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Operation Continuous Function and Lipschitz Function on Partial Metric Space

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Abstract: Ordered pairs form of a metric space (S, d) where d is the metric on a nonempty Set S. Concept of partial metric space is a minimal generalization of a metric space where each x∈S, d(x, x) does not need to be zero in other terms is known as non-self distance. Axiom obtained from the generalization is following properties p(x, x)≤p(x, y) for every x, y∈S. The results of this study are few studies in the form of definitions and theorems concerning continuity function and Lipschitz function of partial metric space. This study also includes a study connection between Lipschitz functions and uniformly continuous functions on partial metric space.

Key words: Continuity, Lipschitz function, metricspace, uniformly continuous, Axiom, generalization

INTRODUCTION

There has been a lot of mathematicians interested in developing the study of metric spaces that appear various generalization of metric spaces. One generalization of a metric space is a partial metric space developed by Matthews (1992). Several studies on partial metric space has ever done in 2005 Romaguera and Schellekens researching on the basic concept of a partial metric space in the quantitative domain theory in journals titled "Partial Monoids and Semivaluation Metric Spaces". Subsequently, Wahyuni (2012) in the journal entitled "Topology of Partial Metric" examines the topology built by base ball open partial metric.

Subsequently, Devi Arintika (2014) have discussed about Banach fixed point theorem applicable on a partial metric spaces in his journal, entitled "Generalitation Banach Fix Point Theorem on Partial Metric Space". Then, Ge Xun and Lin Shou examines the existence and uniqueness theorems for completion of a partial metric space in the journal entitled "Completion of Partial Metric Spaces".

In 2016, researchers have observed about concepts continuity of function on partial metric space which is defined by p(x, y) = |x-y|+|x|+|y|/2. The result of this research are definition of the partial metric subspace, definition and properties of continous function on partial metric space, definition of a bounded sequence on partial metric space and its relation with sequence convergency.

In this study, the researchers will continue about operation of continous function, uniformly continous functions and lipschitz function on partial metric space. The partial metric space will be observed in this study is p(x, y) = (|x-y|+|x|+|y|/2).

MATERIALS AND METHODS

Metric and metric space; Definition 1 (Bartle and Sherbert, 2011): Let S is nonempty Set. A function d: S×S→ℝ called metric if it satisfies the following 4 properties:

- d(x, y)≥0 for all x, y∈S
- d(x, y) = 0 if and only if x = y
- d(x, y) = d(y, x) for all x, y∈S
- d(x, y)≤d(x, z)+d(z, y) for all x, y, z∈S

If d is metric on non-empty set S, then S called metric Space as described in the following definition:

Definition 2 (Thomson *et al.*, 2008): Metric space is a set of pairs (S, d) where S is nonempty Set and d is a metric on S.

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Partial metric and partial metric space: Partial metric space is a generalization of metric spaces. Consider the following definitions.

Definition 3 (Romaguera and Schellekens, 2005): A partial metric on nonempty set S is a function p: $S \times S \rightarrow \mathbb{R}$ such that for every x, y, z$\in$S satisfy the following axioms:

- $p(x, x) = p(x, y) = p(y, y)$ if and only if $x = y$
- $p(x, x) \leq p(y, x)$
- $p(x, y) = p(y, x)$
- $p(x, z) \leq p(x, y)+p(y, z)-p(y, y)$

According to Malhotra *et al.* (2014) says that a partial metric space is a pair (S, p) such that S is nonempty set and p is metric partial. The function on the partial metric is a generalization of the axiom minimal metrics such that for every x$\in$S, d(x, x) does not need to be zero in other terms is known as nonzero self-distance. Axiom obtained from the generalization is following properties $p(x, x) \leq p(y, x)$ (Wahyuni, 2012).

Definition of partial metrics and partial metric spaces more easily understood by observing the following example.

Example 1: Let S = $\mathbb{R}$. Defined functions p: $\mathbb{R} \times \mathbb{R} \rightarrow \mathbb{R}^+$ as follows:

$$p\,(x, y) = \frac{|x-y|+|x|+|y|}{2}$$

$\forall$x, y$\in\mathbb{R}$ show that (S, p) is partial metric space!.

Solution: Given a nonempty set S = $\mathbb{R}$ with p: $\mathbb{R} \times \mathbb{R} \rightarrow \mathbb{R}^+$ where $p(x, y) = |x-y|+|x|+|y|/2$ for all x, y$\in\mathbb{R}$ will be shown that (S, d) is partial metric space, means non-empty S with function p: $\mathbb{R} \times \mathbb{R} \rightarrow \mathbb{R}^+$ where $p(x, y) = |x-y|+|x|+|y|/2$ for all x, y, z$\in\mathbb{R}$ must following four properties from Definition 3. Properties 1-4 and already clearly fulfilled for $p(x, y) = |x-y|+|x|+|y|/2$ so (S, p) is partial metric space. The following definitions of the partial metric subspace which is as follows.

Definition 4 (Aryani *et al*, 2016): If (S, p) is partial metric space and A$\subseteq$S, then (A, p) called partial metric subspace (S, p). The following definitions will explain definition function and properties of continuous function on partial metric space.

Continuity function on partial metric space

Definition 5 (Aryani *et al*, 2016): Let (S_1, p) and (S_2, p) is partial metric space, $A \subseteq S_1$, function f: $A \rightarrow S_2$ and c$\in$A. Function f continous at c, if for any ε>0 there is δ>0 such that if x$\in$A:

$$p(x, c)-p(x, x)<\delta \text{ and } p(x, c)-p(c, c)<\delta$$

Then:

$$p(f(x), f(c))-p(f(x), f(x))<\varepsilon$$

And:

$$p(f(x), f(c))-p(f(c), f(c))<\varepsilon$$

Theorem 1 (Aryani *et al.*, 2016): Let (S_1, p) and (S_2, p) are partial metric space and $A \subseteq S_1$. If f: $A \rightarrow S_2$ and c is limit point of A, then f only have one limit point at c (uniqueness of limits).

After we get concepts of the limit of a function on partial metric space, then we will observe about concepts of continuous functions on partial metric space. The following definition explain about continuous function on partial metric space at a point.

Definition 6 (Aryani *et al.*, 2016): Let (S_1, p) and (S_2, p) are partial metric space, $A \subseteq S_1$, function f: $A \rightarrow S_2$ and c$\in$A. f continuous at c, if for each ε>0 there exist δ >0 such that if x$\in$A:

$$p(x, c)-p(x, x)<\delta \text{ and } p(x, c)-p(c, c)<\delta$$

Then:

$$p(f(x), f(c))-p(f(x), f(x))<\varepsilon$$

And:

$$p(f(x), f(c))-p(f(c), f(c))<\varepsilon$$

To understand continuity of partial metric spaces will be given in the following example.

Example 2: Given (S_1, p) and (S_2, p) is partial metric space, and $A \subseteq S_1$. Show that function f: $A \rightarrow S_2$defined on A with f(x) = x is continuos.

Solution: Given f (x) = x will be shown for any ε>0 there is δ>0 such that if x$\in$A:

$$p(x, c)-p(x, x)<\delta \text{ and } p(x, c)-p(c, c)<\delta$$

Then:

$$p(x, c)-p(x, x)<\varepsilon$$

And:

$$p(x, c)-p(c, c)<\varepsilon$$

If ε>0 is given choose $\delta = \varepsilon$. Consider that if x$\in$A:

$$p(x, c)-p(x, x)<\delta \text{ and } p(x, c)-p(c, c)<\delta$$

Then:

$$p\big(f(x), f(c)\big)-p\big(f(x), f(x)\big) = p(x,c)-p(x,x)<\partial = \varepsilon$$

And:

$$p(f(x), f(c))\text{-}p(f(c), f(c)) = p(x,c)\text{-}p(c,c) < \partial = \varepsilon$$

Therefore, f(x) = x is continous on A. Next will be given definition of bounded sequence on a partial metric space and its relation with sequence convergency.

Definition 7 (Aryani *et al*., 2016): A sequence x = $\{x_n\}$ on partial metric space is bounded if there exist M>0 such that:

$$|x_n| \leq M$$

for all n∈ℕ. Sequence convergent of partial metric space is bounded as described in the following Theorem 2.

Theorem 2 (Aryani *et al*., 2016): A sequence $\{x_n\}$ convergent of partial metric space is bounded.

Lipschitz function on ℝ

Definition 8 (Bartle and Sherbert, 2011): Let A⊆ℝ, function f: A→ℝ. Function f called uniformly continuos on A, if for every ε>0 there exist δ(ε)>0 such that if x, u∈A satisfies |x-u|<δ(ε) then |f(x)-f(u)|<ε. Continuos function on closed bounded interval I is uniformly continous on I. It is describe in Theorem 3.

Theorem 3 (Bartle and Sherbert, 2011): Let I be closed bounded interval and f: I→ℝ continuos on I, then f uniformly continuos on I. Next, we define about Lipschitz function.

Definiftion 9 (Bartle and Sherbert, 2011): Let A⊆ℝ and f: A→ℝ. If there exist a constanta K>0 such that:

$$|f(x)\text{-}f(u)| < K\,|x\text{-}u| \qquad (1)$$

For every x, u∈A, then f is called Lipschitz function on A. If is Lipschitz function then f uniformly continuos, described in Theorem 4.

Theorem 4 (Bartle and Sherbert, 2011): If f: A→ℝ be Lipschitz function, then f uniformly continuos on A. Next, we give example of application of Theorem 4.

Example 3: Given f: A→ℝ with f(x) = 2x. Show that f uniformly continuos on A (use concept of Lipschitz function).

Solution: It is easy to check that |f(x)-f(u)| = |2x-2u|≤2|x-u| for every x, u∈A. Hence, f satisfies Eq. 1 with K = 2, so, f is Lipschitz function. Use Theorem 2, we get f uniformly continuos on A.

RESULTS AND DISCUSSION

Operation continuous function on partial metric space: Sum and difference two functions continuous on partial metric space are continuos which is as follows.

Theorem 5: Let (S_1, p) and (S_2, q) are partial metric space, and $A \subseteq S_1$. Function f: $A \to S_2$, g: $A \to S_2$ and c∈A. If function f and g continuous at c, then:

- f+g continuous at c
- f-g continuous at c

Proof: Let f and g continuous at c will be shown f+g continuous at c. Since, f continous at c, means for any ε>0 there is δ′(ε/2)>0 such that if x∈A:

$$p(x, c)\text{-}p(x, x) < \delta'(\varepsilon/2) \text{ and } p(x, c)\text{-}p(c, c) < \delta'(\varepsilon/2)$$

Then:

$$p(f(x),f(c))\text{-}p(f(x), f(x)) < (\varepsilon\text{-}2p(f(c), f(c)))/2$$

And:

$$p(f(x),f(c))\text{-}p(f(c), f(c)) < (\varepsilon\text{-}2p(f(c), f(c)))/2$$

Next, for g continous at c there is also δ″(ε/2)>0 such that if x∈A:

$$p(x, c)\text{-}p(x, x) < \delta''(\varepsilon/2) \text{ and } p(x, c)\text{-}p(c, c) < \delta''(\varepsilon/2)$$

Then:

$$p(g(x), g(c))\text{-}p(g(x), g(x)) < (\varepsilon\text{-}2p(g(c), g(c)))/2$$

And:

$$p(g(x),g(c))\text{-}p(g(c),g(c)) < (\varepsilon\text{-}2p(g(c),g(c)))/2$$

Taken any ε>0, choose δ = min {δ′(ε/2)}, δ″ (ε/2) if x∈A:

$$p(x, c)\text{-}p(x, x) < \delta \text{ and } p(x, c)\text{-}p(c, c) < \delta$$

Then:

$$p(f(x)+g(x), f(c)+g(c))\text{-}p(f(x)+g(x), f(x)+g(x))$$

$$< \left(\frac{\varepsilon\text{-}2p(f(c), f(c))}{2} + p(f(c), f(c))\right) + \left(\frac{\varepsilon\text{-}2p(g(c), g(c))}{2} + p(g(c), g(c))\right) = \frac{\varepsilon}{2} + \frac{\varepsilon}{2} = \varepsilon$$

And:

$$p\left(f(x)+g(x),f(c)+g(c)\right)-p\left(f(c)+g(c),f(c)+g(c)\right)$$

$$<\left(\frac{\varepsilon-2p\left(f(c), f(c)\right)}{2}+p\left(f(c), f(c)\right)\right)+\left(\frac{\varepsilon-2p\left(g(c), g(c)\right)}{2}+p\left(g(c), g(c)\right)\right)=\frac{\varepsilon}{2}+\frac{\varepsilon}{2}=\varepsilon$$

So, for any $\varepsilon>0$ there is $\delta>0$ such that if $x\epsilon A$:

$$p(x,c)-p(x,x)<\delta \text{ and } p(x,c)-p(c, c)<\delta$$

Then:

$$p(f(x)+g(x),\ f(c)+g(c))-p(f(x)+g(x),\ f(x)+g(x))<\varepsilon$$

And:

$$p(f(x)+g(x),\ f(c)+g(c))-p(f(c)+g(c),\ f(c)+g(c))<\varepsilon$$

Proven, f+g continous at c. The same argument can be used to show that f-g continous at c. Multiplication and division of two functions continuous on partial metric spaces are continous as be explained in the following Theorem 5.

Theorem 5: Let (S_1, p) and (S_2, q) are partial metric spaces and $A\subseteq S_1$. Let functions f: $A\rightarrow S_2$, g: $A\rightarrow S_2$ and $c\epsilon A$. If functions f and g continous at c, then:

- f.g continous at c
- f/g continous at with $g\neq 0$

Proof: Let f and g continous at c will be shown that f.g continous at c. Consider that:

$$p\left(f(x)g(x),\ f(c)g(c)\right)-p\left(f(x)g(x),\ f(x)g(x)\right)\leq\left|g(x)\right|\left(p\left(f(x),\ f(c)\right)-p\left(f(x),\ f(x)\right)\right)+\left|f(c)\right|\left(p\left(g(x),\ g(c)\right)-p\left(g(x),\ g(x)\right)\right)$$

And:

$$p\left(f(x)g(x),\ f(c)g(c)\right)-p\left(f(c)g(c),\ f(c)g(c)\right)\leq\left|g(x)\right|\left(p\left(f(x),\ f(c)\right)-p\left(f(c),\ f(c)\right)\right)+\left|f(c)\right|\left(p\left(g(x),\ g(c)\right)-p\left(g(c),\ g(c)\right)\right)$$

Based on Theorem 1 there is real number $M_1>0$, so that, $|g(x)|<M_1$ for any $x\epsilon S_1$, then select $M = \sup\{M_1, |f(c)|\}$ therefore:

$$p\left(f(x)g(x),\ f(c)g(c)\right)-p\left(f(x)g(x),\ f(x)g(x)\right)\leq M\left(p\left(f(x),\ f(c)\right)-p\left(f(x),\ f(x)\right)\right)+M\left(p\left(g(x),\ g(c)\right)-p\left(g(x),\ g(x)\right)\right)$$

And:

$$p\left(f(x)g(x),\ f(c)g(c)\right)-p\left(f(c)g(c),\ f(c)g(c)\right)\leq M\left(p\left(f(x),\ f(c)\right)-p\left(f(c),\ f(c)\right)\right)+M\left(p\left(g(x),\ g(c)\right)-p\left(g(c),\ g(c)\right)\right)$$

Since, f continous at c, means for any $\varepsilon>0$ there is $\delta'>0$ such that if $x\epsilon A$:

$$p(x,\ c)-p(x,\ x)<\delta' \text{ and } p(x,\ c)-p(c,\ c)<\delta''$$

Then:

$$p\left(f(x),\ f(c)\right)-p\left(f(c),\ f(c)\right)<\frac{\varepsilon}{2M}$$

And:

$$p\left(f(x),\ f(c)\right)-p\left(f(c),\ f(c)\right)<\frac{\varepsilon}{2M}$$

Next, for g continous at c there is also $\delta''>0$ such that if $x\epsilon A$:

$$p(x,\ c)-p(x,\ x)<\delta'' \text{ and } p(x,\ c)-p(c,\ c)<\delta''$$

Then:

$$p\left(g(x),\ g(c)\right)-p\left(g(x),\ g(x)\right)<\frac{\varepsilon}{2M}$$

And:

$$p\left(g(c),\ g(c)\right)-p\left(g(c),\ g(c)\right)<\frac{\varepsilon}{2M}$$

Taken any $\varepsilon>0$, then select $\delta = \min\{\delta', \delta''\}$, if $x\epsilon A$:

$$p(x,\ c)-p(x,\ x)<\delta \text{ and } p(x,\ c)-p(c,\ c)<\delta$$

Then:

$$p\left(f(x)g(x),\ f(c)g(c)\right)-p\left(f(x)g(x),\ f(x)g(x)\right)\leq M\left(p\left(f(x),\ f(c)\right)-p\left(f(x),f(x)\right)\right)+M\left(p\left(g(x),\ g(c)\right)-p\left(g(x),g(x)\right)\right)<M\left(\frac{\varepsilon}{2M}\right)+M\left(\frac{\varepsilon}{2M}\right)=\varepsilon$$

And:

$$p\left(f(x)g(x),\ f(c)g(c)\right)-p\left(f(c)g(c),\ f(c)g(c)\right)\leq M\left(p\left(f(x),\ f(c)\right)-p\left(f(c),f(c)\right)\right)+M\left(p\left(g(x),g(c)\right)-p\left(g(c),g(c)\right)\right)<M\left(\frac{\varepsilon}{2M}\right)+M\left(\frac{\varepsilon}{2M}\right)=\varepsilon$$

So, for any $\varepsilon>0$ there is $\delta>0$ such that if $x\epsilon A$:

$$p(x, c)-p(x, x)<\delta \text{ and } p(x, c)-p|(c, c)<\delta$$

Then:

$$p\big(f(x)g(x), f(c)g(c)\big)-p\big(f(x)g(x), f(x)g(x)\big)<\varepsilon$$

And:

$$p\big(f(x)g(x), f(c)g(c)\big)-p\big(f(c)g(c), f(c)g(c)\big)<\varepsilon$$

Proven, f.g continous at c. The same argument can be show that f/g continuous at c. Next, if $A\subseteq S_1$ and $f(A)\subseteq S_2$. Functions f: $A\rightarrow S_2$ continuous at $c\in A$ and g: $f(A)\rightarrow S_3$ continuous at $b = f(c)\in f(A)$, then function composition $g\circ f$ continuous at c. As be explained in the following theorem.

Theorem 6: Let (S_1, p), (S_2, p) and (S_3, p) are partial metric spaces, $A\subseteq S_1$ and $f(A)\subseteq S_2$. Let functions f: $A\rightarrow S_2$ and g: $f(A)\rightarrow S_3$. If f continous at $c\in A$ and g continous at $b = f(c)\in f(A)$, then $g\circ f$ continous at c.

Proof: Let f continous at $c\in A$ and g continous at $b = f(c)\in f(A)$ will be shown that for any $\epsilon>0$ there is $\delta>0$ such that if $x\in A$:

$$p(x, c)-p(x, x) < \delta \text{ and } p(x, c)-p(c, c) < \delta$$

Then:

$$p\big((g\circ f)(x), (g\circ f)(c)\big)-p\big((g\circ f)(x), (g\circ f)(x)\big)<\varepsilon$$

And:

$$p\big((g\circ f)(x), (g\circ f)(c)\big)-p\big((g\circ f)(c), (g\circ f)(c)\big)<\varepsilon$$

Taken any $\varepsilon>0$, since, g continous at $b = f(c)\in f(A)$, then there is $\mu>0$ such that for any $y\in f(A)$ with:

$$p\big(y, f(c)\big)-p(y, y)<\mu \text{ and } p\big(y, f(c)\big)-p\big(f(c), f(c)\big)<\mu$$

Then:

$$p\big(g(y), g(f(c))\big)-p\big(g(y), g(y)\big)<\varepsilon$$

And:

$$p\big(g(y), g(f(c))\big)-p\big(g(y), g(f(c))\big)<\varepsilon$$

Besides that, f continous at $c\in f$ A, then for $\mu> 0$, there is $\delta>0$ such that for any $\in f(A)$ with:

$$p(x,c)-p(x,x)<\delta \text{ and } p(x,c)-p(c,c)<\delta$$

Then:

$$p\big(f(x), f(c)\big)-p\big(f(x), f(x)\big)<\mu$$

And:

$$p\big(f(x), f(c)\big)-p\big(f(c), f(c)\big)<\mu$$

Thus, obtained:

$$p\big(g(f(x)), g(f(c))\big)-p\big(g(f(x)), g(f(x))\big) = p\big((g\circ f)(x), (g\circ f)(c)\big)-p\big((g\circ f)(x), (g\circ f)(x)\big)<\varepsilon$$

And:

$$p\big(g(f(x)), g(f(c))\big)-p\big(g(f(c)), g(f(c))\big) = p\big((g\circ f)(x), (g\circ f)(c)\big)-p\big((g\circ f)(c), (g\circ f)(c)\big)<\varepsilon$$

So, for any $\epsilon>0$ there is $\delta>0$ such that if x, $u\in A$:

$$p(x, c)-p(x, x)<\delta \text{ and } p(x, c)-p(c, c)<\delta$$

Then:

$$p\big((g\circ f)(x), (g\circ f)(c)\big)-p\big((g\circ f)(x), (g\circ f)(x)\big)<\varepsilon$$

And:

$$p\big((g\circ f)(x),(g\circ f)(c)\big)-p\big((g\circ f)(c),(g\circ f)(c)\big)<\varepsilon$$

Proven, $g\circ f$ continous at c. After defining some concepts of continous functions on partial metric spaces will be determined concepts of Lipschitz functions on partial metric spaces.

Lipschitz function of partial metric space: Lipschitz functions of partial metric space linked to the uniformly continuous function of partial metric space. Therefore, before defining the function Lipschitz of partial metric spaces will be given first definition of uniformly continuous function of partial metric spaces. Consider the following definitions.

Definition 10: Let (S_1, p) and (S_2, p) are partial metric space, $A\subseteq S_1$ and function f: $A\rightarrow S_2$. Function f uniformly continuous on A, if for any $\varepsilon>0$ there is $\delta>0$ such that if x, $u\in A$:

$$p(x,u)-p(x, x)<\delta \text{ and } p(x,u)-p(u,u)<\delta$$

Then:

$$p\big(f(x), f(u)\big)-p\big(f(x), f(x)\big)<\varepsilon$$

And:

$$p\big(f(x), f(u)\big)-p\big(f(u), f(u)\big)<\varepsilon$$

Furthermore, it would be given the definition of Lipschitz functions on artial metric spaces which are as follows.

Definition 11: Let (S_1, p) and (S_2, p) are partial metric space, $A \subseteq S_1$ and function f: $A \rightarrow S_2$. If there is constants K>0 such that:

$$p(f(x), f(u)) - p(f(x), f(x)) < K(p(x,u) - p(x,x)) \quad (2)$$

And:

$$p(f(x), f(u)) - p(f(u), f(u)) < K(p(x,u) - p(u,u)) \quad (3)$$

For each x, u∈A, then f called Lipschitz function on A. As described at the beginning that Lipschitz functions of partial metric space linked to the uniformly continuous function of partial metric space. This is explained in the following Theorem 7.

Theorem 7: Let (S_1, p) and (S_2, p) are partial metric space and $A \subseteq S_1$. If function f: $A \rightarrow S_2$ is Lipschitz function on A, then f uniformly continuous on A.

Proof: Given f is Lipschitz function will be shown that f uniformly continuous on A, means it will be shown that for any ε>0 there is δ>0 such that if x, u∈A:

$$p(x, u) - p(x, x) < \delta \text{ and } p(x, u) - p(u, u) < \delta$$

Then:

$$p(f(x), f(u)) - p(f(x), f(x)) < \varepsilon$$

And:

$$p(f(x), f(u)) - p(f(u), f(u)) < \varepsilon$$

Because f Lipschitz function, means there is a constants K such that:

$$p(f(x), f(u)) - p(f(x), f(x)) < K(p(x, u)) - p(x, x)$$

And:

$$p(f(x), f(u)) - p(f(u), f(u)) < K(p(x, u)) - p(u, u)$$

for every x, u∈A. Taken for any ϵ>0, select δ = ϵ/K such that if x, u∈A:

$$p(x,u) - p(x,x) < \delta \text{ and } p(x,u) - p(u,u) < \delta$$

Then:

$$p(f(x), f(u)) - p(f(x), f(x)) \le K(p(x,u) - p(x,x)) < K\left(\frac{\varepsilon}{K}\right) = \varepsilon$$

And:

$$p(f(x), f(u)) - p(f(u), f(u)) \le K(p(x,u) - p(u,u)) < K\left(\frac{\varepsilon}{K}\right) = \varepsilon$$

Proven f uniformly continuous on A. To understand Theorem 5 will be given in the following example.

Example 4: Let (S_1, p) and (S_2, p) is partial metric space, $A \subseteq S_1$. Prove that the function f: $A \rightarrow S_2$ defined on A with f(x) = 2x is uniformly continuous.

Solution: Consider that:

$$p(f(x), f(u)) - p(f(x), f(x)) = p(2x, 2u) - p(2x, 2x) = 2(p(x, u) - p(x, x))$$

And:

$$p(f(x), f(u)) - p(f(u), f(u)) = p(2x, 2u) - p(2u, 2u) = 2(p(x, u) - p(u, u))$$

For each x, u∈A. Based on definition 7, f satisfy Eq. 2 and 3 with K = 2, so that, f is Lipschitz function. Based Theorem 6, obtained that f uniformly continuous on A. Proven that f(x) = 2x uniformly continuous on A.

CONCLUSION

Some of the concept of continuity function on partial metric space is as follows. Let (S_1, p) and (S_2, p) are partial metric space (A)$\subseteq S_1$, suppose function f: $A \rightarrow S_2$, g: $A \rightarrow S_2$ and c∈A. If f and g continuous at c, then function:

- f+g continuous at c
- f-g continuous at c
- f.g continuous at c
- f/g continuous at c with g≠0

Let (S_1, p), (S_2, p) and (S_3, p) are partial metric space, $A \subseteq S_1$ and f(A)$\subseteq S_2$. Suppose function f: $A \rightarrow S_2$ and g: $A \rightarrow S_3$. If f continuous at c∈A and g continuous at b = f(C)∈f(A), then g∘f continuous at c. Let (S_1, p) and (S_2, p) are partial metric space and $A \subseteq S_1$. If function f: $A \rightarrow S_2$ is Lipschitz function on A, then f uniformly continuous on A.

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