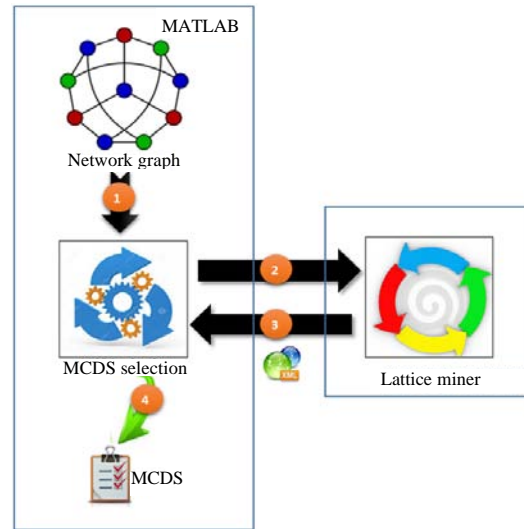


Fig. 5: Simulation environment

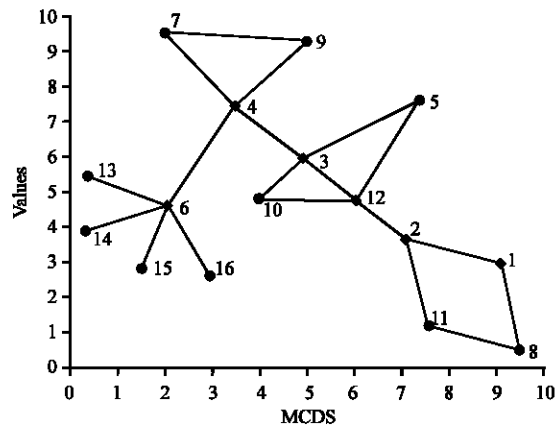


Fig. 6: The selected MCDS is {6,4,3,12,2,1}

- Phase 2: the adjacency matrix is created and sent to lattice Minner as a concept
- Phase 3: at the Lattice Minner side, the Lattice of the graph context is created and sent back to the Matlab side as an XML file
- Phase 4: select the MCDS

We first execute our scheme on the graph of the previous section. The result of MCDS selection is given in Fig. 6. We can note that the selected set is optimal. To evaluate our scheme many tests have been performed. Figure 7 shows the results of using our techniques to select MCDS in small networks. We can note that the results are optimal. Figure 8-10 show the results of using our techniques to select MCDS in random networks. We can note that the results are optimal.

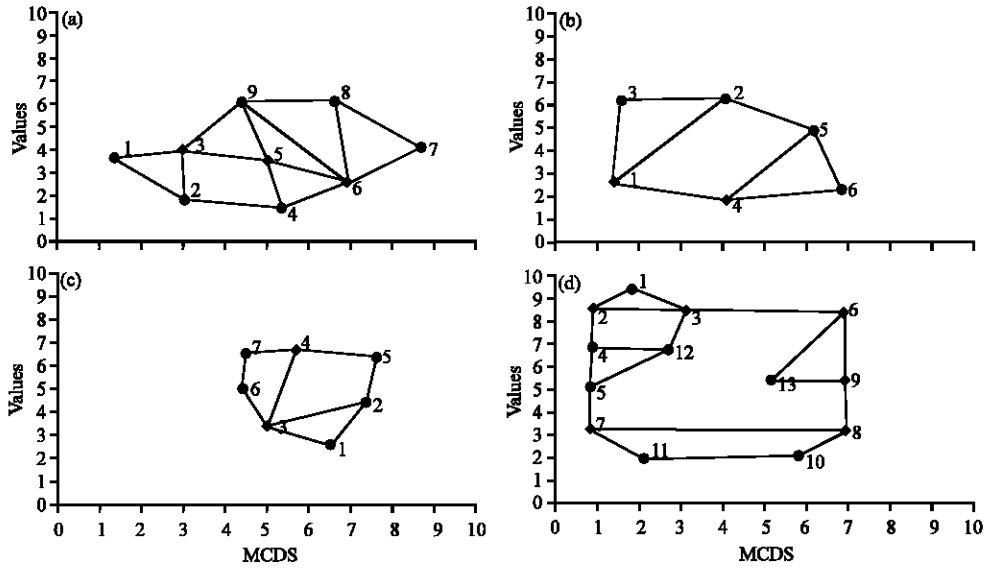


Fig. 7: Selected MCDS

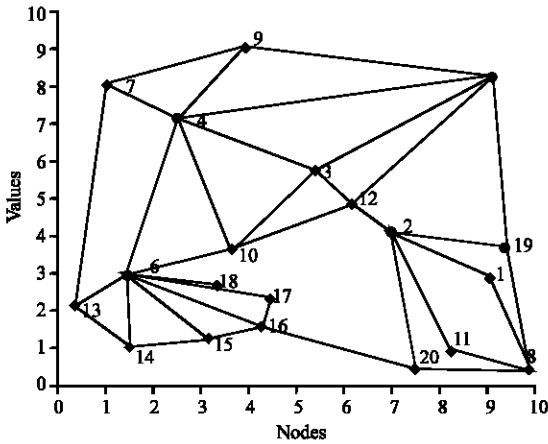


Fig. 8: Selected MCDS with 20 nodes

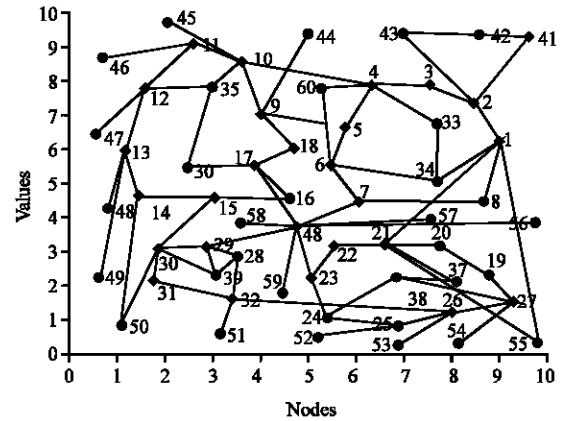


Fig. 10: Selected MCDS with 60 nodes

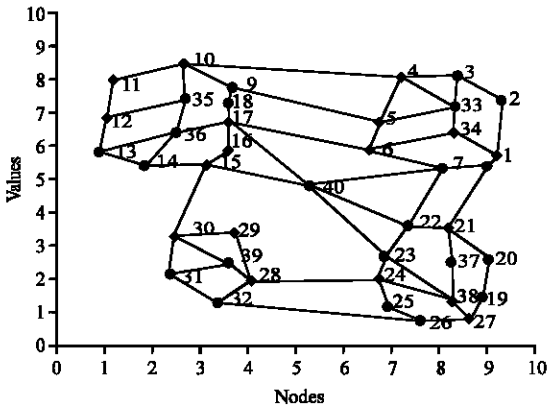


Fig. 9: Selected MCDS with 40 nodes

### CONCLUSION

Many researches have focused on selecting MCDS in AD HOC and sensor networks. Indeed MCDS are used to minimize the communication costs in terms of energy minimization, throughput and end-to-end delay. As a NP-hard problem, many heuristics and approximation techniques have been proposed by the research community to solve this problem. In this study, we use FCA to achieve this goal. To the best of our knowledge, exploring FCA technique to construct MCDS is an innovative approach. According to various test we have made, our proposed technique seems to provide optimal solutions. Nevertheless, a Mathematical proof of what we propose can be the subject of a possible extension of this research.

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