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Organizational Practices to Reduce the Risk of Loan Portfolios of Commercial Banks

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Abstract: According to the laws of the probability theory, enlargement and simultaneous differentiation of portfolio are underwriters of risk reduction. At the same time, commercial banks are forced and in some way are obliged to limit the most risky portion of credit assets, especially assets that are in greatest need of risk limitation. This study contains the initial stage of a study on the rationale of creating an organization that unites the most risky credit assets of different commercial banks.

Key words: Credit risk, banking cooperative, loan portfolio, credit risk mitigation techniques, commercial banks

INTRODUCTION

The researchers believe that some parts of corporate financial and lending legislation are inconsistent and sometimes even contrary to the natural law of existence and development of social and economic systems in Russia. Some of these laws, for example, time value of money are simply ignored others may still be unknown.

Mathematical science has accumulated great knowledge about the natural properties of random events. However, the shape and the boundaries of the random events in financial and lending situations were not studied enough. The ignorance of the "Law of Large Numbers" in lending may lead to erroneous assessment of the overall risk of the bank, its financial stability and the need for reserves.

Finally, the lack of knowledge of the fundamental properties of such an integral part of the financial and credit relations as "credit portfolio", prevents the purposeful modification of the properties with natural methods.

The outcome of almost any credit transaction is that the loan will be either repaid or not. This outcome will always have some degree of uncertainty. This uncertainty might become the main uncertainty of the financial result of the bank. Thus, when stabilizing the revenue, a bank faces non-trivial problem: how to decrease the uncertainty of financial result when it is impossible to influence uncertainty of credit environment.

It is important to note for our study that the portfolio which is relatively small in volume and weight does not allow the use of all risk reduction measures (Vasicek, 2002; DeYoung *et al.*, 2005). Moreover, the small size of the portfolio does not allow its holder to use the positive manifestation of completely natural properties of random events (Muravetsky and Kuntashev, 2013). This property is "Law of Large Numbers".

MATERIALS AND METHODS

The study is based on scientific methods, economic and mathematical modeling, statistical methods and functional analysis. On certain stage, the study implies the use of marginal analysis.

The main part: Defining the mathematically posing problem of reduction of uncertainty of credit transaction (portfolio risk) as search of the conditions when the variance of the yield of loan portfolio would be minimum. Such models have become classics of the analysis of a portfolio of securities. However, these models are not common among loan portfolios yet.

Let's assume that bank issues a loan in Q the amount of credit rate i_{kp}, the probability of default is denoted p. For simplicity, we assume the loan period is 1 year. Repayment of the loan together with interest made at the end of the loan term. Let's denote interest income as D and yield of the loan as R. Both variables, interest income D and loan yield R are discrete random variables (Kremer, 2004) that can take two possible values for the two scenarios, i.e.,the repayment of the loan and loan default (Table 1).

The expectation of a discrete random variable is calculated as the sum of values of the probability of this scenario:

$$m(D) = Q\times(1-p)\times i_{kp} - Q\times p \quad (1)$$

$$m(R) = i_{kp}\times(1-p) + (-1)\times p \quad (2)$$

The variance of a discrete random variable is calculated as the sum of the squares of the differences and the magnitude of its mathematical expectation, multiplied by the probability of this scenario:

$$\sigma^2(R) = \left(i_{kp} - m(R)\right)^2 \times (1-p) + \left(-1 - m(R)\right)^2 \times p \quad (3)$$

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Table 1: The value of revenue and return on a credit depending on the outcome of credit transaction.

Loan	Probability	D	R
Repaid	1-p	$Q \times i_{kp}$	i_{kp}
Not repaid	p	-Q	-1

As a measure of credit risk, in addition to the probability of default, you can use the standard deviation of a random variable of return:

$$\sigma(R) = \sqrt{\sigma^2(R)} \quad (4)$$

Another measure of credit risk is the coefficient of variation that takes into account the risk per unit of return:

$$k(R) = \frac{\sigma(R)}{m(R)} \quad (5)$$

To investigate the return on the portfolio and the risk of the portfolio, we will first consider the case of a portfolio with two credit loans. All the fundamental aspects will be identified. Bank issues two loans based on credit rates. For simplicity, we assume the terms of loans to be 1 year. The bank gives two loans in the amounts of Q_1 , Q_2 with corresponding credit rates i_1, i_2. Let's assume that the loan term for both credits is 1 year. The revenue from the credits will be denoted as D_1, D_2, revenues from the portfolio will be D_p. The return of the loans will be R_1, R_2 and return on portfolio will be R_p. All the above variable are discrete random variables. The relationship between revenue and return is expressed by the following equation:

$$\begin{aligned} D_1 &= R_1 \times Q_1;\ D_2 = R_2 \times Q_2; \\ D_p &= D_1 + D_2 = R_1 \times Q_1 + R_2 \times Q_2 \end{aligned} \quad (6)$$

After dividing the revenues from the portfolio on capital of portfolio loan, we see that random value of of return on the portfolio R_p equals to weighted average returns of individual loans R_1, R_2:

$$R_p = \frac{D_p}{Q_p}; Q_p = Q_1 + Q_2; R_p = x_1 \times R_1 + x_2 \times R_2 \quad (7)$$

Where x_1, x_2 are shares of portfolio capital invested in corresponding loans:

$$x_1 = \frac{Q_1}{Q_p};\ x_2 = \frac{Q_2}{Q_p} \quad (8)$$

RESULTS AND DISCUSSION

In probability theory to study the simultaneous behavior of two discrete random variables R_1, R_2, the concept of random variables or two-dimensional random variable (R_1, R_2) is introduced. The behavior of this variable is described in Table 2.

Table 2: Behavior of two-dimensional random variable

$R_2/\sqrt{R_1}$	R_2^B	R_2^H
R_1^B	p_{BB}	p_{BH}
	p_{HB}	

The first column of this table shows the possible values of the yield of return of the first loan R_1 depending, whether, it is repaid or not. The first line shows the possible values of the yield of return of the second loan R_2 depending whether it is repaid or not.

Inside Table 2, there are probability of simultaneous realization of the scenario of the first and second loans. For example, p_{HB} is the probability that the first loan is not repaid and the second loan is repaid. The total probability of default for the first loan and the probability of default for the second loan, respectively calculated according to the table:

$$p_1 = p_{HB} + p_{HH};\ p_2 = p_{BH} + p_{HH} \quad (9)$$

The sum of all probabilities in table = 1. There exists a notion of covariance of two discrete random variables X,Y or two-dimensional random variable (X,Y) in the probability theory:

$$\sigma(X,Y) = \sum_i \sum_j (X_i - m(X)) \times (Y_j - m(Y)) \times p_{ij} \quad (10)$$

Where p_{ij} is a probability that $X = X_i$, $Y = Y_j$ will happen simultaneously. Using this formula and data from the previous table we can calculate the covariance of yields of return of two loans $\sigma(R_1, R_2)$. Variance can be noted as a special case of covariance:

$$\sigma^2(X) = \sigma(X,X) = \sum_i (X_i - m(X))^2 \times p_i \quad (11)$$

We estimate the expected return of the portfolio of loans $m(R_p)$ as the expectation of return of the portfolio. We take into account the properties of the linearity of the operation of taking the mathematical expectation. As you can see, it is equal to the weighted average value of the expected returns of individual loans:

$$\begin{aligned} m(R_p) &= m(x_1 \times R_1 + x_2 \times R_2) \\ m(R_p) &= x_1 \times m(R_1) + x_2 \times m(R_2) \end{aligned} \quad (12)$$

To assess the risk of the loan portfolio, we first calculate variance of the return of the portfolio using Eq. 10 and 11 of discrete random variables and using the property of linearity of the covariance for each of the two arguments:

$$\sigma^2(R_p) = \sigma(R_p, R_p) = \sigma(x_1 \times R_1 + x_2 \times R_2, x_1 \times R_1 + x_2 \times R_2),$$
$$\sigma^2(R_p) = \sigma(x_1 \times R_1; x_1 \times R_1) + \sigma(x_1 \times R_1; x_2 \times R_2) + \sigma(x_2 \times R_2; x_1 \times R_1) + \sigma(x_2 \times R_2; x_2 \times R_2) \quad (13)$$

Let's rewrite this result in a form convenient for further generalizations, on the portfolio of an arbitrary number of loans using the summation over two indexes:

$$\sigma^2(R_p) = \sum_{i=1}^{2} \sum_{j=1}^{2} x_i x_j \sigma(R_i, R_j) \quad (14)$$

Now let's evaluate returns of loans N in the amounts of $Q_1, Q_2, ..., Q_N$ with ratings $i_1, i_2, ..., i_N$ and maturity term of 1 year. We can see that Eq. 7, 12 and 14 of portfolio risk and return get generalized:

$$R_p = x_1 \times R_1 + x_2 \times R_2 + ... + x_N \times R_N \quad (15)$$

$$m(R_p) = x_1 \times m(R_1) + x_2 \times m(R_2) + ... + x_N \times m(R_N) \quad (16)$$

$$\sigma^2(R_p) = \sum_{i=1}^{N} \sum_{j=1}^{N} x_i x_j \sigma(R_i, R_j) \quad (17)$$

Now, our goal is to identify the beneficial effects of diversification of the loan portfolio which allow to identify opportunities to reduce the overall risk of the portfolio by increasing the amount of loans, i.e., the number of borrowers, while limiting the loan amount.

For accurate mathematical evaluations consider the case of return on all loans granted are pairwise independent random variables. This means that the repayment or default of a loan with number i does not depend on the repayment or default of another j. In this case, covariance for non-matching indexes equal to zero in Eq. 17 while covariance for matching indexes by definition equal to variance:

$$\sigma(R_i, R_j) = 0, \text{ if } i \neq j$$
$$\sigma(R_i, R_j) = \sigma^2(R_i), \text{ if } i = j \quad (18)$$

Now Eq. 17 of the variance of return looks like that:

$$\sigma^2(R_p) = \sum_{i=1}^{N} x_i^2 \sigma^2(R_i) \quad (19)$$

Now let's examine the case, when the amounts of loans are the same:

$$x_j = \frac{1}{N}; \sigma^2(R_p) = \sum_{j=1}^{N} \left(\frac{1}{N}\right)^2 \times \sigma^2(R_j) \quad (20)$$

The variance of returns of all loans is bounded by a constant σ_{max} as a result there is a chain of valid estimates of inequalities:

$$\sigma^2(R_p) = \sigma(R_p, R_p) = \sigma(x_1 \times R_1 + x_2 \times R_2, x_1 \times R_1 + x_2 \times R_2),$$
$$\sigma^2(R_p) = \sigma(x_1 \times R_1; x_1 \times R_1) + \sigma(x_1 \times R_1; x_2 \times R_2) + \sigma(x_2 \times R_2; x_1 \times R_1) + \sigma(x_2 \times R_2; x_2 \times R_2) \quad (21)$$

Having extracted the square root, we obtain the following estimate for the risk of the loan portfolio:

$$\sigma(R_p) \leq \frac{\sigma_{max}}{\sqrt{N}}; \text{ if } N \to \infty, \text{ mo } \sigma(R_p) \to 0 \quad (22)$$

This means that the risk of a portfolio of loans is reduced in proportion to the square root of the number of loans. In the case, where revenues on loans are independent random variables, the theoretical risk of the loan portfolio of the bank can be almost eliminated.

In reality, the impact of the totality of the micro and macroeconomic factors can sometimes make unwanted differentiation of bank loan portfolios (Acharya *et al*., 2006; Behr *et al*., 2007; Berger *et al*., 2010), however, the most general case corresponds to the previous conclusion that the risk of the loan portfolio can be greatly reduced by increasing the number of loans (Diamond, 1984; Bebczuk and Galindo, 2008; Tabak *et al*., 2011).

CONCLUSION

Constructed mathematical model suggests that if the results of lending to each individual client are independent of each other, than bank lending falls under the scope of the law of large numbers. According to its natural manifestation with the increase of number of random events of their cumulative effect becomes less random. Nevertheless, the researchers believe (only hypothetically) that small and medium-sized banks' loan portfolios are not able to take advantage of the positive effects of the manifestation of this law.

SUGGESTIONS

As an organizational solution, we suggest the creation of an organization, more specifically cooperative bank that will combine loans with the most uncertain outcomes (risks) in one portfolio.

The approximate scheme of the cooperative work is the following. At first, the standardized tools to assess the credit worthiness of the borrower are developed. Banks that enter the cooperative are obliged to use these tools to assess the borrowers whose loans would be recommended for portfolio. In case the bank wants to keep the borrower as its own client only, the bank may assess the borrower using its own tools. However, the final decision on inclusion a certain loan in a portfolio and the rationale behind that decision should be prepared in universally accepted way. Further, the bank receives a confirmation and the recommended conditions of the loan agreement which is formed automatically on the basis of the scoring systems of banking. At the next stage, the bank enters into an agreement with the borrower and sends electronic copies of all documents to a common database. An agreement with the borrower should be formed in universally accepted ways.

Issuance of the approved loan amount to the borrower is made from the funds of the bank that signed a contract with the borrower. Fulfillment of the obligations of the borrower is completely controlled by the same bank.

Repayment of the loan amount and the interest is made on a special account specified in the contract. Information on the status of such account and its history is made available to the commission of the cooperative bank and all members of the cooperative. However, the right to dispose of the funds in these accounts up to a certain point belongs only to the bank who signed the contract with the borrower. Periodically, for example, once a month, quarter, 6 months or a year, the average percentage of income received throughout the combined portfolio can be calculated based on actual payments received by the banks. Further, this percentage is redistributed between banks participants of the cooperative. Payment made to cover the principal of the loan are subject to redistribution as well. The model of the total portfolio of risky loans of commercial banks at is presented in the most general terms at the moment. We need to continue our studies and refinement of methodological, organizational and legal aspects. Lastly, it is necessary to confirm the assumption of adequate independence of the results of individual credit borrowers.

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