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Estimation of the Heat Load of a Spherical Product from Abrasive Action

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Abstract: Abrasion is widely used in technological equipment in different industries. The biggest problem of abrasion process is a high temperature of object being processed. This can be solved by a periodic contact of the treated object with the abrasive. In such cases, the nutritional value (vitamins, the native state of a protein, enzymes, etc.) can be saved by preventing overheating. To solve the problem of observing a certain temperature during the process it is necessary to carry out some analytical assessments. This research proposes the method for determining the critical time where by raw materials retain their beneficial properties during the abrasion.

Key words: Abrasion, periodic contact with the abrasive, nutritional value, preventing overheating, technological equipment, high temperature

INTRODUCTION

The treatment of material's trajectory inside specific working chambers provides evidence that it contacts with abrasive tools at various points. According to, this fact it is possible to conclude that thermal contact with tools occurs periodically and accidentally.

The physical model that was described above can be formalized in the form of the heat transfer problem through a sphere which undergoes periodic point contacts with a source of heat (Karlosou and Eger, 1964). Assume that contact is carried out at infinite number of points, temperature field can be defined as:

$$u_t = \alpha^2 u_{xx},\ 0 \le x \le \infty,\ 0 \le t \le \infty \tag{1}$$

With an initial conditions:

$$u(x,0) = u_0,\ 0 \le x \le \infty \tag{2}$$

With a boundary conditions:

$$u_x(0,t) - u(0,t) = 0,\ 0 \le t \le \infty \tag{3}$$

Using the Laplace transform to solve boundary value problem with respect to the variable t (Lykov, 1967; Alexeev *et al*., 2015; Horenstein, 1945; Kaye, 1955). At the first stage, we can define partial derivatives from Eq.1:

$$L[u_t] = \int_0^\infty u_t(x,t)e^{-st}dt = u(x,t)e^{-st}\Big|_0^\infty + s\int_0^\infty u(x,t)e^{-st}dt =$$

$$sU(x, s) - u(x,0) \quad L[u_x] = \int_0^\infty u_x e^{-st}dt = \frac{\partial}{\partial x}U(x, s)$$

$$L[u_x] = \int_0^\infty u_{xx}e^{-st}dt = \frac{\partial^2}{\partial x^2}U(x, s)$$

Combining partial derivatives and Eq. 1, we obtain ordinary differential equation (Martynenko, 1990):

$$sU(x) - u_0 = \frac{d^2U}{dx^2},\ 0 \le x \le \infty \tag{4}$$

Using Eq. 2, we get initial conditions for Eq. 4:

$$\frac{dU}{dx}(0) = U(0)$$

Equation 4 is second order differential equation, therefore, to solve this boundary-value problem it is necessary to add additional boundary conditions. From the analysis of physical model, it is obvious that $U(x) \to 0$ as $x \to +\infty$. Summing general solution of the homogeneous differential equation:

$$\frac{d^2U}{dx^2} - sU(x) = 0$$

And particular solution of the nonhomogeneous differential equation (Voronenko and Klyuchkin, 1997):

$$U(x) = c_1 e^{\sqrt{sx}} + c_2 e^{-\sqrt{sx}} + \frac{u_0}{s} \tag{5}$$

Using physical properties of the model, we define constant of integration. And from the boundary

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conditions for Eq. 4 it follows that the constant is a solution of equation that was obtained by a substitution Eq. 5 in 4:

$$-c_2\sqrt{s} = c_2 + \frac{u_0}{s},\ c_2 = -\frac{u_0}{s\left(\sqrt{s}+1\right)} \tag{6}$$

Finally, substituting and in Eq. 7, we obtain:

$$U(x) = u_0\left(1 - \frac{1}{s(\sqrt{s}+1)}\right) \tag{7}$$

Temperature field can be defined by inverse Laplace transform (Kartashov 1999):

$$u(x,t) = L^{-1}\left[U(x,s)\right] = \int_{c-i\infty}^{c+i\infty} U(s)e^{st}ds = \int_{c-i\infty}^{c+i\infty}\left(\frac{1}{s} - \frac{1}{s\left(\sqrt{s}+1\right)}\right)e^{st}ds \tag{8}$$

The result of integration:

$$u(x,t) = u_0 - u_0\left[\operatorname{erfc}\left(\frac{x}{2\sqrt{t}}\right) + \operatorname{erfc}\left(\sqrt{t} + \frac{x}{2\sqrt{t}}\right)e^{x+t}\right] \tag{9}$$

Where:

$$\operatorname{erfc}(x) = \frac{2}{\sqrt{\pi}}\int_x^{\infty} e^{-\xi^2}d\xi$$

is a complementary error function. Thus, Eq. 9 is a solution of boundary value problem Eq. 1. The results obtained confirm the infinite number of contacts with the ab rasive throughout semi-infinite specified domain of parameters.

MATERIALS AND METHODS

If the object under consideration is sphere that is heated by friction against abrasive surfaces then the thermal process can be defined with a heat equation in sphere:

$$\frac{\partial t}{\partial \tau} = a\left(\frac{\partial^2 t}{\partial r^2} + \frac{2}{r}\cdot\frac{\partial t}{\partial r}\right),\ 0 \le r \le R,\ \tau > 0 \tag{10}$$

Where:

$t(r, \tau)$ = Temperature at any point r and any time τ
r = Radius of the processed product
R = Thermal diffusivity
a = Thermal diffusivity

Suppose the initial temperature distribution in the sphere is constant:

$$t(r,0) = t_0 = \text{const} \tag{11}$$

The process proceeds under first-type boundary conditions (Kartashov, 2001):

$$t(R,\tau) = \begin{cases} t_1 = t_{min}, & 0<\tau<\tau_1 \\ t_2 = t_{mp} = t_{max}, & \tau_1<\tau<\tau_2\ (t_1>t_2) \end{cases}$$

Since, sphere has central symmetr it follows that second-type boundary conditions:

$$\frac{\partial t(0,\tau)}{\partial r} = 0 \tag{12}$$

Condition of the temperature limitation in the center of the object:

$$t(0,\tau) < \infty \tag{13}$$

$$\text{period}\quad T = \tau_2,\ 0<\tau_2<T,\ \Delta\tau = \tau_1 - \tau_2 \tag{14}$$

$$\text{number of cycle}\quad N = \frac{2\pi R}{L} \tag{15}$$

where, L is a circumference of the product processed by abrasive tool for the period T. Boundary value problem of that kind can be solved with an integral Laplace transform. Temperature distribution can be defined in dimensionless form (Kartashov and Kudinov, 2012):

$$T(X,Fo) = 1 - \frac{Fo_1}{Fo_2} + \sum_{n=1}^{\infty}(-1)^n \frac{2(1-\exp(-(n\pi)^2 Fo_2\left(1-\frac{Fo_1}{Fo_2}\right))}{1-\exp(-(n\pi)^2 Fo_2)}\frac{\sin(n\pi X)}{\pi n X}\cdot \exp(-(n\pi)^2 Fo_1) - \sum_{m=1}^{\infty}\frac{\left(P_1\cos\left(\frac{2\pi m}{Fo_2}\left(Fo - \frac{Fo_1}{2}\right)\right) + P_2\sin\left(\frac{2\pi m}{Fo_2}\left(Fo - \frac{Fo_1}{2}\right)\right)\right)}{ch\left(2\sqrt{\frac{\pi m}{Fo_2}}\right) - \cos\left(2\sqrt{\frac{\pi m}{Fo_2}}\right)}\cdot\frac{\sin\left(\pi m\frac{Fo_1}{Fo_2}\right)}{\pi m X} \tag{16}$$

where, $T(X,F_0) = \frac{t(r,\tau)-t_1}{t_2-t_1}$ is dimensionless relative temperature:

$$P_1 = sh\sqrt{\frac{\pi m}{Fo_2}}\cos\sqrt{\frac{\pi m}{Fo_2}}sh(\sqrt{\frac{\pi m}{Fo_2}}X)\cos(\sqrt{\frac{\pi m}{Fo_2}}X) + ch\sqrt{\frac{\pi m}{Fo_2}}\sin\sqrt{\frac{\pi m}{Fo_2}}ch(\sqrt{\frac{\pi m}{Fo_2}}X)\sin(\sqrt{\frac{\pi m}{Fo_2}}X)$$

$$P_2 = ch\sqrt{\frac{\pi m}{Fo_2}}\sin\sqrt{\frac{\pi m}{Fo_2}}sh(\sqrt{\frac{\pi m}{Fo_2}}X)\cos(\sqrt{\frac{\pi m}{Fo_2}}X) - sh\sqrt{\frac{\pi m}{Fo_2}}\cos\sqrt{\frac{\pi m}{Fo_2}}ch(\sqrt{\frac{\pi m}{Fo_2}}X)\sin(\sqrt{\frac{\pi m}{Fo_2}}X)$$

where, $X = \frac{r}{R}$ is dimensionless coordinate; $Fo = \frac{a\tau}{R^2}$ is Fourier number, $Fo_i = \frac{a\tau_i}{R^2}$.

RESULTS AND DISCUTION

The results obtained give the opportunity to predict the temperature distribution along the depth of the processed object for the physical model that was chosen. Some types of processed food raw materials (e.g., coffee beans) can tolerate high temperatures without negative changing in its properties but others (e.g., cereal grains, potato tubers) can't retain its quality.

CONCLUSION

The relations obtained allow to define the parameters of the process at which the negative effect on foodstuffs will be minimized. The solutions obtained can also be used for the inverse problem to define the critical time at which an impermissible temperature exceedance is reached. This in turn will lead to the reducing energy consumption.

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