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The Building Control Systems Through One-Parameter Structurally Stable Mapping

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Abstract: This study devoted to problems of building robust stability of control system for dynamic objects in a class of one-parameter structurally stable mapping. Problem of building control systems positions central role in creation of automatic and automated control systems widely used nearly in all areas of production and technology: engineering, textile industries, transport and other technologies, etc. This approach allowing to fully increase potential of robust stability. Concept of building control system with increased potential of robust stability in dynamic objects based on conclusions of Catastrophe theory where structurally stable mapping is deduced. Research of robust stability of control systems based on a new approach to Lyapunov function. Efficiency of control systems is clearly illustrated on the example of building control systems for technological drying process of materials in textile industry. A detailed case provided to demonstrate efficiency of control systems with greater robust stability. The results of numerical experiment prove theoretical principles. This method shows stability of control systems. Actually, the results of creating control systems with greater potential of robust stability allow to provide dynamic safety and operating capacity of control systems in engineering and technologies at their initial design and operation stages.

Key words: Robust stability, structurally stable mapping, lyapunov function, asymptotical stability, technological drying process

INTRODUCTION

Problem of building control systems positions central role in creation of automatic and automated control systems widely used nearly in all areas of production and technology: engineering, economic power industry, electronics, chemical and biological, metallurgy and textile industries, transport and robotechnics, aviation, space systems, high-precision military hardware and technologies, etc.

It is now widely accepted that majority of real control systems operate in varying degrees of uncertainty. Yet, uncertainty might be attributable to lack of awareness of initial contents in control objects and their unpredictable change over time (during operation). That is why, the problem of robust stability is the most acute in theory of ruling and of a great practical interest. In general, it points to delimitation of system's parametric variation while its stability endured. Obviously, these delimitations defined by stability range of object's undetermined parameters and by settings of control devices.

Notable methods of building control systems of objects with undetermined parameters mostly devoted to examination of system's robust stability with given structure and linear law. These prevent from designing control systems with wider area of robust stability conditioned by object's undetermined parameters and motion of its characteristics at greater ranges. Now a days scientific contributions lack research and development of control systems with wider area of robust stability.

This study devoted to actual problems of building robust stabile control system of dynamic objects with undetermined parameters while addressing control system in a class of one-parameter structurally-stable mapping, allowing to fully increase potential of robust stability.

Concept of building control system with increased potential of robust stability in dynamic objects based on conclusions of catastrophe theory where structurally stable mapping is deduced.

Literature review: The theory of robust control began in the late 1970s and early 1980s and soon developed a number of techniques for dealing with bounded system uncertainty. The Robust stability is closely linked to the fundamental studies (Barbashin, 1967; Malkin, 1966; Siljak, 1989; Polyak, 2010) and today we see many works in this field (Siljak, 1989; Polyak and Scherbakov, 2002a, b; Polyak, 2010; Gilmore, 1981; Voronova and Matrosova, 1987; Pupkov and Egugov, 2004; Gantmacher, 2002; Strejc,

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1985). However, many of them address either linear systems or nonlinear systems with specific constraints. Successful results were reported when Lyapunov theory (Polyak and Scherbakov, 2002a,b; Polyak, 2010; Gilmore, 1981; Voronova and Matrosova, 1987) was employed to achieve robust stability of control systems with uncertain parameters. In particular (Gilmore, 1981; Voronova and Matrosova, 1987; Pupkov and Egugov, 2004; Gantmacher, 2002) presents both analysis and synthesis steps of the process.

The robustness is assumed as an ability to maintain system availability in a condition of parametric and nonparametric indeterminateness describing control objects. The most important idea in the study of robust stability is to specify constraints for changes in control system parameters that preserve stability. Studies (Siljak, 1989; Polyak and Scherbakov, 2002a, b), etc., dedicated to study robust stability of control systems.

Nevertheless, many of them focused on studies of robust stability of linear continuous and discrete control systems specifically of characteristic polynomial, frequency characteristics and Lyapunov matrix equation.

The results reported by Abitova *et al.* (2012a, b), Yermekbayeva (2013), Yermekbayeva *et al.* (2014), Beisanbi and Yermekbayeva (2013), Beisenbi and Abdrakhmanova (2013), Abdrakhmanova and Beisenbi (2014), Beisenbi and Uskenbayeva (2014), Beisenbi and Yermekbayeva (2014), Beisenbi and Mukataev (2014) and Beisenbi *et al.* (2014) are of particular interest where increased robustness based on catastrophe theory lead to structurally stable systems.

Special attention is given to dynamic systems where processes of self-organizing of Physical-Chemical and Biological systems addressed (Yermekbayeva, 2013; Yermekbayeva *et al.*, 2014). Models of these systems represented in a form of structurally stable mapping from catastrophe theory and examined as multi-purpose arithmetic model of evolution and self-assembly in a wildlife. For this reason, it is of a particular interest in conditions of uncertainty of object's control parameters with generation of determined chaos. Building of automated control system in a class of structurally stable mapping with arithmetic models corresponding to complex system behavior specifically resulting in multiply consequent and stable solutions.

MATERIALS AND METHODS

Research model: Suppose control system is given by state equation:

$$\dot{x} = Ax + Bu,\ y = cx,\ x \in R^n,\ y \in R \qquad (1)$$

Where:

$$A = \begin{Vmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \dots & 0 \\ \dots & \dots & \dots & \dots & . \\ 0 & 0 & 0 & \dots & 1 \\ a_n & -a_{n-1} & -a_{n-2} & \cdots & -a_1 \end{Vmatrix}$$

$$B = \begin{Vmatrix} 0 & 0 & \dots & 0 & 0 \\ 0 & 0 & \dots & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ \dots & \dots & \dots & & \\ 1, & 1,\dots, & 1, & , & 1 \end{Vmatrix} \quad c = \begin{Vmatrix} 1, & 0, & \dots, & 0 & 0 \end{Vmatrix}$$

Let us consider set of equations with input stimulus u (t)ϵR^n. It appears from object's controllability that number of null item at the end line of the matrix B corresponds to non-controlled coordinate.

In this case, it is assumed that all state variables of Eq. 1 are controlled and the only output coordinate is x_1. This implies that controlled object with matrix A by inducing to the control path of a regulator with control law in a form of one-parameter structurally-stable mapping of (Polyak, 2010):

$$u_i = -x_i^3 + k_i x_i,\ i = 1,\dots,n \qquad (2)$$

Could be reclassified to any pre-designed position. Let us show that Eq. 1 and 2 allows to define stability areas of control system at controlled variables and provides limitary wide area of stability at undefined parameters. Equation 1 by taking in account (Eq. 2) could be presented in the expanded form of:

$$\begin{cases} \dfrac{dx_1}{dt} = x_2,\ \dfrac{dx_2}{dt} = x_3, \\ \dots\ \dots\ \dots\ \dots\ \dots\ \dots, \\ \dfrac{dx_n}{dt} = -x_1^3 + (k - a_n)\, x_1 - a_n - x_2^3 + \\ (k_2 - a_{n-1})\, x_2 -, \dots, -x_n^3 + (k_n - a_1)\, x_n \end{cases} \qquad (3)$$
$$y = x_1$$

Let us consider steady status of the equation:

$$\begin{cases} x_{2s} = 0, \\ x_{3s} = 0, \\ \dots\dots\dots\dots, \\ x_{ns} = 0 \\ -x_{1s}^3 + (k_1 - a_n)\, x_{1s} - x_{2s}^3 + \\ (k_2 - a_{n-1})\,(x_{2s} -, \dots, -x_{ns}^3 + (k_n - a_1)\, x_n = 0 \end{cases} \qquad (4)$$

From Eq. 4 implies steady-state of Eq. 3:

$$x_{1s} = x_{2s} = \dots x_{ns} = 0 \tag{5}$$

Other steady-states of the (Eq. 3) shall be defined by solution of the equation:

$$-x_{is}^2 + (k_i - a_{i-m+1}) = 0,\ i = 1,\dots\dots,n \tag{6}$$

If negative k_i-a_{n-i+1} (k_i-a_{n-i+1}<0) i = 1, 2, ..., n, this equation has i (imaginary) solution that does not correspond to any physically feasible situation. At k_i-a_{n-i+1}>0, i = 1, 2, ..., n, Eq. 6 allows following steady states:

$$\begin{aligned} & x_{is}^2 = \sqrt{k_i - a_{n-i+1}} \\ & x_{is}^3 = -\sqrt{k_i - a_{n-i+1}}, x_{js} = 0 \\ & i \neq j, i = 1,\dots,n; j = 1,\dots,n \end{aligned} \tag{7}$$

These states (Eq. 7) of the equation merge with (Eq. 5) at control parameter and bifurcate from it at k_i-a_{n-i+1}>0.

RESULTS

Data analysis

To examine robust stability of steady-state conditions of (Eq. 5 and 7), let us consider fundamental principle of Lyapunov function: For asymptotical stability of equilibrium states and Lyapunov stability, it is necessary and sufficient (if and only if) a positive definite Lyapunov function V (x) to exist so that it's total derivative to time V (x) in solution of state differential (Eq. 3) is a negative definite function, i.e.,:

$$\dot{V}(x) = \frac{\partial V(x)}{\partial x}\frac{dx}{dt} < 0$$

Herein, total derivative in time of Lyapunov function with regard to state equation is geometrically defined as a scalar product of gradient vector of Lyapunov function (∂V (x)/∂x)on velocity vector (dx/dt). Besides, gradient vector of Lyapunov function is sidelong to ultimate growth of the function, i.e. from origin of coordinates to ultimate growth of Lyapunov function. It shall be also noted that while examining state stability (Voronova and Matrosova, 1987; Pupkov and Egugov, 2004) the origin of coordinates corresponds to defined motion or steady-state system. Equation of steady-state (Eq. 1 or 3) is to be always written in depart of from steady-state condition X_s (x = Δx = X-X_s).

Thus, Eq. 1 or 3, reflects the rate of change of deflection vector x (t) and we could assume that vector of velocity deflation leads to the origin of coordinates in state stability. Let us consider Lyapunov function V (x) given by vector-function (Voronova and Matrosova, 1987; Pupkov and Egugov, 2004; Gantmacher, 2002; Strejc, 1985) (V_1 (x), V_2 (x), ..., V_n (x)) and from geometric interpretation let us consider antigradients from components of Lyapunov function (-∂V_i (x)/∂x, i = 1, 2, ..., n) equal to components of velocity vector dx/dt, i.e.,:

$$\begin{aligned} & -\frac{dx_i}{dt} = \frac{\partial V_i(x)}{\partial x_1} + \frac{\partial V_i(x)}{\partial x_2} + \dots + \frac{\partial V_i(x)}{\partial x_n}, \\ & i = 1,2,\dots,n \end{aligned} \tag{8}$$

It follows from Eq. 8 that:

$$\begin{cases} -\dfrac{dx_1}{dt} = \dfrac{\partial V_1(x)}{\partial x_2} = -x_2\ -\dfrac{dx_2}{dt} = \dfrac{\partial V_2(x)}{\partial x_3} = -x_3 \\ \dots \quad \dots \quad \dots \\ -\dfrac{dx_{n-1}}{dt} = \dfrac{\partial V_{n-1}(x)}{\partial x_n} = -x_n \\ -\dfrac{dx_n}{dt} = \dfrac{\partial V_n(x)}{\partial x_1} + \dfrac{\partial V_n(x)}{\partial x_2} +,\dots,+ \dfrac{\partial V_n(x)}{\partial x_n} = \\ = -x_1^3 + (k_1 - a_n)x_1 - x_2^3 + (k_2 - a_{n-1})x_2 - \\ ,\dots, -x_n^3 + (k_n - a_1)x_n \end{cases}$$

Then, total derivatives in time from components of Lyapunov vector-function for steady-state condition (Eq. 5) equal to:

$$\begin{cases} \dfrac{dV_1(x)}{dt} = -x_2^2 \\ \dfrac{dV_1(x)}{dt} = -x_3^2 \\ \dots \quad \dots \\ \dfrac{dV_{n-1}(x)}{dt} = -x_n^2 \\ \dfrac{dV_n(x)}{dt} = \\ = -\begin{bmatrix} -x_1^3 + (k_1 - a_n)x_1 - \\ -x_2^3 + (k_2 - a_{n-1})x_2 -,\dots, \\ -x_n^3 + (k_n - a_1)x_n \end{bmatrix}^2 \end{cases} \tag{9}$$

From Eq. 9, it follows that total derivatives in time from components of Lyapunov vector-function will always be a negative function. Also, total derivatives in time of Lyapunov function represented by sum of ($\dot{V}(x) = \dot{V}_1(x) + \dot{V}_2(x) + \dots + \dot{V}_n(x)$) shall be obtained as:

$$\frac{dV(x)}{dt} = -x_2^2 - x_3^2 -,...,-x_n^2 - \begin{bmatrix} -x_1^3 + (k_1 - a_n)x_1 - \\ -x_2^3 + (k_2 - a_{n-1})x_2 -,..., \\ -x_n^3 + (k_n - a_1)x_n \end{bmatrix}^2$$

Lyapunov function could be formed by vector-function (Polyak, 2010) with components:

$$V_1(x) = (0, -\frac{1}{2}x_2^2, 0, ..., 0)$$

$$V_2(x) = (0, 0, -\frac{1}{2}x_3^2, ..., 0)$$

$$... \quad ... \quad ...$$

$$V_{n-1}(x) = (0,0,0,..., -\frac{1}{2}x_n^2)$$

$$V_n(x) = \begin{pmatrix} \frac{1}{4}x_1^4 + \frac{1}{2}(a_n - k_1)x_1^2, \\ \frac{1}{4}x_2^4 + \frac{1}{2}(a_{n-1} - k_2)x_2^2, \\ ......, \\ \frac{1}{4}x_n^4 + \frac{1}{2}(a_1 - k_n)x_n^2 \end{pmatrix}$$

Herein components of Lyapunov vector-function (V_i, i-1, 2, ..., n) formed by components of gradient vector:

$$\frac{\partial V_1(x)}{\partial x_1} = 0, \frac{\partial V_1(x)}{\partial x_2} = -x_2, \frac{\partial V_1(x)}{\partial x_3} = 0, ..., \frac{\partial V_1(x)}{\partial x_n} = 0$$

$$\frac{\partial V_2(x)}{\partial x_1} = 0, \frac{\partial V_2(x)}{\partial x_2} = 0, \frac{\partial V_2(x)}{\partial x_3} = -x_3, ..., \frac{\partial V_2(x)}{\partial x_n} = 0$$

$$\frac{\partial V_{n-1}(x)}{\partial x_1} = 0, \frac{\partial V_{n-1}(x)}{\partial x_2} = 0, \frac{\partial V_{n-1}(x)}{\partial x_3} = 0, \frac{\partial V_{n-1}(x)}{\partial x_n} = -x_n$$

$$\frac{\partial V_n(x)}{\partial x_1} = x_1^3 + (a_n - k_1)x_1$$

$$\frac{\partial V_n(x)}{\partial x_2} = x_2^3 + (a_{n-1} - k_2)x_2, \frac{\partial V_n(x)}{\partial x_n} = x_n^3 + (a_1 - k_n)x_n$$

Lyapunov function in a scalar form is represented by:

$$V(x) = \frac{1}{4}x_1^4 + \frac{1}{2}(a_n - k_1^2)x_1^2 + \frac{1}{4}x_2^4 + \frac{1}{2}(a_{n-1} - k_2 - 1)x_2^2 +,...,+ \frac{1}{4}x_n^4 + \frac{1}{2}(a_1 - k_n - 1)x_n^2 \quad (11)$$

Condition for stability of steady state (Eq. 5) of the (Eq. 3) shall be achieved with negative definiteness of the (Eq. 10) conditioned by positive definiteness of Lyapunov (Eq. 11) as:

$$\begin{cases} a_n - k_1 > 0 \\ a_{n-1} - k_2 - 1 > 0 \\ ... \quad ... \quad ... \\ a_1 - k_n - 1 > 0 \end{cases} \quad (12)$$

Or inequality (Eq. 12) could be re-stated as:

$$\begin{cases} -\infty < k_1 < a_n \\ -\infty < k_2 < a_{n-1} - 1 \\ ... \quad ... \quad ... \\ -\infty < k_n < a_1 - 1 \end{cases}$$

As a result, steady state Eq. 5 of the Eq. 3 will be asymptotically stable if conditions of (Eq. 12) met.

Let us examine stability of steady state (Eq. 7) and for the (Eq. 3) in deviation with respect to steady-state (Eq. 7): To generalize the reasoning, if we assume that all equation parameters are undetermined and flow simultaneously from negative domain of parameters to positive, we shall result in:

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = x_3 \\ ... \quad ... \\ \dot{x}_{n-1} = x_n \\ \dot{x}_n = -x_1^3 + 2(a_n - k_1)x_1 - \\ x_2^3 + 2(a_{n-1} - k_2)x_2 \\ ... \quad ... \\ -x_n^3 + 2(a_1 - k_n)x_n \end{cases} \quad (13)$$

$$y = x_1$$

Determine components of gradient vector from component of Lyapunov vector-function:

$$\frac{\partial V_1(x)}{\partial x_1} = 0, \frac{\partial V_1(x)}{\partial x_2} = -x_2, \frac{\partial V_1(x)}{\partial x_3} = 0, ..., \frac{\partial V_1(x)}{\partial x_n} = 0$$

$$\frac{\partial V_2(x)}{\partial x_2} = 0, \frac{\partial V_2(x)}{\partial x_1} = 0,$$
$$\frac{\partial V_2(x)}{\partial x_3} = -x_3, ..., \frac{\partial V_2(x)}{\partial x_n} = 0$$

$$\begin{aligned} &\frac{\partial V_n(x)}{\partial x_1} = x_1^3 - 2(a_n - k_1)\, x_1, \\ &\frac{\partial V_n(x)}{\partial x_2} = x_2^3 + 2(a_{n-1} - k_2)\, x_2 \end{aligned} \quad (14)$$

$$\frac{\partial V_n(x)}{\partial x_3} = x_3^3 - 2(a_{n-2} - k_3)\, x_3, ...,$$
$$\frac{\partial V_n(x)}{\partial k_n} = x_n^3 - 2\,(a_1 - k_n)\, x_n$$

Total derivative in time of Lyapunov vector-function is prescribed by:

$$\begin{aligned} \dot{V}(x) = &-x_2^2 - x_3^2 -, ..., -x_n^2 - \\ &\left[x_1^3 - 2(a_n - k_1)\right. x_1 + x_2^3 - \\ &2(a_{n-1} - k_2)x_2 +, ..., + x_n^3 - 2(a_1 - k_n)x_n]^2 \end{aligned} \quad (15)$$

It is obvious from (Eq. 15) that sufficient condition for asymptotic stability of Lyapunov theorem will be always fulfilled, i.e., total derivatives of Lyapunov vector-function shall be a sign-negative function. By gradient let us build Lyapunov function in a scalar form (Pupkov and Egugov, 2004):

$$\begin{aligned} V(x) = &V_1(x) + V_2(x) +, ..., + V_n(x) = \\ &= \frac{1}{4}x_1^4 + \frac{1}{2}(a_n - k_1)x_1^2 + \frac{1}{4}x_2^4 + \\ &\frac{1}{2}(a_{n-1} - k_2 - \frac{1}{2})x_2^2 + \\ &\frac{1}{4}x_3^4 + (a_{n-2} - k_3 - \frac{1}{2})x_3^2 +, ..., + \\ &\frac{1}{4}x_n^4 + (a_1 - k_n - \frac{1}{2})x_n^2 \end{aligned} \quad (16)$$

From Eq. 16 equilibrium state of Eq. 7 is a necessary condition of asymptomatic stability, i.e., condition of positive definiteness of Lyapunov function for equilibrium state of Eq. 7 that only exists at:

$$k_i - a_{n-i+1} > 0, i = 1, ..., n$$

If true:

$$a_n - k_n > 0,\ a_{n-1} - k_2 - \frac{1}{2} > 0,$$
$$a_{n-1} - k_2 - \frac{1}{2} > 0, ..., a_1 - k_n - \frac{1}{2} > 0$$

Thus, it is noted that state Eq. 1 after introducing control law in a form of one-parameter structurally-stable mapping, attains qualities of robust stability in wider ranges of undefined motion of the parameter a_i, i = 1, ..., n. It appears that state x_{1s} = x_{2s} = ... x_{ns} is global asymptotically stable if conditions of (Eq. 12) met and unstable if conditions fail, state (Eq. 7) asymptotically stable. If k = 0, then bifurcation occurs with new stable bifurcations, i.e., arises new opportunity to build stable control system at any changes of undetermined parameters.

Considering all, a new method for building robust stable control system in a class of one-parameter structurally-stable mapping (Beisanbi and Yermekbayeva, 2013; Beisenbi and Yermekbayeva, 2014) of dynamic objects with undetermined parameters was presented, this will allow to fully increase the potential of robust stability.

DISCUSSION

The Examination of control system stability of drying process, built in a class of structurally stable mapping by Lyapunov function: Efficiency of control systems with increased potential of robust stability is clearly illustrated on the example of building control systems in a form of one-parameter structurally-stable mapping for technological drying process of materials in textile industry. Let us examine control system stability of drying process, build in a class of structurally stable mapping by Lyapunov function (Voronova and Matrosova, 1987; Pupkov and Egugov, 2004). Equation ACS shall be presented in a problem state as (Eq. 1) where:

$$A = \begin{Vmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & -T_1 & -T_1T_0 \end{Vmatrix},$$
$$B = \begin{Vmatrix} 0 \\ 0 \\ k_0k_1 \end{Vmatrix} \quad u = k_p x_1, c = \begin{Vmatrix} 1 & 0 & 0 \end{Vmatrix}$$

In detailed view of state equation:

$$\begin{cases} \dfrac{dx_1}{dt} = x_2 \\ \dfrac{dx_2}{dt} = x_3 \\ \dfrac{dx_3}{dt} = -k_0k_1x_1^3 + k_0k_1k_px_1 - T_1x_2 - T_0T_1x_3 \end{cases} \quad (17)$$

Let us determine steady state of the (Eq. 17):

$$\begin{cases} x_{1s} = 0 \\ x_{2s} = 0 \\ x_{3s} = -k_0 k_1 (x_{1s}^3 - k_p x_{1s}) - T_1 x_{2s} - T_0 T_1 x_{3s} = 0 \end{cases} \quad (18)$$

From Eq. 18, we identify steady state of Eq. 17:

$$x_{1s} = x_{2s} = x_{3s} = 0 \quad (19)$$

Other steady states of Eq. 17 shall be defined by solution of the equation:

$$-x_{1s}^2 + k_p = 0,\ x_{2s} = 0,\ x_{3s} = 0$$

If k_p ($k_p<0$) is negative, this equation has i (imaginary) solution. If $k_p>0$, the following steady conditions are allowed in the equation:

$$x_{1s}^2 = \sqrt{k_p},\ x_{2s} = x_{3s} = 0 \quad (20)$$

$$x_{1s}^3 = -\sqrt{k_p},\ x_{2s} = x_{3s} = 0 \quad (21)$$

Conditions Eq. 20 and 21 of Eq. 17 merge with Eq. 19 if parameter $k_p = 0$ and bifurcate if.To examine robust stability of steady states (Eq. 19-21), let us use fundamental principles of Lyapunov function, we shell then define components of gradient-vector:

$$\frac{\partial V_1(x_1,x_2,x_3)}{\partial x_1} = 0,\ \frac{\partial V_1(x_1,x_2,x_3)}{\partial x_2} = -x_2,$$
$$\frac{\partial V_1(x_1,x_2,x_3)}{\partial x_3} = 0$$
$$\frac{\partial V_2(x_1,x_2,x_3)}{\partial x_1} = 0,\ \frac{\partial V_2(x_1,x_2,x_3)}{\partial x_2} = 0,$$
$$\frac{\partial V_2(x_1,x_2,x_3)}{\partial x_3} = -x_3$$
$$\frac{\partial V_3(x_1,x_2,x_3)}{\partial x_1} = k_0 k_1 x_1^3 - k_0 k_1 k_p x_1,$$
$$\frac{\partial V_3(x_1,x_2,x_3)}{\partial x_2} = T_1 x_2,\ \frac{\partial V_3(x_1,x_2,x_3)}{\partial x_3} = T_0 T_1 x_3$$

Total derivatives in time of components from Lyapunov vector-function shall be:

$$\frac{dV_1(x)}{dt} = -x_2^2,\ \frac{dV_2(x)}{dt} = -x_3^2,\ \frac{dV_3(x)}{dt} = -k_0^2 k_1^2 (x_1^3 - k_p x_1)^2 - T_1^2 x_2^2 - T_0^2 T_1^2 x_3^2$$

Or total derivatives in time from scalar Lyapunov function could be stated as:

$$\frac{dV(x)}{dt} = -k_0^2 k_1^2 (x_1^3 - k_p x_1)^2 - (T_1^2 + 1) x_2^2 - (T_0^2 T_1^2 + 1) x_3^2 \quad (22)$$

Total derivative in time of Lyapunov function is a sign-negative function. We shall then achieve components of Lyapunov function as:

$$V_1(x) = -\frac{1}{2} x_2^2,\ V_2(x) = -\frac{1}{2} x_3^2,\ V_3(x) = -\frac{1}{4} k_0 k_1 x_1^4 - \frac{1}{2} k_0 k_1 k_p x_1^2 + \frac{1}{2} T_0 T_1 x_3^2$$

Lyapunov function in scalar form is represented as:

$$V(x) = \frac{1}{4} k_0 k_1 x_1^4 - \frac{1}{2} k_0 k_1 k_p x_1^2 + \frac{1}{2} (T_0 T_1 - 1) x_3^2 \quad (23)$$

Stability conditions of zero steady state of Eq. 19 identified with negative definiteness of Eq. 22 from a condition of positive definiteness of Eq. 23 $k_0k_1>0$, $-k_0k_1k_p>0$, $T_1-1>0$, $T_0T_1-1>0$, when $k_0>0$, $k_1>0$, $T_1>0$, $T_0>0$, this condition shall be feasible if $k_0<0$, so that zero equilibrium will be stable at negative values of coefficient $k_p<0$ while other steady stable conditions do not exist.

The study stability of steady condition (Eq. 20): Let us examine stability of steady condition (Eq. 20) for such equation of drying process (Eq. 17) in deviation with respect to steady state (Eq. 20) we provide the following (Beisenbi and Yermekbayeva, 2014; Beisenbi and Mukataev, 2014; Beisenbi *et al*., 2014):

$$\begin{cases} \frac{dx_1}{dt} = x_2 \\ \frac{dx_2}{dt} = x_3 \\ \frac{dx_3}{dt} = -k_0 k_1 x_1^3 - 3k_0 k_1 \sqrt{k_p} x_1^2 - \\ 2k_0 k_1 k_p x_1 - T_1 x_2 - T_0 T_1 x_3 \end{cases}$$

We shall identify components of gradient vector of Lyapunov vector-function components:

$$\frac{\partial V_1(x_1,x_2,x_3)}{\partial x_1}=0,\frac{\partial V_1(x_1,x_2,x_3)}{\partial x_2}$$
$$=-x_2,\frac{\partial V_1(x_1,x_2,x_3)}{\partial x_3}=0;$$
$$\frac{\partial V_2(x_1,x_2,x_3)}{\partial x_1}=0,\frac{\partial V_2(x_1,x_2,x_3)}{\partial x_2}$$
$$=0,\frac{\partial V_2(x_1,x_2,x_3)}{\partial x_3}=-x_3;$$
$$\frac{\partial V_3(x_1,x_2,x_3)}{\partial x_1}=$$
$$k_0k_1x_1^3+3k_0k_1\sqrt{k_p}x_1^2+2k_0k_1k_px_1,$$
$$\frac{\partial V_3(x_1,x_2,x_3)}{\partial x_2}=T_1x_2$$

Total derivative in time of Lyapunov vector-function we identify as:

$$\dot{V}(x)=\frac{\partial V(x)}{\partial x}\frac{dx}{dt}=-x_2^2-x_3^2-(k_0k_1x_1^3+ 3k_0k_1\sqrt{k_p}x_1^2+2k_0k_1k_px_1)^2-T_1^2x_2^2-T_0^2T_1^2x_3^2 \quad (24)$$

Full derivative Eq. 24 from Lyapunov vector-function is a sign-negative function. By gradient the following Lyapunov function is formed:

$$V(x)=V_1(x)+V_2(x)+V_3(x)=-\frac{1}{2}x_2^2-\frac{1}{2}x_3^2+ k_0k_1x_1^4+k_0k_1\sqrt{k_p}x_1^3+k_0k_1k_px_1^2+ \frac{1}{2}T_1x_2^2+\frac{1}{2}T_0T_1x_3^2=k_0k_1x_1^4+k_0k_1\sqrt{k_p}x_1^3+ k_0k_1k_px_1^2+\frac{1}{2}(T_1-1)x_2^2+\frac{1}{2}(T_0T_1-1)x_3^2 \quad (25)$$

As per Moro's lemma Eq. 25 could be replaced by quadric form:

$$V(x)=k_0k_1k_px_1^2+\frac{1}{2}(T_1-1)x_2^2+\frac{1}{2}(T_0T_1-1)x_3^2 \quad (26)$$

The condition of positive definiteness of Eq. 25 or 26 shall be reached by:

$$k_0k_1k_p>0,\ T_1-1>0,\ T_0T_1-1>0$$
$$k_0>0,\ k_1>0,\ k_p>0,\ T_1>0\ \ T_0>0 \quad (27)$$

From Eq. 27, we conclude that steady state of Eq. 20 shall be asymptotically stable. The overall the transition process and phase portrait of the study stability of steady condition Eq. 20.

The stability of steady condition (Eq. 21):

$$\begin{cases}\frac{dx_1}{dt}=x_2\\ \frac{dx_2}{dt}=x_3\\ \frac{dx_3}{dt}=-k_0k_1x_1^3+3k_0k_1\sqrt{k_p}x_1^2-\\ -2k_0k_1k_px_1-T_1x_2-T_0T_1x_3\end{cases}$$

In this case, equation of deviation state represented as following Eq. 17. Let us determine gradient vector of components of Lyapunov vector-function:

$$\frac{\partial V_1(x_1,x_2,x_3)}{\partial x_1}=0,\ \frac{\partial V_1(x_1,x_2,x_3)}{\partial x_2}=-x_2,$$
$$\frac{\partial V_1(x_1,x_2,x_3)}{\partial x_3}=0;$$
$$\frac{\partial V_2(x_1,x_2,x_3)}{\partial x_1}=0,\ \frac{\partial V_2(x_1,x_2,x_3)}{\partial x_2}=0,$$
$$\frac{\partial V_2(x_1,x_2,x_3)}{\partial x_3}=-x_3;$$
$$\frac{\partial V_3(x_1,x_2,x_3)}{\partial x_1}=k_0k_1x_1^3-$$
$$3k_0k_1\sqrt{k_p}x_1^2+2k_0k_1k_px_1,$$
$$\frac{\partial V_3(x_1,x_2,x_3)}{\partial x_2}=T_1x_2\ \frac{\partial V_3(x_1,x_2,x_3)}{\partial x_1}=T_0T_1x_3$$

Total derivative in time from Lyapunov vector-function is reached by:

$$\dot{V}(x)=\frac{\partial V(x)}{\partial x}\frac{dx}{dt}=-x_2^2-x_3^2-(k_0k_1x_1^3+ 3k_0k_1\sqrt{k_p}x_1^2+2k_0k_1k_px_1)^2-T_1^2x_2^2-T_0^2T_1^2x_3^2 \quad (28)$$

Total derivative Eq. 28 of Lyapunov vector-function is a sign-negative function. By gradient the following Lyapunov function is formed:

$$V(x)=V_1(x)+V_2(x)+V_3(x)=k_0k_1x_1^4- k_0k_1\sqrt{k_p}x_1^3+k_0k_1k_px_1^2+\frac{1}{2}(T_1-1)x_2^2+\frac{1}{2}(T_0T_1-1)x_3^2 \quad (29)$$

As per Morros lemma (Eq. 29) could be replaced by quadric form:

$$V(x)=k_0k_1k_px_1^2+\frac{1}{2}(T_1-1)x_2^2+\frac{1}{2}(T_0T_1-1)x_3^2 \quad (30)$$

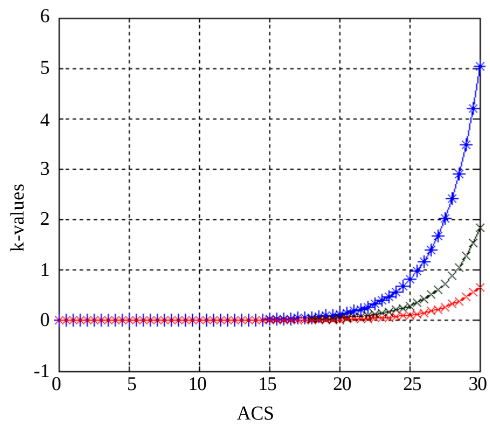

Fig. 1: The transient-response curve in ACS with proportional control laws for parameters: $k_0 = 0.8$; $k_p = 2$; $k_1 = 2$; $T_1 = 10$; $T_0 = 2$

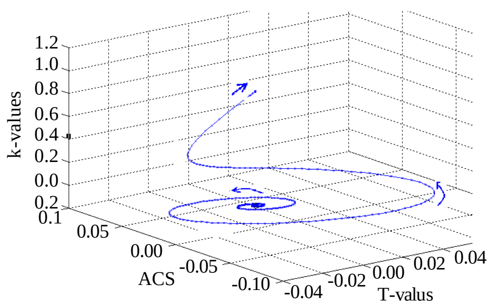

Fig. 2: The phase portrait of ACS with proportional control laws for parameters: $k_0 = 0.8$; $k_p = 2$; $k_1 = 2$; $T_1 = 10$; $T_0 = 2$

The condition of positive definiteness of Eq. 29 or 30 shall be reached by:

$$\begin{aligned} &k_0k_1k_p > 0, T_1 - 1 > 0,\ T_0T_1 - 1 > 0 \\ &k_0 > 0, k_1 > 0, k_p > 0, T_0 > 0, T_1 > 0 \end{aligned} \tag{31}$$

From Eq. 31, we conclude that steady state of Eq. 31 shall be asymptotically stable if conditions of Eq. 31 met. The overall the transition process and phase portrait of the study stability of steady condition (Eq. 21).

Case study: Conditions of robust stability have discussed in our paper can be demonstrated by simulation experiments in case study. Figure 1 and 2 shows trajectories (phase portrait and transient-response curve in ACS) of the state variables of the system with a proportional gain controller. But Fig. 3 and 4 shown with robust stability with conditions when parameter k_p assumes both with conditions positive and negative values of the system. Figure 3-6 shows phase portrait and

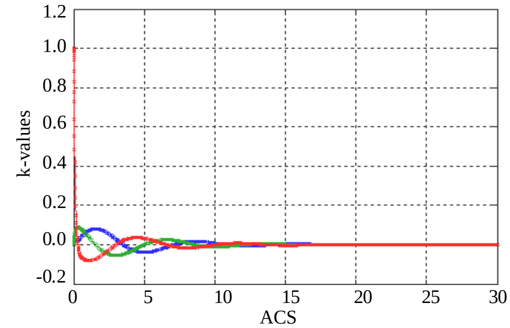

Fig. 3: The transient-response curve in ACS with greater potential of robust stability in drying process for parametres: $k_0 = 1$; $k_p = \pm 2$; $k_1 = 2$; $T_1 = 5$; $T_0 = 2$

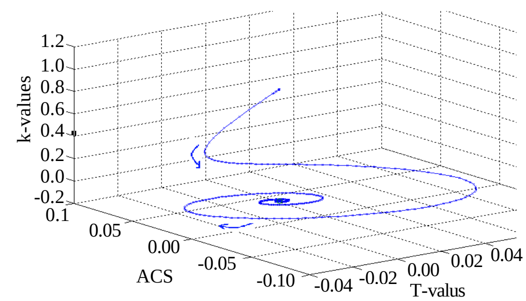

Fig. 4: The phase portrait of ACS with greater potential of robust stability in drying process for parametres: $k_0 = 1$; $k_p = \pm 2$; $k_1 = 2$; $T_1 = 10$; $T_0 = 2$

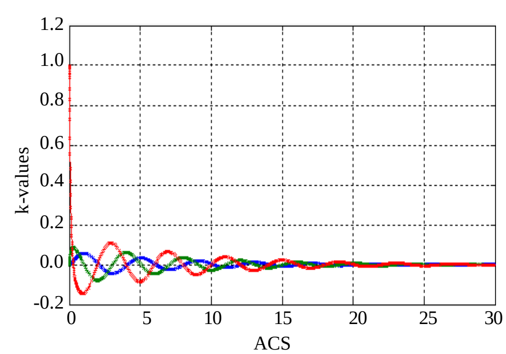

Fig. 5: The transient-response curve in ACS with greater potential of robust stability in drying process for parameters: $k_0 = 1$; $k_p = \pm 6$; $k_1 = 2$; $T_1 = 5$; $T_0 = 2$

transient-response curve in ACS with greater potential of robust stability in drying process. Transient-response curve for state variables ACS with greater potential of robust stability in drying process. Figure 5 and 6 shown with robust stability with conditions, when parameter k_p will grow. The experimental results of the proposed robust control system and comparing the satisfied results obtained.

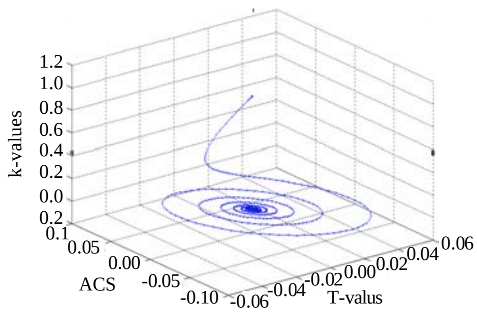

Fig. 6: The Phase portrait of ACS with greater potential of robust stability in drying process for parametres: $k_0 = 1$; $k_p = \pm 6$; $k_1 = 2$; $T_1 = 5$; $T_0 = 2$

CONCLUSION

Now a days significance of building control systems with greater potential of robust stability is determined by contemporary demands of science and technology. Case studies related to formation and design of control processes in technology, economics, biology and other spheres in conditions of essential parametric indefiniteness of greater potential of robust stability is one of the key factors that guarantees prevention from chaotic motion and guarantees applicability of models and operational reliability of designed control systems. De facto, the results of creating control systems with greater potential of robust stability allow to provide dynamic safety and operating capacity of control systems in engineering and technologies at their initial design and operation stages.

The study justifies building control systems with greater potential of robust stability for linear objects with undetermined parameters of choicely control law in a class of one-parameter structurally-stable mapping. It was demonstrated that system has asymptotically stable steady state at negative and positive domains of variation of control objects' undetermined parameters.

While undetermined parameters of control objects pass through zero, a bifurcation occurs and new stable bifurcations arise. These steady states do not exist simultaneously and the opportunity to build a system which is stable at any variations of undetermined parameters, occurs.

Use of one-parameter structurally-stable mapping though building control systems for technological process of drying shows that transient system at any value of undetermined parameter not only stabilizes but also has no limits on variation of undetermined parameters of drying process.

The results of numerical experiment prove theoretical principles. This method shows stability of control systems at negative and positive domains of undetermined parameters of system.

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