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On the Total Irregularity Strength of M-Copy Cycles and M-Copy Paths

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Abstract: Let G = (V, E) be a graph. A totally irregular total k-labeling f: $V \cup E \rightarrow \{1, 2, ..., k\}$ of a graph G is a total labeling such that for any different vertices x and y of G, their weights wt. (x) and wt. (y) are distinct and for any different edges x_1x_2 and y_1y_2 of G, their weights wt. (x_1x_2) and wt (y_1y_2) are distinct. The weight wt (x) of a vertex x is the sum of the label of x and the labels of all edges incident with x. The weight wt. (x_1x_2) of an edge x_1x_2 is the sum of the label of edge x_1 x_2 and the labels of vertices x_1 and x_2. The minimum k for which a graph G has a totally irregular total k-labeling is called the total irregularity strength of G, denoted by ts(G). In this study, we determine the total irregularity strength of M-copy cycles and M-copy paths.

Key words: Totally irregular total k-labeling, total irregularity strength, M-copy cycles, M-copy paths, weight, irregular

INTRODUCTION

A labeling of a graph is a function that carries graph elements to the numbers, usually to the positive or non-negative integers that satisfies certain requirements. The most common choices of domain are the set of all vertices (vertex labeling), the set of all edges (edge labeling) or the set of all vertices and edges (total labeling). Other domain are possible.

Baca *et al*. (2007) introduced two kinds of irregular total labelings, namely, vertex irregular total labeling and edge irregular total labeling. Let G = (V, E) be a graph. A vertex irregular total k-labeling f: $V \cup E \rightarrow \{1, 2, 3, ..., k\}$ of G is a total labeling such that for any two different vertices x and y of G, their weights wt (x) and wt (y) are distinct, where the weight wt (x) of a vertex x is the sum of the label of x and the labels of all the edges incident with x. The minimum k for which a graph G has a vertex irregular total k-labeling is called the total vertex irregularity strength of G, denoted by tvs(G). An edge irregular total k-labeling f: $V \cup E \rightarrow \{1, 2, ..., k\}$ of G is a total labeling such that for any two different edges x_1x_2 and y_1y_2 of G, their weights wt. $(x_1 x_2)$ and wt. $(y_1 y_2)$ are distinct where the weight wt. (x_1x_2) of an edge x_1x_2 is the sum of the label of x_1x_2 and the labels of the vertices x_1 and x_2. The minimum k for which a graph G has an edge irregular total k-labeling is called the total edge irregularity strength of G, denoted by tes(G).

Baca *et al*. (2007) obtained lower bound and upper bound of total vertex irregularity strength of a graph G such as the following theorem.

MATERIALS AND METHODS

Theorem 1.1; Baca *et al*. (2007): Let G be a graph (p, q) with minimum degree δ and maximum degree Δ, then:

$$\left\lceil \frac{p+\delta}{\Delta+1} \right\rceil \le tvs(G) \le p+\Delta-2\delta+1$$

Total vertex irregularity strength of t-copy of path has been determined by Nurdin *et al*. (2009) such as the following theorem.

Theorem 1.2; Nurdin *et al*. (2009): Let graph tP_n be t-copy of path with n vertices where $t \ge 2$, then:

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$$tvs(tP_n) = \begin{cases} t, & \text{for } n = 1; \\ t+1, & \text{for } 2 \le n \le 3; \\ \left\lceil \frac{nt+1}{3} \right\rceil, & \text{for } n \ge 4 \end{cases}$$

Baca *et al*. (2007) also, determined lower bound and upper bound of total edge irregularity strength of a graph G as stated in the following theorem.

Theorem 1.3; Baca *et al.* (2007): Let G = (V, E) be a graph with the set of vertices V and the non empty set of Edges E:

$$\left\lceil \frac{|E|+2}{3} \right\rceil \le tes(G) \le |E|$$

In 2012, combination of the both labelings, namely totally irregular total k-labeling has introduced by Marzuki *et al*. (2013). Let G = (V, E) be a graph. A totally irregular total k-labeling f: V∪E→{1, 2, ..., k} of G is a total labeling such that for any two different vertices x and y of G, the weights wt. (x) and wt. (y) are distinct and for any two different edges x_1x_2 and y_1y_2 of G, the weights wt. (x_1 x_2) and wt. (y_1y_2) are distinct. The minimum k for which a graph G has a totally irregular total k-labeling is called the total irregularity strength of G, denoted by ts (G).

Theorem 1.4; Marzuki *et al.* (2013): For every graph G, we have maxftes(G); tυs(G)g≤ts(G). After that, Ramdani *et al*. (2015a, b) also, observed about the total irregularity strength of M-copy of stars by Ramdani (2014) total irregularity strength of three families of graph by and total irregularity strength of regular graph. Two theorems which have resulted by Ramdani *et al*. (2015) are:

Theorem 1.5; Ramdani *et al.* (2015a, b): Let P_2 be a path with 2 vertices. Then, $ts(mP_2) = m+1$ for $m \ge 1$.

Theorem 1.6; Ramdani *et al.* (2015 a, b): Let Cn be a cycle of order n. For n≥3 and n ≡ (0mod3), $ts(mC_n) = \lceil mn+2/3 \rceil$.

RESULTS AND DISCUSSION

Graph mP_3 is a graph which is obtained by copying graph P_3 as much as m times in which the set of vertices of each copy are disjoint. Let the ith copy results of graph P_3 is υ_i, 1_{ei}, $1\upsilon_i$, 2_{ei}, 2_{ui}, 3 where e_i, j = $\upsilon_{i,j}$ $\upsilon_{i,j}$+1 for $1 \le i \le m$ and $1 \le j \le 3$. The following theorem will be discussed about total irregularity strength of graph mP_3, for positive integer m with $m \ge 2$.

Theorem 2.1: Let mP_3 be m-copy of path with three vertices. For $m \ge 2$, $ts(mP_3) = m+1$.

Proof: Note that the number of edges of graph mP_3 is $|E(mP_3)| = 2m$. According to theorem (1.3), we have tes $(mP_3) \ge \lceil 2m+2/3 \rceil$. While based on Theorem (1.1), we get tυs $(mP_3) \ge m+1$.

According to theorem (1.4), we have $ts(mP_3) \ge \max\{\lceil 2m+2/3 \rceil, m+1\}$. Because of $m+1 > \lceil 2m+2/3 \rceil$ for every $m \in Z^+$ then $ts(mP_3) \ge m+1$.

Next, we will prove that $ts(mP_3) \le m+1$ by showing that there is a totally irregular total (m+1)-labeling on graph mP_3 such as the following:

$$\lambda(\upsilon_{i,j}) = \begin{cases} 1, & \text{for } 1 \le i \le m \text{ and } j = 1; \\ m, & \text{for } 1 \le i \le m \text{ and } j = 2; \\ m+1-i, & \text{for } 1 \le i \le m \text{ and } j = 3; \end{cases}$$

$$\lambda(e_{i,j}) = \begin{cases} i, & \text{for } 1 \le i \le m \text{ and } j = 1; \\ m+1, & \text{for } 1 \le i \le m \text{ and } j = 2 \end{cases}$$

Easy to check that the weights of vertices and edges of graph mP_3 under labeling λ are as follows:

$$wt(e_{i,j}) = \begin{cases} m+1+i, & \text{for } 1 \le i \le m \text{ and } j = 1; \\ 3m+2-i, & \text{for } 1 \le i \le m \text{ and } j = 2; \end{cases}$$

$$wt(\upsilon_{i,j}) = \begin{cases} 1+i, & \text{for } 1 \le i \le m \text{ and } j = 1; \\ 2m+2-i, & \text{for } 1 \le i \le m \text{ and } j = 2; \\ 2m+2-i, & \text{for } 1 \le i \le m \text{ and } j = 3 \end{cases}$$

Observe that λ is a labeling from $V(mP_3) \cup E(mP_3)$ into {1, 2, ..., m+1} such that no two vertices have the same weight and no two edges have the same weight. So, λ is a totally irregular total (m+1)-labeling. We conclude that $ts(mP_3) \le m+1$.

Let mC_n is a graph which is obtained by copying graph C_n as much as m times in which the set of vertices of each copy are disjoint. In the following theorem, we will discuss about total irregularity strength of mC_n for m = 2, 3 and n = 1 (mod3).

Theorem 2.2: Let mC_n be m-copy of cycle with n vertices, then for m = 2, 3 and n ≡ 1 (mod3), $ts(mC_n) = \lceil mn+2/3 \rceil$.

Proof: The number of edges of graph mC_n is $|E(mC_n)| = mn$. Based on Theorem (1.3), we have tes $(mC_n) \ge \lceil mn+2/3 \rceil$. Next, according to Theorem (1.1), we get tυs $(mC_n) \ge \lceil mn+2/3 \rceil$. While based on Theorem (1.4), we have $ts(mC_n) \ge \max\{tes(mC_n), t\upsilon s(mC_n)\}$. Because of tes $(mC_n) \ge \lceil mn+2/3 \rceil$ and $t\upsilon s(mC_n) \ge \lceil mn+2/3 \rceil$ for every m = 2, 3 and n ≡ 1 (mod3), then ts $(mC_n) \ge \lceil mn+2/3 \rceil$.

Next, we will prove that $ts(mC_n) \geq \lceil mn+2/3 \rceil$ by showing that there is a totally irregular total $\lceil mn+2/3 \rceil$ labeling on graph mC_n.

Let the set of vertices of graph mCn is; $V(mC_n) = \{\upsilon_i | 1 \leq i \leq mn$ with m = 2, 3 and n = 1 (mod3)$\}$ and the set of edges of graph mCn is; $E(mC_n) = \{\upsilon_i | 1 \leq i \leq mn$ with m = 2, 3 and n = 1 (mod3)$\}$, construct a labeling f such as the following:

$$f(\upsilon_i) = f(e_i) = \begin{cases} \left\lceil \frac{i+1}{3} \right\rceil, & \text{for } 1 \leq i \leq 2n\text{-}1 \\ \left\lceil \frac{i+2}{3} \right\rceil, & \text{for } 2n \leq i \leq 3n \end{cases}$$

Based on this labeling, we have the weights of vertices and edges of graph mC_n are as follows:

$$wt(\upsilon_i) = wt(e_i) = \begin{cases} 3, & \text{for } i = 1; \\ \left\lceil \frac{i+1}{3} \right\rceil + \left\lceil \frac{i}{3} \right\rceil + \left\lceil \frac{i+2}{3} \right\rceil, & \text{for } 2 \leq i \leq n\text{-}1; \\ 2\left\lceil \frac{n+1}{3} \right\rceil + \left\lceil \frac{n}{3} \right\rceil, & \text{for } i = n; \\ 2\left\lceil \frac{n+2}{3} \right\rceil + \left\lceil \frac{n+3}{3} \right\rceil, & \text{for } i = n+1; \\ \left\lceil \frac{2n}{3} \right\rceil + \left\lceil \frac{2n+1}{3} \right\rceil + \left\lceil \frac{2n+2}{3} \right\rceil, & \text{for } i = 2n\text{-}1; \\ 2n+2, & \text{for } i = 2n; \\ 2n+4, & \text{for } i = 2n+1; \\ \left\lceil \frac{i+2}{3} \right\rceil + \left\lceil \frac{i+1}{3} \right\rceil + \left\lceil \frac{i+3}{3} \right\rceil, & \text{for } i = 2n+2 \leq i \leq 3n\text{-}1; \\ 3n+3, & \text{for } i = 3n \end{cases}$$

The weights of vertices and edges of graph mC_n for m = 2, 3 and n ≡ 1 (mod3) are:

- For i = 1, the weights of vertices and edges of graph mC_n are 3
- For $2 \leq i \leq n\text{-}1$, the weights of vertices and edges of graph mC_n are consecutive integer from 4 until $\lceil n/3 \rceil + \lceil n\text{-}1/3 \rceil + \lceil n+1/3 \rceil = n+1$
- For i = n, the weights of vertices and edges of graph mC_n are $2\lceil n+1/3 \rceil + \lceil n/3 \rceil = n+2$
- For i = n+1, the weights of vertices and edges of graph mC_n are $2\lceil n+2/3 \rceil + \lceil n+3/3 \rceil = n+3$
- For $n+2 \leq i \leq 2n\text{-}2$, the weights of vertices and edges of graph mCn are consecutive integer from $\lceil n+3/3 \rceil + \lceil n+2/3 \rceil + \lceil n+4/3 \rceil = n+4$ until $\lceil 2n\text{-}1/3 \rceil + \lceil 2n/3 \rceil = 2n$
- For i = 2n-1, the weights of vertices and edges of graph mC_n are $\lceil 2n/3 \rceil + \lceil 2n\text{-}1/3 \rceil + \lceil 2n+2/3 \rceil = 2n+2$
- For i = 2n, the weights of vertices and edges of graph mCn are 2n+3
- For i = 2n+1, the weights of vertices and edges of graph mC_n are 2n+4
- For $2n+2 \leq i \leq 3n\text{-}1$, the weights of vertices and edges of graph mC_n are consecutive integer from $2\lceil 2n+4/3 \rceil + \lceil 2n+5/3 \rceil = 2n+5$ until 3n+2
- For i = 3n, the weights of vertices and edges of graph mC_n are 3n+3

So, the weights of vertices and edges of graph mC_n for m = 2, 3 and n ≡ 1 (mod3) are consecutive integer from 3 until 2n and consecutive integer from 2n+2 until 3n+3.

Observe that f is a labeling from $V(mC_n) \cup E(mC_n)$ into $\{1, 2, ..., \lceil mn+2/3 \rceil\}$ such that no two vertices have the same weight and no two edges have the same weight. So, f is a totally irregular total $\lceil mn+2/3 \rceil$-labeling. We conclude that $ts(mC_n) \leq \lceil mn+2/3 \rceil$ for m = 2, 3 and n ≡ 1 (mod3).

Let mC_4 is a graph which is obtained by copying graph C_4 as much as m times in which the set of vertices of each copy are disjoint. Let the ith copy results of graph C_4 is $\upsilon_{4i}\text{-}3$, $e_{4i}\text{-}3$, $\upsilon_{4i}\text{-}2$, $e_{4i}\text{-}1$, υ_{4i}, e_{4i}, $\upsilon_{4i}\text{-}1$, $e_{4i}\text{-}2$, $\upsilon_{4i}\text{-}3$ for $1 \leq i \leq m$. The following theorem will be discussed about total irregularity strength of graph mC_4 for positive integer m with $m \leq 4$.

Theorem 2.3: Let mC_4 be m-copy of cycle of order 4. For $m \leq 4$, $ts(mC_4) = \lceil 4m+2/3 \rceil$.

Proof: According to Theorem (1.1), $t\upsilon s(mC_4) \leq \lceil 4m+2/3 \rceil$. While based on Theorem (1.3), $t\upsilon s(mC_4) \leq \lceil 4m+2/3 \rceil$. By Theorem (1.4), we have $t\upsilon s(mC_4) \leq \lceil 4m+2 \rceil$. Next, we will prove that $t\upsilon s(mC_4) \leq \lceil 4m+2 \rceil$ by showing there is a totally irregular total $\lceil 4m+2 \rceil$-labeling on mC_4 such as following: For m ≡ 1 (mod3) and m ≡ 2 (mod3):

$$\lambda(\upsilon_i) = \lambda(e_i) = \begin{cases} \left\lceil \frac{i+2}{3} \right\rceil, & \text{for } i \equiv 8(\text{mod}12); \\ \left\lceil \frac{i+1}{3} \right\rceil, & \text{for } i \equiv 9(\text{mod}12);\ i \equiv 10(\text{mod}12), \\ & \text{atau } i \equiv 0(\text{mod}12); \\ \left\lceil \frac{i+5}{3} \right\rceil, & \text{for } i \equiv 11(\text{mod}12); \\ \left\lceil \frac{i+1}{3} \right\rceil, & \text{for others} \end{cases}$$

For m≡ 0 (mod 3):

$$\lambda(v_i) = \lambda(e_i) = \begin{cases} \left\lceil \frac{i+2}{3} \right\rceil, & \text{for } i = 8 \text{ or } i = 20; \\ \left\lceil \frac{i-1}{3} \right\rceil, & \text{for } i = 9, 10, 12 \text{ or} \\ & i \equiv 6 (\text{mod}12) \text{ with } i \geq 30; \\ \left\lceil \frac{i+5}{3} \right\rceil, & \text{for } i = 11; \\ \left\lceil \frac{i+4}{3} \right\rceil, & \text{for } i = 11(\text{mod}12) \text{ and } i \neq 11; \\ \left\lceil \frac{i+3}{3} \right\rceil, & \text{for } i \equiv 2(\text{mod}12) \text{ with } i \geq 26 \\ & \text{or } i \equiv 4(\text{mod}12) \text{ with } i \geq 28; \\ \left\lceil \frac{i+1}{3} \right\rceil, & \text{for } i \equiv 7(\text{mod}12) \text{ with } i \geq 31; \\ \left\lceil \frac{i+3}{3} \right\rceil, & \text{for others} \end{cases}$$

By using this labeling, we get the weights of vertices and edges such as follows: For m = 1 (mod 3) and m = 2 (mod3):

$$wt(v_i) = wt.(e_i) = \begin{cases} 2\left\lceil \frac{i-1}{3} \right\rceil + \left\lceil \frac{i}{3} \right\rceil, & \text{for } i \equiv 9(\text{mod}12); \\ 2\left\lceil \frac{i+1}{3} \right\rceil + \left\lceil \frac{i+2}{3} \right\rceil, & \text{for } i \equiv 1(\text{mod } 12) \text{ or} \\ & i \equiv 5(\text{mod}12); \\ \left\lceil \frac{i-1}{3} \right\rceil + \left\lceil \frac{i-2}{3} \right\rceil + \left\lceil \frac{i+6}{3} \right\rceil, & \text{for } i \equiv 10(\text{mod } 12); \\ \left\lceil \frac{i+5}{3} \right\rceil + \left\lceil \frac{i-2}{3} \right\rceil + \left\lceil \frac{i}{3} \right\rceil, & \text{for } i \equiv 11(\text{mod } 12); \\ \left\lceil \frac{i+1}{3} \right\rceil + \left\lceil \frac{i}{3} \right\rceil + \left\lceil \frac{i+3}{3} \right\rceil, & \text{for } i \equiv 7(\text{mod } 12); \\ \left\lceil \frac{i+1}{3} \right\rceil + \left\lceil \frac{i}{3} \right\rceil + \left\lceil \frac{i+2}{3} \right\rceil, & \text{for } i \equiv 2(\text{mod } 12), i \equiv 3 \\ & (\text{mod } 12), i \equiv 6(\text{mod } 12); \\ 2\left\lceil \frac{i+2}{3} \right\rceil + \left\lceil \frac{i}{3} \right\rceil, & \text{for } i \equiv 8(\text{mod } 12); \\ 2\left\lceil \frac{i-1}{3} \right\rceil + \left\lceil \frac{i+4}{3} \right\rceil, & \text{for } i \equiv 0(\text{mod } 12); \\ 2\left\lceil \frac{i+1}{3} \right\rceil + \left\lceil \frac{i}{3} \right\rceil, & \text{for } i \equiv 4(\text{mod } 12) \end{cases}$$

For m≡ 0 (mod3):

$$\begin{aligned} wt(v_i) \\ = wt(e_i) \end{aligned} = \begin{cases} 2\left\lceil \frac{i-1}{3} \right\rceil + \left\lceil \frac{i}{3} \right\rceil, & \text{for } i = 9; \\ 3\left\lceil \frac{i+1}{3} \right\rceil + 1, & \text{for } i \equiv 1(\text{mod}12) \text{ with } i \geq 25; \\ 2\left\lceil \frac{i+1}{3} \right\rceil + \left\lceil \frac{i}{3} \right\rceil, & \text{for } i \equiv 4,\ i = 4, i = 16 \text{ or} \\ & i \equiv 5(\text{mod}12) \text{ with } i \geq 29; \\ 2\left\lceil \frac{i+1}{3} \right\rceil + \left\lceil \frac{i+2}{3} \right\rceil, & \text{for } i = 1, 5, 13, 17 \text{ or} \\ & (i \equiv 9(\text{mod}12) \text{ and } i \neq 9); \\ \left\lceil \frac{i-1}{3} \right\rceil + \left\lceil \frac{i}{3} \right\rceil + \left\lceil \frac{i+1}{3} \right\rceil + 2, & \text{for } i \equiv 6(\text{mod}12) \text{ with } i \geq 30; \\ \left\lceil \frac{i-1}{3} \right\rceil + \left\lceil \frac{i-2}{3} \right\rceil + \left\lceil \frac{i}{3} \right\rceil + 2, & \text{for } i = 10; \\ \left\lceil \frac{i+1}{3} \right\rceil + \left\lceil \frac{i}{3} \right\rceil + \left\lceil \frac{i+2}{3} \right\rceil + 1, & \text{for } i \equiv 10(\text{mod}12) \text{ and } i \neq 10; \\ & \text{or } i \equiv 11(\text{mod}12) \text{ and } i \neq 11; \\ 2\left\lceil \frac{i}{3} \right\rceil + \left\lceil \frac{i+2}{3} \right\rceil + 1, & \text{for } i \equiv 2(\text{mod}12) \text{ with } i \geq 26; \\ \left\lceil \frac{i+2}{3} \right\rceil + \left\lceil \frac{i-2}{3} \right\rceil + \left\lceil \frac{i}{3} \right\rceil + 1, & \text{for } i = 11; \\ 2\left\lceil \frac{i+1}{3} \right\rceil + 2\left\lceil \frac{i}{3} \right\rceil + 1, & \text{for } i = 7 \text{ or } i = 19; \\ \left\lceil \frac{i}{3} \right\rceil + \left\lceil \frac{i-2}{3} \right\rceil + \left\lceil \frac{i+2}{3} \right\rceil + 2, & \text{for } i \equiv 7(\text{mod } 12) \text{ with } i \geq 31; \\ 2\left\lceil \frac{i+1}{3} \right\rceil + \left\lceil \frac{i+2}{3} \right\rceil + 1, & \text{for } i \equiv 3(\text{mod } 12) \text{ with } i \geq 27; \\ & \text{or } i \equiv 8(\text{mod } 12) \text{ and } i \geq 32; \\ \left\lceil \frac{i+1}{3} \right\rceil + \left\lceil \frac{i}{3} \right\rceil + \left\lceil \frac{i+2}{3} \right\rceil, & \text{for } i = 2, 3, 6, 14, 15 \text{ or } 18; \\ 2\left\lceil \frac{i+2}{3} \right\rceil + \left\lceil \frac{i}{3} \right\rceil, & \text{for } i = 8 \text{ or } 20; \\ 2\left\lceil \frac{i-1}{3} \right\rceil + \left\lceil \frac{i+4}{3} \right\rceil, & \text{for } i = 12; \\ 3\left\lceil \frac{i}{3} \right\rceil + 2, & \text{for } i = 49(\text{mod}12) \text{ with } i \geq 28; \\ 2\left\lceil \frac{i+1}{3} \right\rceil + \left\lceil \frac{i}{3} \right\rceil + 1, & \text{for } i \equiv 0(\text{mod } 12) \text{ and } i \neq 12 \end{cases}$$

CONCLUSION

Observe that λ is a labeling from V (mC_4) ∪ E (mC_4) into {1, 2, ..., ⌈4m+2/3⌉} such that no two vertices have the same weight and no two edges have the same weight. So, λ is a totally irregular total $\lceil 4m+2/3 \rceil$-labeling. We conclude that tvs $(mC_4) \leq \lceil 4m+2/3 \rceil$.

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