

## Evaluation of Faculty Employment Policies Using Hybrid Markov Chain Model

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**Abstract:** A hybrid model based on Markov chain and data interpolation is proposed for evaluating the faculty recruitment policy in higher learning institutions. The question arise is how to evaluate the new recruitment policy in a quantitative projection approach. The data contains about 1033 individuals are collected from the registrar office at one university, the detail of academic staff such age and status ranks are used in the model design. The movement of this academic staff from one state to another categorized by age and status rank is developed using matrix transition diagram of Markov chain. The transition in each state is recomputed using interpolation. The revised transition matrix of Markov chain based on interpolation can be used as an equation solver to calculate mean time estimation for each category of faculties. The hybrid model results are then compared to the classical Markov chain results for both old and new policies by means of mean time estimation. Two scenarios were considered in the study; scenario 1 was based on historical data pattern between year 1999-2014 and scenario 2 was based on the new policies. The results showed the possibility average length of stay by position for both scenarios. The hybrid model shows the projection number of faculties in order to reduce the mean time at each category, if the new policies were to introduced. This study analyses the result of mean time estimation for faculties using classical Markov chain and hybrid Markov chain model. The numerical results showed that the hybrid Markov chain model presented lower mean time estimation than the classical Markov chain model. This approach presents an alternative yet effective way of the use of Markov chain technique in the planning of manpower.

**Key words:** Markov chains, excel spreadsheet, matrix transition diagram, interpolation, model

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### INTRODUCTION

Faculty employment policies have always been a continuous subject of discussion among higher learning institution nationally. The shortages of faculty members are relatively due to increasing number of faculty leave the institution through resignation for some reason. The problem will be particularly serious especially in a high demand academic programmes.

When the academic department face the faculty shortage, more teaching workload were assigned to the present faculty members. This phenomena will decrease their capability to focus on their research and publication. Therefore, monitoring faculty flow becomes increasingly important and the analysis of the effects of hiring decisions on the faculty recruitment becomes more critical.

To meet the demand for higher education and to stem the outflow of foreign exchange, the government during the Sixth Malaysian Plan, 1991-1995, adopted a policy of expanding the role of the private sector as a provider of higher education. This was within the wider policy framework of promoting the private sector as a provider of higher education. Because of this policy, more private universities appear and have become competitors to the

public universities. Interesting offers from private universities and business organization attract more lecturers to resign from public universities. If careful attention is not given to forecast the impact of this scenario, public higher learning institution may face problem in recruiting and maintaining their academic staff of good quality. Indirectly this will influence their student's academic performance.

Obviously manpower is one of the crucial factor in achieving the mission and also an aid to the institutional planning process. Earliest manpower analysis models in higher learning institutions were developed in Berkeley campus of University California (Bartholomew and Smith, 1988). The analysis contributes issues of staff recruitment, retirement and retention towards improving recruitment success, faculty retention and better planning for future campus faculty hiring needs. By understanding the manpower behaviour, management of the university can make a plan for the future. Central aspect of manpower planning problems lays In lecturers' shortages (Bartholomew *et al.*, 1991). These shortages are reasoned by no commensurate of supply and demand of lecturers as it comes from poor recruitment by management in flow of lecturers in faculty.

**Literature review:** Any investigation on the faculty flow of an institution must take account for institutional policies and plans (Hackett *et al.*, 1999; Bandmir and Mehrpouyan, 2016). A model begins in analysing with the presence of historical data and also parameters for alternative personnel plans and the policies which is also known as variables (Hackett *et al.*, 1999). The variables are distinguished by three planning variables which are the activity, stock and price variables. In terms of supply and demand, planning variables are categorized as activity variables. Activity variables are defined as almost anything that goes on at a university during a particular period of time. Since activity variables can be measured in terms of quantity per unit of time, they should be viewed as “flows” (Hopkins, 1981). Thus, planning variables are important in the faculty flow. Hopkins (1974) stated that, the purpose of using planning variables is to investigate the influence of variables to the faculty flow. In this research, several variables will need to be considered in developing a faculty flow model. From previous studies, there are many variables used in faculty flow model. In Berkeley Campus of the University of California, variables used in the faculty flow model are tenured, untenured and retirement (Oliver, 1969). Eddy and Morrill (1975) developed a faculty cost and tenure model which involves the variables which are rank status, percentage at each rank, years of service required for tenure at each rank, present salary by rank, percentage of salary allocated for fringe benefits and average salary increases by rank. Meanwhile, several researches that have developed faculty flow models for Stanford University, Oregon State University, Capitol Campus (Pennsylvania State University), Michigan State University as well as Auburn University found that these universities have similar variables which are tenured, untenured, rank status, age, resignation, retirement, quit and death (Bleau, 1982; Hopkins, 1981; Hackett *et al.*, 1999). In this study, we focus on rank status and age as these two variables are related to the change of policy that is currently being the issue. Many studies model manpower flow using Markov chain in variety of field such as personnel planning by Ossai and Uche (2009) military (Skulj *et al.*, 2008), management (Heneman and Sandver, 1977), healthcare (Liu *et al.*, 1991) and education (Hackett *et al.*, 1999; Adeleke *et al.*, 2014; Supratman, 2015).

Other method to model manpower flow can be found in studies such as equilibrium model (Oliver, 1969; Bleau, 1982) Bowen and Sosa model (Shapiro, 2001), Cohort model (Marshall, 1973) and simulation model (Skulj *et al.*, 2008; Hanna and Ruwanpura, 2007) System dynamic (Dimitriou and Tsantas, 2010). Most studies focus on the application of Markov chain model in flow projection and mean time analysis.

Recently, studies in Markov chain model have been on improving the forecasting ability by hybridising it with other potential method such as by Jamal, Krampe *et al.* (2010) and Dindarloo *et al.* (2015). The hybrid approaches are all meant to supplement the Markov chain capabilities in data flow forecasting.

Therefore in this study, we integrate the interpolation technique in the Markov chain model for modeling manpower flow in order to identify the recruitment and promotion behavior for academic staff in higher learning institutions.

A Markov chain model is considered for modeling manpower flow in order to identify the recruitment and promotion behavior for academic staff in higher learning institutions. To increase the model flexibility a matrix form or high order Markov structure is used which takes into account a few number of faculty categories in order to determine the next faculty flow. The underlying rationale behind this approach is that although the promotion of staff is random, it generally follows a probabilistic pattern which might be captured by a rich Markov model. Therefore high order model, in which the next flow depends on the recent history, say for the last few years seems more appropriate, as a modeling tool. The Mixture Transition Distribution (MTD) was introduced by Raftery for high order Markov chain model. It is flexible and can represent a wide range of dependence pattern Raftery 1985; Raftery and Tavare, 1994; Berchtold and Raftery 2002), however MTD model assumed homogeneous transient states. Later (Ching *et al.*, 2013) extended Raftery model with non-homogeneous transient states. We consider the non-homogeneous transient states for generating the possible states transition diagram when certain states required a fixed change. The problem with the current application of Markov chain is when changes required for some categories other categories are assumed to remain the same. Practically, this may seem impossible in real situation. Our basic approach to overcoming the constant states where some states are modified according to policy change has two key components assign the respective state value to a new value as required by a new policy, use interpolation technique to interpolate other states value. The steps identify new states value and form a matrix transition diagram of interpolated states value that can be used for forecasting. Early studies of this approach can be found by Rahim *et al.* (2012, 2013).

## **MATERIALS AND METHODS**

The data contains all academic staff and their details are collected from the registrar office at the university, the

Status rank	A	A	A	A	B	B	B	B	B	C	C	C	C	TOTAL
age interval	22-27	28-33	34-39	40-45	46-51	28-33	34-39	40-45	46-51	52-57	34-39	40-45	46-51	52-57
A:22-27	346	123			6									475
A:28-33		72	23		1	49					3			148
A:34-39			33	3		10	21				39			106
A:40-45				1			3	8						12
A:46-51								2	2					4
B:22-27														0
B:28-33														0
B:34-39														0
B:40-45														0
B:46-51														0
C:34-39						13								13
C:40-45							17							17
C:46-51											1			1
C:52-57												11	6	17

Fig. 1: States transition diagram of scenario 1

information required from the academic staff such age and status ranks are used in the model design. The data contains about 1033 individuals and for each of the user there are three categories of rank and five sub-categories of age interval. Considering these categories as 3 blocks of 5 sub-blocks.

The block of staff categories are treated as training data. Suppose that training data consist of 5 sub-blocks including 5A (lecturer), 5B (senior lecturer) and 5C (associate professor). Then,  $K = 3$  and  $M = \{s_1 = 22-27 \text{ year}, s_2 = 34-39 \text{ year}, s_4 = 40-45 \text{ year}, s_5 = 46-51 \text{ year}\}$ . Let  $\{C_t; t = 1, 2, \dots\}$  be data arranged in sequence taking values in the state space:

$$M = \{s_1, s_2, s_3, s_4, s_k\}$$

The transition probabilities of an l-th order Markov chain as proposed by Raftery (1985) and Raftery and Tavare (1994) can be written as:

$$P(C_t = s_{i0} / C_{(t-1)} = s_{1,C(t-2)} = s_{i2,\dots,C(t-1)} = s_{i1} = \sum_{(j=1)}^1 \alpha_{jq} (s_{i0} / s_{ij}) \quad t=1 \quad (1)$$

Where:

$$Q = \{q(s_i/s_j); I, j = 1, 2, \dots, K$$

And:

$$\varphi = \{\alpha_i, I = 1, 2, \dots, l\} \text{ satisfy } q(s_i/s_j) \geq 0, i, j = 1, \dots, K$$

And:

$$\sum_{i=1}^K q(s_i/s_j) = 1, \quad \forall j=1, \dots, K, \alpha_i \geq 0, i = 1, 2, \dots, l$$

And:

$$\sum_{i=1}^l \alpha_i = 1$$

**Data collection and states transition diagram:** We have prepared the worksheet displays the states transition matrix of 5 years transition of staff categorized by rank and age as shown in Fig. 1. Two alternative scenarios were proposed in this study. The first scenario considers the policy of promoting the academic staff remain the

x	f(x)	Interpolated x
1	2.00	2.00
2		3.72
3		5.44
6		10.57
8		13.96
10		17.34
11		19.02
13		22.37
17	29.00	29.00
21		35.56
23		38.81
33		54.77
39		64.11
49		79.30
72		112.41
123		176.75
346	311	310.98

Fig. 2: Interpolated state of scenario 2

same as in year 1999-2014. Therefore, the transition probabilities developed from the previous 5 year of data would hold. The second scenario considered the new policy suggested by the government that the appointment of lecturers and associate professors should be increased by 70%. In this model, the transition probabilities would change according to the respective change in the number of academic staff appointed.

The data frequency in the transition probability matrix of scenario 1 is arranged ascendingly and the minimum, median and maximum data are selected. The data is then referred to which category of staff they are in and later assigned to new frequency stated by the rule in scenario 2. For example, median is 17 and falls on state B(40-45).

The new policy indicated that the category should be raised by 70%. Therefore the median is assigned to new data frequency, 29. Based on the observed frequency and the three assigned frequency, a new set of interpolated data are generated using Lagrange equation as following:

$$f(x) = 0.002407x^2 + 1.730826x + 0.271581$$

Later, the obtained interpolated data (Fig. 2) are arranged accordingly to their states and a new states

Status rank age interval	A 22-27	A 28-33	A 34-39	A 40-45	A 46-51	B 28-33	B 34-39	B 40-45	B 46-51	B 52-57	C 34-39	C 40-45	C 46-51	C 52-57	TOTAL
A:22-27	311	177				11									499
A:28-33		113	39			2	80				6				240
A:34-39			55	6			18	36			64				179
A:40-45				2				6	14						22
A:46-51									4	4					8
B:22-27															0
B:28-33															0
B:34-39															0
B:40-45															0
B:46-51							23								0
C:34-39								29							23
C:40-45															29
C:46-51												2			2
C:52-57													20	11	31

Fig. 3: Interpolated States Transition matrix categorized by state such as age interval and status

Status rank age interval	A 22-27	A 28-33	A 34-39	A 40-45	A 46-51	B 28-33	B 34-39	B 40-45	B 46-51	B 52-57	C 34-39	C 40-45	C 46-51	C 52-57
A:22-27	0.623246	0.354709	0	0	0	0.022044088	0	0	0	0	0	0	0	0
A:28-33	0	0.470833	0.1625	0	0	0.008333333	0.333333333	0	0	0	0.025	0	0	0
A:34-39	0	0	0.307263	0.03352	0	0	0.100556659	0.201117318	0	0	0.357542	0	0	0
A:40-45	0	0	0	0.090909	0	0	0	0.272727273	0.636363636	0	0	0	0	0
A:46-51	0	0	0	0	0	0	0	0	0.5	0	0	0	0	0
B:22-27	0	0	0	0	0	0	0	0	0	0	0	0	0	0
B:28-33	0	0	0	0	0	0	0	0	0	0	0	0	0	0
B:34-39	0	0	0	0	0	0	0	0	0	0	0	0	0	0
B:40-45	0	0	0	0	0	0	0	0	0	0	0	0	0	0
B:46-51	0	0	0	0	0	0	0	0	0	0	0	0	0	0
C:34-39	0	0	0	0	0	0	1	0	0	0	0	0	0	0
C:40-45	0	0	0	0	0	0	0	1	0	0	0	0	0	0
C:46-51	0	0	0	0	0	0	0	0	0	0	0	2	0	0
C:52-57	0	0	0	0	0	0	0	0	0	0	0	0	0.645161	0.354839

Fig. 4: Transition Probability matrix of Interpolated data categorized by state such as age interval and status

transition matrix is developed as shown in Fig. 3. The new developed interpolated data transition matrix is transformed into transition probability matrix as shown in Fig. 4 using Eq. 1.

**RESULTS AND DISCUSSION**

Matrix transition probability for data distribution of year 1999-2014 was used to project faculty distribution. The assumption was the historical patterns for 1999-2014 will continue if policy of faculty appointment remains the same. The transition probability matrix indicates the probability that a faculty will move from one state to another within one period (5 years). Figure 5 provides the average length of stay for each level of position. The important use of Markov Chain is to predict future manpower distributions if there is policy changes in the current policy. As stated earlier, regarding the new policy, which is the recruitments of tutor and lecturer will be reduced by 10% and the appointment of senior lecturer, associate professor and professor will be increased by 70%, a new formation of matrix transition diagram is realized. We consider this policy change as scenario 2 and the old policy as scenario 1. Additionally, we proposed quadratic interpolation to predict the expected probability for each transition value so that it can be used as a guideline to monitor the transition of each state as given in Fig. 4. Estimation of average length of stay, N for each category of staff is calculated using inverse matrix operation given by:

$$N = (I - Q)^{-1}$$

Finally, the total estimation of average length of stay, for each state category is obtained by matrix multiplication NB where B is the total frequency of each state in the state transition matrix. The average length of stay for each category of staff is calculated for the three scenarios, which are scenario 1: the distribution of staff faculties remain as year 1999-2014, scenario 2: the percentage of certain staff faculties has been changed with respect to the new policy while others remain, scenario 3: the percentage of certain staff faculties has been changed with respect to the new policy while others are interpolated with respect to minimum, median and maximum data of scenario 2. The analysis is done by comparing the average length of stay between the three category of states.

Figure 6 shows the comparative results between the average length of stay value for each category using classical transition probability matrix of Markov chain and the proposed interpolated TPM of Markov chain for the old and new policy. The results for interpolated TPM has shown a better estimates of average mean queue length of stays by status for states towards a higher range of age and status. Prediction for the number of faculty member that should be either recruited or promoted using interpolation can be used by the management to consider in their decision making as this would reduce the average length of time stay by the faculty in the present position.

	2.654255	1.779192	0.417357	0.015389	0	0.073337242	0.828735429	0.088134596	0.009792733	0	0.193702	0	0	0
	0	1.889764	0.443294	0.016345	0	0.015748031	0.88023876	0.093611887	0.010401321	0	0.20574	0	0	0
	0	0	1.443548	0.053226	0	0	0.661290323	0.10483871	0.033870968	0	0.516129	0	0	0
	0	0	0	1.1	0	0	0	0.3	0.7	0	0	0	0	0
	0	0	0	0	1	0	0	0	0.5	0.5	0	0	0	0
	0	0	0	0	0	1	0	0	0	0	0	0	0	0
	0	0	0	0	0	0	1	0	0	0	0	0	0	0
$(I-Q)^{-1} \times N$	0	0	0	0	0	0	0	0	1	0	0	0	0	0
	0	0	0	0	0	0	0	0	1	0	0	0	0	0
	0	0	0	0	0	0	1	0	0	0	1	0	0	0
	0	0	0	0	0	0	0	1	0	0	0	1	0	0
	0	0	0	0	0	0	0	2	0	0	0	2	1	0
	0	0	0	0	0	0	0	2	0	0	0	2	1	1.55

Fig. 5: Average length of stay of interpolated data states for each level of position

	Old Policy	New Policy	Interpolate Data With New Policy
A:22-27	7.162696447	6.059895752	6.400788126
A:28-33	3.601527168	3.555143510	2.973587047
A:34-39	3.03113325	3.012903226	2.812769629
A:40-45	2.090909091	2.1	2.052631579
A:46-51	2	2	2
B:22-27	1	1	1
B:28-33	1	1	1
B:34-39	1	1	1
B:40-45	1	1	1
B:46-51	1	1	1
C:34-39	2	2	2
C:40-45	2	2	2
C:46-51	3	5	3
C:52-57	4.545454545	6.55	4.526315789

Fig. 6: Comparative average length of stay between scenario 1: old policy, scenario 2: new policy and scenario 3: interpolated data with new policy

**CONCLUSION**

Based on the results obtained, the Markov chain model developed in this study is an appropriate evaluation tool for policy change concerning the appointment of faculty. This study demonstrates that if new policy is implemented, there will be a high impact on the number of academic staff by diverse rank especially towards more senior faculty members. Mean time for the faculty remain in current state does not show much difference between old and new policy. Based on the results, it is predicted the proposed policy will not have much changes if it were to be implemented. Otherwise a modified predicted approach is required such as interpolated data estimators approach as proposed in this study. This study can be further extended to incorporate other factors such as gender or ethnic to model the flow from different perspective and increase diversity.

This study presents the potential use of interpolation technique for predicting better estimate for projecting transition data in Markov chains. The states transition of staff faculties categories can easily be constructed using spreadsheet and the calculation of matrix performance can be done simply using excel built-in function. However the interpolated data could not automatically be filled into the respective cell in the spreadsheet to update the classical matrix transition diagram. This limitation will be studied further in the future work.

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