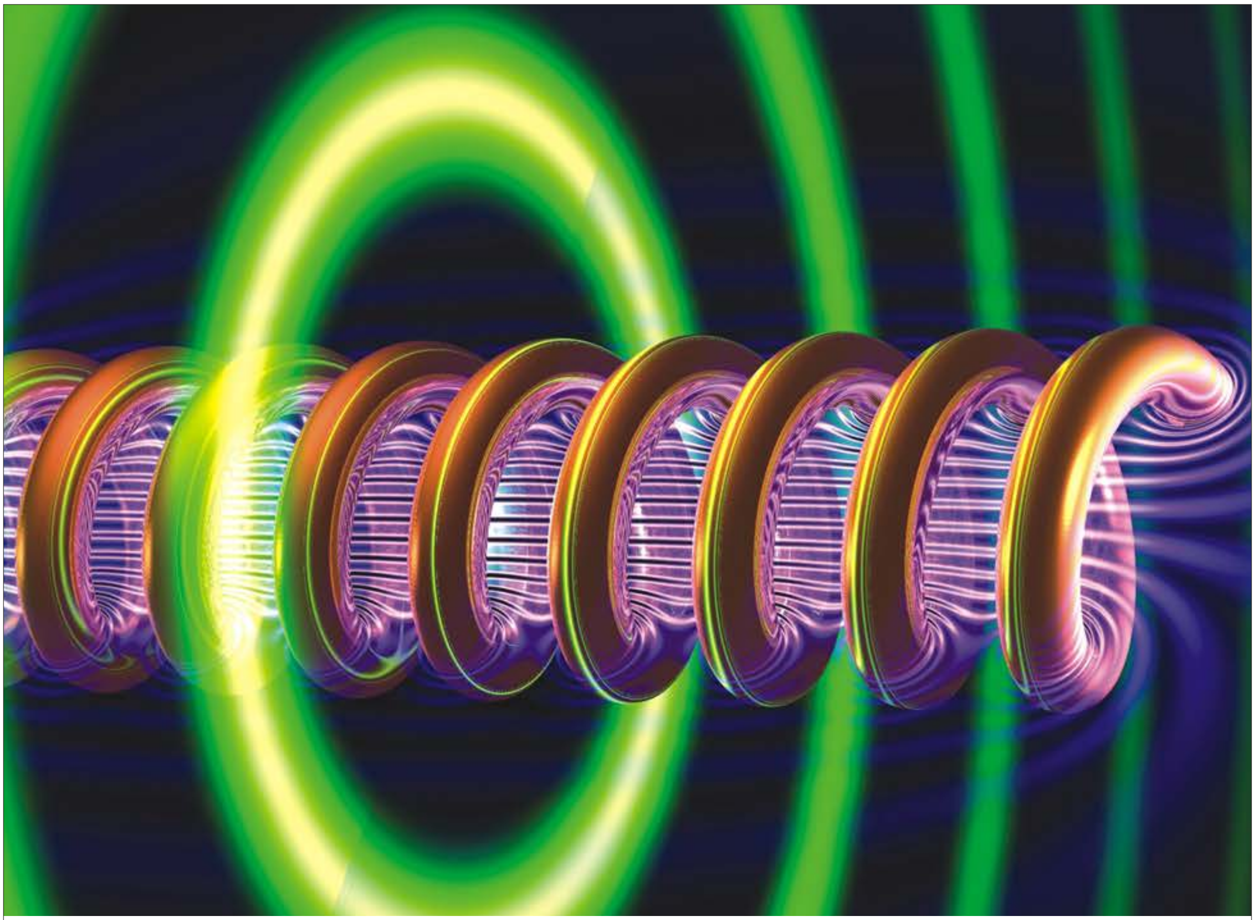

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Study of Quantum Turbulence with the Exponential Potentials

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ABSTRACT

The aim of this study, is using exponential potential in the non-linear Schrodinger equation well-known as Ginsburg-Pitaevski-Gross equation to study quantum turbulence. The non-linear Schrodinger equation which arises in quantum turbulence is also important in various fields of physics such as optics, elementary particle physics and mathematical physics. We assume exponential potentials such as Morse and generalized Woods-Saxon potentials in this equation and use Runge-Kutta-Fehlberg approximation method to obtain wave functions. We found that, the wave function has periodic behavior for all exponential form of potentials.

Key words: Quantum turbulence, non-linear schrodinger equation, wave function, exponential potential

INTRODUCTION

Traveled light through the atmosphere affected by a number of phenomena such as scattering, absorption and turbulence (Mohammadein and Abu-Bakr, 2010; Momeni and Moslehi-Fard, 2008; Lewandowski, 2003). Turbulence has been investigated not only in applied sciences but also in basic science research, such as physics and mathematics research. Turbulence is a complicated dynamical phenomenon which based on strong nonlinearity. This phenomenon is far from an equilibrium state and may be understand in context of vortices. However, classical description of vortices are not well-defined. Therefore quantum turbulence (Vinen, 2006; Tsubota, 2008; Kobayashi and Tsubota, 2005) will be more convenient. Comparing quantum turbulence and classical turbulence reveals definite differences which demonstrates the importance of studying quantum turbulence. Turbulence in a classical viscous fluid admitted vortices, but these vortices are unstable. Moreover, in order to have conserved circulation we need quantum turbulence.

Thus, quantum turbulence is an easier system to study than classical turbulence and has a much simpler model of turbulence than classical turbulence (Vinen and Niemela, 2002). A vortex in superfluid with circular quantization is called a quantized vortex. Quantized vortices also appropriate to study any rotational motion of a superfluid which is different from a classical vortex in viscous fluid. Thermal counter flow of superfluid turbulence has been studied experimentally, where the normal fluid and superfluid flow assumed in opposite directions. By using an injected heat current one can obtain flow which suggests that the superflow becomes dissipative if the relative velocity between the two fluids exceeds a critical value (Gorter and Mellink, 1949).

Since, the dynamics of quantized vortices is nonlinear and non-local, one can understand vortex dynamics observations quantitatively. Superfluid turbulence is often called quantum turbulence, which indeed study quantized vortices. Turbulence phenomenon also affect on the laser beam. The subject of turbulence is also important in optics and laser researches. Some of the important effects of turbulence on the laser beam are for example phase-front distortion, scintillation and beam broadening. Information about the turbulence profile is crucial to assist the tomographic process in wide field Adaptive Optics (AO) system. Also study of the turbulent layers may be used to reduce the impact of the delays which exists in AO systems (Poyneer *et al.*, 2009).

QUANTUM TURBULENCE

The turbulent flow of a fluid is a phenomenon, widely extend in the nature. The air circulation in the lungs and also gas movement in the interstellar medium are examples of turbulent flows.

Here, we interest to quantum turbulence in superfluid which is microscopic theory of superfluid 4He that will provide a proper description of its behavior on the small scales. In that case we deals with an equation describing the static and dynamic behavior of the condensate wave function which is the non-linear Schrodinger equation, or Ginsburg-Pitaevski-Gross equation (Pitaevskii, 1961; Gross, 1963):

$$i\hbar\frac{\partial\psi}{\partial t} = -\frac{\hbar^2}{2m}\nabla^2\psi + (U0\,|\,\phi\,|^2 + V0[\psi]^2 - \mu m + V)\psi \quad (1)$$

where, ψ is the condensate wave function of particle and ϕ is the wave function of electron. Also $V_0 = 4\pi d\hbar^2/m$ and $U_0 = 2\pi l\hbar^2/\mu$ are measures of the repulsive interatomic forces in the fluid, where, l is the boson-impurity scattering length and d is the boson diameter. Moreover, μ is the chemical potential. We assume $U_0 | V_0$ and set $\hbar = 1$ for simplicity. A single-quantum rectilinear vortex along r = 0 in cylindrical polar coordinates is described by the following function:

$$\psi = f(r)e^{i\theta} \quad (2)$$

where, f(r) = 0 and $f(\infty) = f_0$. Therefore, non-linear differential Eq. 1 reduced to the following equation:

$$f'' - 8\pi d f^3 + 2m(\mu m - V(r))f = 0 \quad (3)$$

where, prime denote derivative with respect to r. In this study, we would like to solve the Eq. 3 for various famous exponential potentials. First of all we consider constant potential. Then, we examine Morse and Wood-Saxon potentials and also a general exponential form of potential.

CONSTANT POTENTIAL

In the simplest case we assume:

$$V = E \quad (4)$$

where, E is a constant. This situation is special form of exponential function, Ee^T, when r | 1. In that case the Eq. 3 has the following solution:

$$f(r) = B[\text{jacobi SN}(B\bar{r}, D)] \quad (5)$$

where:

$$\bar{r} = c_1 + r\sqrt{2\mu m^2 - 2mE - 4\pi d} \quad (6)$$

and

$$B = c_2\sqrt{\frac{m(\mu m - E)}{\mu m^2 + 2\pi d c_2^2 - mE - 2\pi d}} \quad (7)$$

$$D = c_2\sqrt{\frac{c_2^2\,\pi d(2\mu m^2 - 2mE - 4\pi d)}{2\pi d + mE - \mu m^2}} \quad (8)$$

where, c_1 and c_2 are integration constants. In the Fig. 1, we can see that the wave function is periodic.

MORSE POTENTIAL

After studying the harmonic oscillator as a representation of molecule vibration, one notice that a diatomic molecule which was actually bound using a harmonic potential would never dissociate. The Morse potential realistically leads to dissociation, making it more useful than the Harmonic potential. The Morse potential is the simplest representative of the potential between two particles where dissociation is possible. The Morse potential may be written in the following form:

$$V(r) = C(e^{-2ar} - 2e^{-ar}) \quad (9)$$

where, C and a are arbitrary constants.

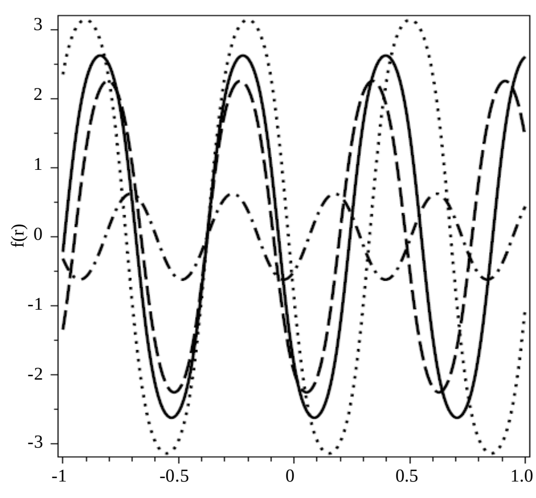

Fig. 1: Wave function in terms of r with constant potential for E = 4 (dotted line), E = 5 (solid line), E = 6 (dashed line) and E = 25 (dash-dotted line)

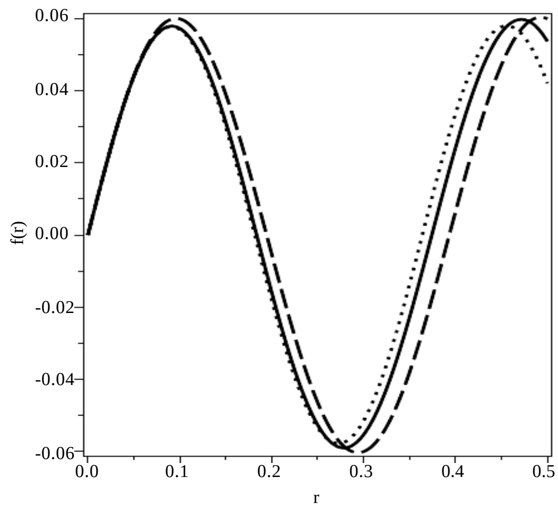

Fig. 2: Wave function in terms of r with Morse potential for a = 1 (dotted line), a= 5 (solid line) and a = 25 (dashed line)

Numerically, we find behavior of the wave function in the Fig. 2, which shows periodic feature.

SIMPLE EXPONENTIAL POTENTIAL

Here, we consider simple exponential function which may be serves as a toy model for interatomic potentials. In that case we assume that:

$$V(r) = Ae^{\gamma r} \tag{10}$$

where, A and γ are arbitrary constants. Numerically, we find behavior of the wave function in the Fig. 3, which shows periodic feature. We find that the value of constant γ should be negative.

We can see that the solution with potential Eq. 10 is similar to solution with the Morse potential.

GENERALIZED WOODS-SAXON POTENTIAL

Woods and Saxon introduced a potential to study elastic scattering (Woods and Saxon, 1954). The Woods-Saxon potential plays an important role in microscopic physics, since it can be used to describe the interaction of a nucleon with the heavy nucleus. This potential is utilized to represent the mean field which is felt by valance electron in Helium model (Dudek *et al.*, 2004). Generalized Woods-Saxon potential may be written as the following form:

$$V(r) = \frac{v}{1+e^{\varepsilon r}} + \frac{\tau}{(1+e^{\varepsilon r})^2} \tag{11}$$

where, v, τ and ε are arbitrary constants. We can see that the generalized Woods-Saxon potential with $\tau = 0$, v = A and $e^{\varepsilon r} \gg 1$ limit yields to the potential (10). Numerically, we find behavior of the wave function in the Fig. 4, which shows periodic feature. We can see that the solution with

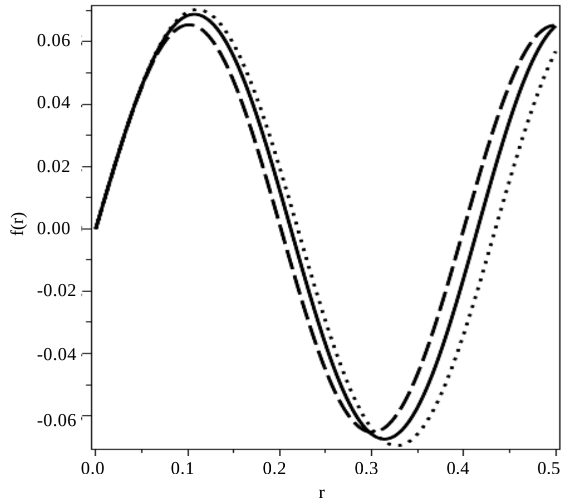

Fig. 3: Wave function in terms of r with exponential potential for γ = -1 (dotted line), γ = -5 (solid line) and γ = -25 (dashed line)

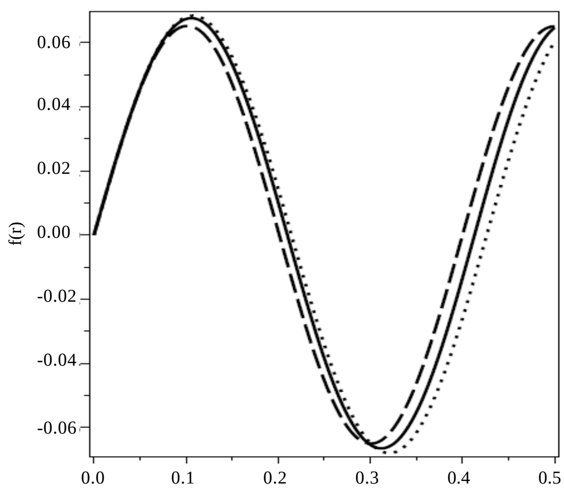

Fig. 4: Wave function in terms of r with various potentials. Constant potential (dash-dotted line), Morse potential (dashed line), Exponential potential (dotted line), Generalized Woods-Saxon potential (solid line)

potential (10) is similar to solution with the Morse and generalized Woods-Saxon potentials. We compare all solution in a single plot to find differences of various models (Fig. 4).

CONCLUSION

In this study, we considered quantum turbulence and calculated wave function from non-linear Schrodinger equation which is known as Ginsburg-Pitaevski-Gross equation with various exponential potentials such as Morse and generalized Woods-Saxon potentials. We found that the wave function has periodic behavior for exponential form of potentials. There are still many interesting potentials which may be used in the non-linear Schrodinger equation such as

Dirac-Morse, Rosen-Morse, Dirac-Rosen-Morse, Dirac-Eckart and Dirac-Scarf potentials (Alhaidari, 2001, 2003, 2004a, b).

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