The Social Sciences 12 (9): 1516-1521, 2017
ISSN: 1818-5800

How Well Do University Level Courses Prepare Students to Be Mathematical Thinkers?

Parmjit Singh, Syazwani Rasid, Nurul Akmal, Teoh Sian Hoon and Cheong Tau Han
Universiti Teknologi MARA, Kampus Puncak Alam, Selangor, Malaysia

Abstract: University students in the field of sciences do indeed take a fairly large number of Mathematics courses (such as Calculus 1 and 2, Algebra, etc.) in order to graduate for their respective degrees. However, have we tried to determine the quality of this knowledge? The quality of student's Mathematics knowledge is always a crucial matter. In this light, we use problem solving as an assessment tool because it is the means by which Mathematics can be applied to a variety of unfamiliar situations to assess student's mathematical thinking. Using a descriptive design method, a paper and pencil test comprising 16 items was administered to 120 students (majoring in Mathematics, Physics and Engineering) among semester 5-6 in a college in Klang Valley. All these students have at least taken courses such as Calculus 1, 2 and Algebra as the requirement of their respective courses. The overall means score obtain by the students was 10.50 (SD = 7.72) from a maximum score of 49. The types of errors made by university students were similar as the types made by lower secondary students based on previous research. The findings indicate the university level Mathematics courses taken by students did not reciprocate with their level of expected mathematical thinking that should be displayed by them. It seems to indicate that the current notion of university Mathematics courses is based almost exclusively on formal mathematical procedures and concepts that of their nature are very remote from the conceptual world of the students who are to learn them.

Key words: Thinking, Mathematics, college Mathematics, algorithms, routine

INTRODUCTION

One of the major aims of mathematical learning is the development of mathematical thinking. The common misconception is that "doing Mathematics" is the same as getting involved in "mathematical thinking". This misconception stems from the pedantic Mathematics education in Malaysian school systems that highlight the mastery of Mathematics through rote memorization of formulaic structures. This outcome has been seriously felt based on Malaysians student's Mathematics performance in TIMSS and PISA studies. Based on the decree of Malaysian student's standard in Mathematics, various concerted effort has been taken by the education ministry to revamp our education system. One of the major outcomes was the introduction of Malaysia Education Blueprint 2013-2015 which endorsed the Science, Technology, Engineering and Mathematics (STEM) education to address this challenge. One of the aims of STEM initiatives in Malaysia is to equip students with higher order thinking and problem-solving skills. To meet this challenge, performance of schools in all levels, the kind of teacher quality and its teaching output became a national priority in addressing the quality of education learners receive (Suan, 2014). While various effort are being made by the education ministry to uplift the standard of Mathematics in schools, what about Mathematics at college level?

Students coming to college needs to unpack their mathematical content knowledge which they brings from school to allow them to examine the undergirding's and interconnections of college Mathematics with other relevant areas of mathematical application such as in Physics, Engineering, Computer Science, etc. The examination of these content knowledge such as algorithms, definitions and properties in such a way will enable them to assess and identify these knowledge if they are applying and understanding them correctly or whether there are any mistakes or misconceptions. The current notion of college Mathematics is based almost exclusively on formal mathematical algorithm, procedures and concepts that of their nature are very remote from the conceptual world of the students who are to learn them (Parmjit, 2002, 2009; Parmjit and Allan, 2006). The finding depicts that basic computation skills have been the focus for competency tests through the years, spawning tutorials sheet and instructional emphases aimed at developing these skills. Students have learned how to do

Corresponding Author: Parmjit Singh, Universiti Teknologi MARA, Kampus Puncak Alam, Selangor, Malaysia

numerical computations at the expense of learning how to think and solve problems. Hanford stated that research conducted over the past few decades shows it's impossible for college students to take in and process all the information presented during a typical lecture and yet, this is one of the primary ways college students are taught. Students see little connection between what they study in the lecture hall and real life. Just having students memorize facts and algorithms is debilitating. While students are memorizing facts which could not possibly hold any meaning for them, they are not constructing relationships and patterns. In fact, they may "stop thinking about the mathematical relationship" altogether (Wheatley, 1991). The consequential impact is negatively felt when such approach is no longer viable and usable in a higher level of tertiary education. To sum up, every student is expected to gear up with knowledge of reasoning and thinking skills in the making informed choices on complex matter. The effort of gearing up within their knowledge comes from many tries which in turn called habits of mind or thinking habits. However, failure to meet the standards of proficiency is a complex matter to pin point the blame even to the learners. There are many variables like instructors quality, quality of instruction, curriculum, financial resources and many more are out of their control (Suan, 2014). Thus, this study was undertaken to assess college students prowess of mathematical thinking per say. But, what is mathematical thinking?

Mathematical thinking is a whole way of looking at things of stripping them down to their numerical, structural or logical essentials and of analyzing the underlying patterns. The significance of mathematical thinking in the quantitative literacy was reported by the national assessment of adult literacy which found that only 13% of adults are deemed proficient in quantitative literacy which requires mathematical thinking; 33% perform at intermediate levels, 33% at basic levels and 22% are below basic student's ability to think mathematically relies on their understanding. Hughes-Hallett highlighted that students first develop skills of solving problems before acquiring the mathematical thinking skills. Leron and Hazza (1997) pointed out that student's ability in solving a mathematical problem is very much affected by their behaviour in solving mathematical tasks taking account of their attempts to understand the task and handling the failures for such attempts.

This study could be useful for curriculum developers and educators, especially in higher education institution context in evaluating the general content, strategies and methods used in teaching and learning of Mathematics in universities. Furthermore, the importance for the development of mathematical thinking, coupled with difficulties in acquiring the skill gives a rationale for assessing student's mathematical thinking in college.

Table 1: Scoring of items

Scorings marks	Items	Score
2	3,5, 8, 12	8
3	1, 2, 4, 9, 11, 13, 16	21
4	6, 7, 10, 14, 15	20
Total	16 items	49

Objectives of the study: This study aims to assess of university students thinking and reasoning capabilities via solving non-routine problems. These students have been formally taught the university level Mathematics courses and this research enabled us to assess the student's quality of mathematical thinking. Specifically, the questions addressed are:

- What are the levels of university student's thinking in Mathematics learning via solving non-routine problems?
- What are the difficulties faced by students in solving the given problems?

MATERIALS AND METHODS

This study employed a quantitative method using a descriptive research design via a paper and pencil test among 120 students from a university in Klang Valley. These student's ages 23-24 are in the fifth and sixth semester of their studies majoring in Mathematics (n = 61), Science (n = 36) and Engineering (n = 23) towards obtaining their Bachelor of Science degree. The rationale for choosing these sample was they have been formally taken the standard Mathematics required courses (over the semesters) namely; Calculus 1, 2. This enabled the researchers to evaluate the student's ability to unpack their knowledge that is to investigate if they could (after taking the various math courses over the semesters) apply their knowledge in solving daily and related problems which required them to think.

A mathematical thinking paper and pencil test instrument comprising 16 items was developed for this purpose, student's responses were categorized on a 2-4 point scale based on the reasoning employed as shown in Table 1.

RESULTS AND DISCUSSION

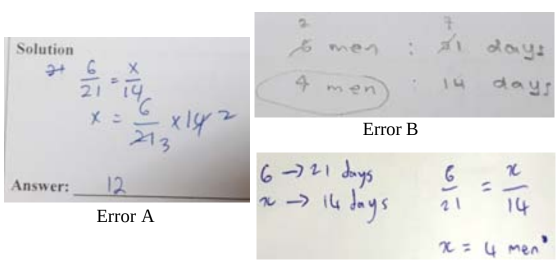
This study details the findings of study. Table 2 depicts the item analysis based on correct and incorrect responses. The findings from Table 2 show students

Table 2: Item analysis of test items

Items	Correct (%)	Incorrect (%)
If it takes six men to paint a house in 21 days, how many men's will be needed to paint the house in 14 days?	3.3	96.7
Find the sum of all the NUMBERS in the sequence 1-2+3-4+5, …, -98+99-100	21.7	78.3
A faulty calculator computes 93×134 as 14 462. Using this information, find the answer for the following number sentence if the faulty calculator is used: 14 462÷930	32.5	67.5
What is the next term for: 8, 18, 36, 49, …,	6.7	93.3
Evaluate $\sqrt{5^5+5^5+5^5+5^5+5^5}$	17.5	82.5
Kareena is 4 times as old as Laila. The 5 years from now, the sum of their ages will be 50. How old is Kareena 5 years from now?	12.5	87.5
Three water hoses are used to fill a children swimming pool. The first hose alone takes 3 h to fill the pool, the second hose alone takes 4 h to fill the pool and the third hose pipe alone takes 12 h to fill the pool If all three hoses are opened at the same time, how long will it take to fill the pool? Please explain	2.5	97.5
The number of football fans in the stadium double every minute. The stadium was full of football fans at 8.45 a.m. When was it half full?	16.7	83.3
The gauge of an airplane indicated that the oil tank was 1/8 full. After 240 L of oil were added to tank, the gauge indicates that the tank was 5/8 full. How many liters of oil do the tank hold, assuming the gauge is accurate?	25.8	74.2
Jason and his dad had a bet in answering Math problems. Jason's fathers said to his son:	19.2	80.8
"I will pay you 16 cent for every answer correct and you must pay me 10 sen for each incorrect answer". Jason accepted the bet	-	-
At the end of 52 problems, neither owes anything to the other. How many problems did Jason solve incorrectly?	-	-
Eva and Alex want to paint the door of their garage. They first mix 2 cans of white paint and 3 cans of black paint to get a particular shade of gray. They add one more can of each. Will the new shade of gray be lighter, darker or are they the same?	25.8	74.2
A student is ranked 13th from right and 8th from left. How many students are there in total?	41.7	58.3
Eighteen people, numbered 1-18 are equally spaced around a round table. What is the number of the person directly across from the person numbered 6?	42.5	57.5
Jason's and Sharon's alarm clock rang at 4.30 a.m. For the remainder of the days Jason's alarm clock will ring every 45 min and Sharon's alarm clock will ring every one hour. What will the time be when both the bell rings together again?	54.2	45.8
There were a number of students in the classroom and a total of 36 handshakes took place among the students. Each student shook hands once and only once with everyone else. How many students were in the classroom?	15.0	85.0
If seven cats catch 7 mice in 7 min, how many cats would it take to catch 105 mice in 49 min?	14.2	85.8

Table 3: Student's overall performance

Variable	N	Mean	SD
Overall	120	10.50	7.72



Error A

Error B

Error C

Fig. 1: Errors in Item 1

faced great difficulty in solving nearly all the items. These items are fundamental task can be administered to students at lower secondary as well. The lowest incorrect responses are obtain for Item 7 (97.5%) followed by Item 1 (96.7%). The item analyses based on common errors made by students are explained in the following section. The overall mean score (Table 3) obtain in the test was 10.50 (max score: 49) with a standard deviation of 7.72. These data can be an important indication of university student's fundamental relational understanding of mathematical concepts.

Item analysis: This study details the fundamentals errors made by students when solving the given problems. Due to constraint of space, only four items will be discussed.

Item 1: If it takes six men to paint a house in 21 days, how many men's will be needed to paint the house in 14 days? The data from Table 2 indicates 96.7% of students obtained an incorrect response for this item.

Majority of the students applied direct proportional thinking method by using the cross multiplication algorithm. The errors made by students (Fig. 1) reveal they failed to see the relationship of the inverse proportional as required. This shows that student's lack of logical thinking that to complete the painting in shorter time one will need more number of men. Furthermore, the error A made by student involve an elementary error which one will not expect from a university student. Since being an inverse proportion, the answer should be nine men.

Item 2: Find the sum of all the Numbers in the series 1-2+3-4+5, ..., 98+99-100. For this item, 78.3% students obtain an incorrect response. Figure 2 indicates the two most common errors made by students. Surprisingly, majority of them applied arithmetic progression's formula to solve this item not taking into consideration its viability (Error A). Similarly as Item 1, they do not tend to read and comprehend if using the formual is viable. Another group f students tried to solve by investigating the pattern involved in the question (Error B). However, the pattern

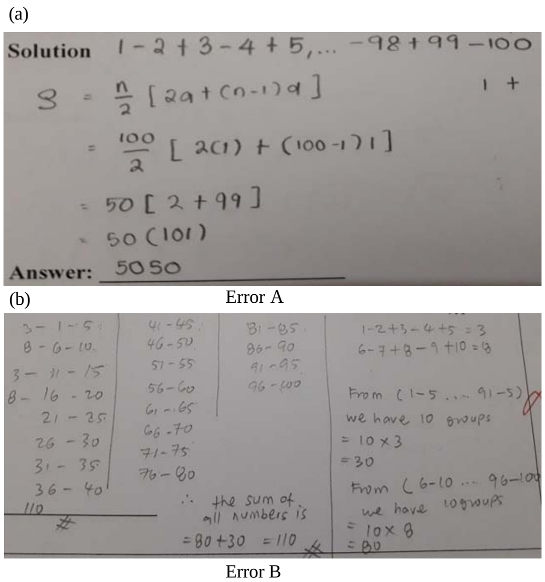


Fig. 2: Errors in Item 2

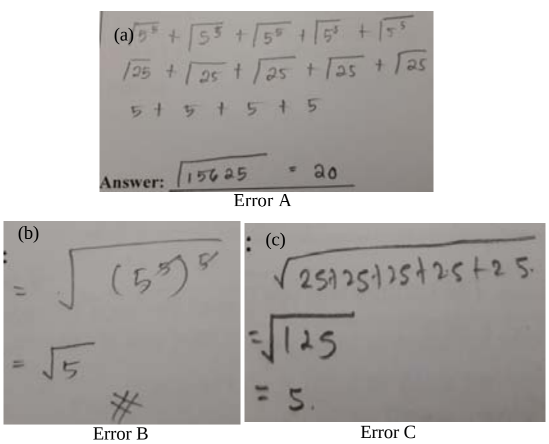


Fig. 3: Errors in Item 3

used by them is incorrect where they tried to use the first four patterns to generalize the whole series. In fact, this item can be easily solved by adding -1 for 50 times.

Item 5:

$$\text{Evaluate} = \sqrt{5^5 + 5^5 + 5^5 + 5^5 + 5^5}$$

Table 1 shows that 82.5% of students failed to answer Item 5. Error A indicates the students did mistake in manipulating the radical expression where the students separated the radical from $\sqrt{5^5+5^5+5^5+5^5+5^5}$ into $\sqrt{5^5}+\sqrt{5^5}+\ldots$. Moreover, students had some indices errors such as $5^5 = 25, \sqrt{5^5+5^5+5^5+5^5+5^5} = \sqrt{(5^5)^5}$ and $\sqrt{(5^5)^5} = \sqrt{5}$. This difficulties faced by students might be related to the over reliance usage of calculators (Fig. 3).

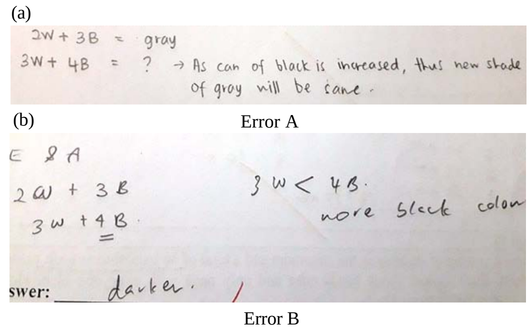


Fig. 4: Errors in Item 11

Item 11: Eva and Alex want to paint the door of their garage. They first mix 2 cans of white paint and 3 cans of black paint to get a particular shade of gray. They add one more can of each. Will the new shade of gray be lighter, darker or are they the same (Fig. 4)?

For this item, 74.2% of the students obtain an incorrect solution for this item where majority of them answered as "same". Students reasoning were based on primitive additive reasoning " if one can of black paint and one can white paint were added, the quantity of both colors will be the same". Hence, the mixture should have the same shade (Error A). Besides, students failed to construct coordination of two ratios simultaneously as 2 white to 3 black and 3 white to 4 black. student's thinking was with greater amount of black paint ratio, the darker the shade is Error B.

The findings from the item analyses shows majority of students are very prone towards algorithm procedures to solve any given problem. Utilizing these procedures simply becomes an act of symbolic manipulation without requiring that individuals make sense of what they are doing. If "doing" Mathematics is an activity in which an individual follows a procedure to obtain an answer then this is an acceptable method for solving problems. However, if "doing" Mathematics is an activity in which an individual constructs patterns and relationships, the method may not provide students with opportunities to develop their mathematical thinking.

One would expect these students, especially those has taken the various university level math courses to be excellent mathematical thinkers. But sad to say this is not so. For example, Item 11, the findings by Parmjit (2009) indicate a similar result where majority of school students used the similar additive reasoning thinking to obtain the incorrect response. In general, the performance of the students was very disappointing and it simply indicates a low level of mathematical thinking attainment among university students. We believe that presently, Math courses in the university place emphasis on the procedures rather than the process of learning. So, when students "practice" these problems via their tutorial sheets, they are practicing to get the correct answer. In other words, they ignore things like context, structure and situations and students do not have the occasion to generate the "richly inter-connected spaces" that Cooper has identified as being crucial for constructing mathematical knowledge for the development of mathematical thinking.

CONCLUSION

The findings of study indicates that students who has taken various university level Mathematics courses (such as Calculus 1, 2, Algebra, Calculus for Engineers, Advances Differential Equation, Numerical Analysis, etc.) for the respective programs does not produce mathematical thinkers. Mathematical thinking here refers to the ability to solve daily problems without the requirement of calculators or formulae's. The data identifies student's inability to unpack the subject knowledge in Mathematics as being a contributory factor in low mathematical thinking attainment.

As a mathematician, it seem obvious that when students are given a problem, the first step in approaching the problem is to identify the parameters and to formulate various heuristic skills based on the parameters. But majority seem to rely on calculators and algorithm (which were not required at all) to solve without even thinking if the solution makes sense.

For example, Item 1: if it takes six men to paint a house in 21 days, how many men will be needed to paint the same house in 14 days? (Assuming that the men are all performing at the same rate and all working for the entire time). In this item, 96.7% of the students failed to see an inverse proportion relationship and solved the question by utilizing a cross multiplicative structure. Many of them used the following heuristics:

- 21 days, 6 men
- 14 day-x
- $x/6 = 14/21$; $21x = 84$; $x = 84/21 = 4$

In fact, majority of the students utilized mechanical reasoning as shown above. Here, they did not reason what each number represents and what they were actually computing. Logically, they should have realized that the answer they produced 4 was illogical and implied that more men take a longer time to finish painting the house! It seems that students have learnt how to do numerical computation at the expense of learning how to think and solve problems.

We strongly believe that possible problems in university Mathematics may be due to the procedural paradigm orientation in most of the Mathematics courses and the conventional style of instructors in the lecture hall which do not provide sufficient opportunities for students to develop conceptual understanding. In order for students to develop this understanding, we should perhaps emphasize on giving them experiences that can create a solid foundation for these concepts (Kieran, 1994). This can be created by introducing a problem solving course where the problems are interesting, stimulating and yet challenging without the aid of any calculators or even any sophisticated formulae's which inadvertently will make students to think. Consequently, the emphasis in instruction will be shifted from learning the rules for operations to understanding of mathematical concepts. This possible solution is to encourage the transition by providing students with "problem solving tools" that would allow them to be accommodative to changing needs towards the development of mathematical thinking. To operationalize this development, instructors should shift their approach from the traditional computation and routine based one to a conceptual one. The former method involves teaching of rules and procedures rather than the conceptual thinking of Mathematics. Development of conceptual mathematical thinking could be developed by getting students to think about Mathematics and representing topics in ways other than procedures in a more meaningful manner as compared to current practice.

As the focus of the university education shifts from imitation and impractical exercise to critical production and innovation, more authentic and creative manner of solving problems are needed in resolving real life problems be it theoretical, mechanical, industrial or philosophical. These observations seem to point that there is a disparity between university Mathematics, where success is guaranteed in conformist formulaic approach and true mathematical thinking that requires "thinking outside-the-box" which would be more valuable to university students and professionals. Rallying to such argument, there is a widespread agreement that Mathematics should be taught as a thinking activity (Burton, 1984). What is the direction of Mathematics learning in higher education institution in Malaysia? What is even more important to ask is "How well do the university level courses prepare students to be mathematical thinkers?

ACKNOWLEDGEMENTS

The researchrs would like to express their gratitude to Institute of Research Management and Innovation

(IRMI), Universiti Teknologi MARA for financing this research and support rendered for the RIG Mathematical Thinking for Teaching Research Group.

REFERENCES

Burton, L., 1984. Mathematical thinking: The struggle for meaning. J. Res. Math. Educ., 5: 35-49.

Kieran, T., 1994. Multiple Views of Multiplicative Structure. In: The Development of Multiplicative Reasoning in the Learning of Mathematics, Harel, G. and J. Confrey (Eds.). State University of New York Press, New York, USA., ISBN:0-7914-1764-6, pp: 387-397.

Leron, U. and O. Hazzan, 1997. The world according to johnny: A coping perspective in mathematics education. Educ. Stud. Math., 32: 265-292.

Parmjit, S. and W. Allan, 2006. Unpacking first year university students mathematical content knowledge through problem solving. Asean J. Univ. Educ., 2: 33-56.

Parmjit, S., 2002. Orientations of school mathematics in Malaysia. J. Pendidikan, 18: 58-64.

Parmjit, S., 2009. Variation in first year college students understanding on their conceptions of and approaches to solving mathematical problems. Asean J. Univ. Educ., 5: 95-118.

Suan, J.S., 2014. Factors Affecting Underachievement in Mathematics. Proceeding of the Global Summit on Education GSE, March 4-5, 2014, Global, Kuala Lumpur, Malaysia, ISBN:978-967-11768-5-6, pp: 13-20.

Wheatley, G.H., 1991. Constructivist perspectives on science and mathematics learning. Sci. Educ., 75: 9-21.