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Prediction of Single Well Land Subsidence Due to Ground Water Drainage

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Abstract: Nowadays so many parts of the world are involving with subsidence. Subsidence is settling of the earth surface because of different factors. Ground movements, mining activities, gas, oil and water withdrawal are some examples that can causes ground subsidence. In recent years it has been proven that in dry areas because of extensive ground water withdrawal, the rate of subsidence increases rapidly (more than 10 centimetres in a year). A decrease in ground water level will causes an increase in effective stresses at clay layers which results consolidation of lower layers. The behaviour can be modelled using finite element technique to predict the future settlement. In this study the relationship between classical soil parameters and subsidence parameters is driven for a pumping single well. It is possible to approximate the model by assuming elastic time dependent behaviour due to decrease in water table level that calculates with a computer software that's name is WTAQ (Water Table Aquifer). For this purpose specialised finite element model was established and related to classical soil mechanics consolidation parameters. For modelling of any single well, correlation between classical parameters, water free surface, and parameters for numerical analysis was developed. A simple correlation between these parameters was found. Based on the prediction model the variation of settlement around a single well can be seen. The results show good comparison with field data.

Key words: Subsidence, finite element, single well, WTAQ, groundwater

INTRODUCTION

In recent decades land subsidence because of its destructive results such as differential settlement and earth fissures in many part of the world, such as Kerman province and some other parts of the world become a major consideration. Earth fissures and subsidence around a single pumping well of Sirjan land in Kerman province are shown respectively in Fig. 1 and 2. In this study excessive ground water withdrawal considered as the main cause of land subsidence.

It has been confirmed that extensive groundwater withdrawal and heavy pumping for agriculture development will tend to increasing soil layers consolidation, different land subsidence and finally earth fissures (Rahmanian, 1986, 1994).

This investigations was continued providing a prediction model in order to simulate the land settlement (Toufigh and Shafiei, 1996). Also a single well under operation was modeled based on finite element formulation (Toufigh and Q`marsi, 2002). In this study Water level decline will be determined by a computer program (WTAQ). This software shows the water level around a single well using a numerical method. In order to simulate and predict the subsidence in a given single well, the authors developed an axisymmetric fully coupled finite element model, using the water table data from WTAQ. Formulation of finite element was based on Biot's three-dimensional consolidation theory (Biot, 1941) assuming elastic behavior of soil skeleton.

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Fig.1: Earth fissures caused by differential settlement of Sirjan land

Fig.2: Subsidence around a pumping well in Sirjan

MATERIALS AND METHODS

The basic formulation presented here is based on Biot's consolidation theory. In Biot's theory the soil skeleton treated as a porous elastic solid and the laminar pore fluid are coupled by the conditions of compressibility and continuity.

In the computations cylindrical coordinates were assumed, and when water is pumped out from the aquifer through wells, both radial and axial flow can take place, which are symmetric. In order to simulate this condition by finite element method the exact behavior should be achieved by actual mathematical equations (Reddy, 1984; Smith and Griffiths, 1992). In Biot's theory the main governing equation is as follows:

$$C_r\left(\frac{\partial^2 u_e}{\partial r^2} + \frac{1}{r}\frac{\partial u_e}{\partial r}\right) + C_z\frac{\partial^2 u_e}{\partial z^2} = \frac{\partial u_e}{\partial t} - \frac{\partial P}{\partial t} \qquad (1)$$

where, u_e = excess pore water pressure, P = mean total stress, z and r = axial and radial directions, t = time and C_y, C_z = coefficient of consolidation in radial and axial directions, respectively.

The equilibrium equation with assumption of zero volumetric force can be written as follows:

$$\frac{\partial \sigma'_r}{\partial r} + \frac{\partial \tau_{rz}}{\partial z} + \frac{\partial u_e}{\partial r} = 0$$

$$\frac{\partial \tau_{rz}}{\partial r} + \frac{\partial \sigma'_z}{\partial z} + \frac{\partial u_e}{\partial z} = 0 \qquad (2)$$

The stress-strain relations for such condition can be written as follows:

$$\begin{Bmatrix} \sigma'_r \\ \sigma'_z \\ \tau_{rz} \\ \sigma'_\theta \end{Bmatrix} = \frac{E(1-\upsilon)}{(1-\upsilon)(1-2\upsilon)} \times$$

$$\begin{bmatrix} 1 & \frac{\upsilon}{1-\upsilon} & 0 & \frac{\upsilon}{1-\upsilon} \\ \frac{\upsilon}{1-\upsilon} & 1 & 0 & \frac{\upsilon}{1-\upsilon} \\ 0 & 0 & \frac{1-2\upsilon}{2(1-\upsilon)} & 0 \\ \frac{\upsilon}{1-\upsilon} & \frac{\upsilon}{1-\upsilon} & 0 & 1 \end{bmatrix} \begin{Bmatrix} \varepsilon_r \\ \varepsilon_z \\ \gamma_{rz} \\ \varepsilon_\theta \end{Bmatrix} \qquad (3)$$

where:

E = Modules of elasticity,

v = Poisson's ratio,

σ = Effective stress,

ε = Strain and

$$\begin{Bmatrix} q_r \\ q_z \end{Bmatrix} = \frac{1}{\gamma_w} \begin{bmatrix} K_r & 0 \\ 0 & K_z \end{bmatrix} \begin{Bmatrix} \dfrac{\partial u_e}{\partial r} \\ \dfrac{\partial u_e}{\partial z} \end{Bmatrix} \qquad (4)$$

where q_y, q_z volumetric flow rates per unit area into and out of the element, K $_y$K ç̧oefficient of permeability in redial and axial directions, respectively.

For fully saturated soil and incompressible fluid, outflow from an element of soil equals the reduction in volume of element. Hence:

$$\frac{\partial q_r}{\partial r} + \frac{\partial q_z}{\partial z} = \frac{d}{dt}\left(\frac{\partial u}{\partial r} + \frac{\partial v}{\partial z}\right) \qquad (5)$$

where u and v = displacements in r and z directions, respectively.

Combining Eq. 4 and 5 we have:

$$\frac{K_r}{\gamma_w}\frac{\partial^2 u_e}{\partial r^2} + \frac{K_z}{\gamma_w}\frac{\partial^2 u_e}{\partial z^2} + \frac{d}{dt}\left(\frac{\partial u}{\partial r} + \frac{\partial v}{\partial z}\right) = 0 \qquad (6)$$

As usual in a displacement method σ, ε are eliminated in terms u, of v so that the final coupled variables are u, v, u_e

Discretizing in normal way we have:

$$\begin{aligned} u &= Nu \\ v &= Nv \\ u_e &= Nu_e \end{aligned} \qquad (7)$$

where N is the vector of shape function.

When discretization and the Galerkin process are completed, Eq. 2 and 6 lead to the pair of equilibrium and continuity equations, which are:

$$\begin{aligned} KM_r + Cu_e &= F \\ C^T\frac{dr}{dt} - KPu_e &= 0 \end{aligned} \qquad (8)$$

where, for a four-nodded element,

$$\begin{aligned} r &= \{u_1, \quad v_1, \quad u_2, \quad v_2, \quad u_3, \quad v_3, \quad u_4, \quad v_4\}^T \\ u_e &= \{u_{e1}, \quad u_{e2}, \quad u_{e3}, \quad u_{e4}\}^T \end{aligned} \qquad (9)$$

KM is the elastic stiffness matrix and is

$$KM = \iint B^T DB r\, dr\, dz \qquad (10)$$

where, B = AN,
N = vector of shape function and

$$A = \left\{\begin{array}{cc} \dfrac{\partial}{\partial r} & 0 \\ 0 & \dfrac{\partial}{\partial r} \\ \dfrac{\partial}{\partial z} & \dfrac{\partial}{\partial r} \\ \dfrac{1}{r} & 0 \end{array}\right\} \qquad (11)$$

KP is the fluid stiffness matrix is

$$KP = \iint \left(C_r\frac{\partial N_i}{\partial r}\frac{\partial N_j}{\partial r} + C_z\frac{\partial N_i}{\partial z}\frac{\partial N_j}{\partial z}\right) r\, dr\, dz \qquad (12)$$

C is a rectangular coupling matrix, can be written as follows:

$$C = \iint N_i \frac{\partial N_j}{\partial r} dr dz \qquad (13)$$

and F is the external loading vector.

Equation 8 could be integrated in time. To integrate Eq. 8 with respect to time, there are many methods available, but we consider only the simplest linear interpolation in time using finite difference, thus:

$$\theta KMr_1 + \theta Cu_{e1} = (\theta - 1)KMr_\theta + (\theta - 1)Cu_{e0} + F$$
$$\theta C^T r_1 - \theta^2 \Delta t KPu_{e1} = \theta C^T r_0 - \theta(\theta - 1)\Delta t KPu_{e0} \qquad (14)$$

In above equations, if $\theta \geq 0.5$, the system will be stable without any condition, in the Crank-Nicolson type of approximation, θ is made equal to 0.5, or in the Galerkin approximation θ is equal to 0.67. By using $\theta = 0.5$ in Crank-Nicolson method, Eq. 14 can be written as follows:

$$\begin{bmatrix} KM & C \\ C^T & -\dfrac{\Delta t}{2}KP \end{bmatrix} \begin{bmatrix} r_{n+1} \\ u_{e_{n+1}} \end{bmatrix} = \begin{bmatrix} -KM & -C \\ C^T & \dfrac{\Delta t}{2}KP \end{bmatrix} \begin{bmatrix} r_n \\ u_{e_n} \end{bmatrix} + \begin{bmatrix} 2F \\ 0 \end{bmatrix} \qquad (15)$$

Therefore unknown values can be calculated at time based $t = t_n + t$ on known parameters at time $t = t_{n..}$. For initial conditions at time $t = 0$ all values are known.

After finding governing matrix equations for a single element, the assembled matrices for total elements can be obtained and boundary conditions can be introduced.

Solving such equations at any time, horizontal and vertical deformations (u,v) at various nodal points can be found and strain values for each element can be calculated.

The equivalent external load due to water table decline can be computed from Fig. 3 that shows of Sirjan aquifer properties.
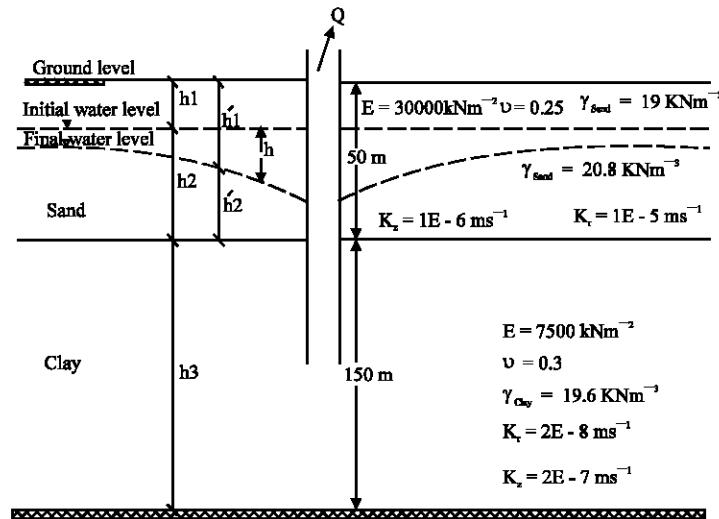
Where h in Fig. 3 is the water table drops, then:

Fig. 3: Declined water level and sirjan aquifer soil profile

$$h = h'_1 - h_1 = h_2 - h'_2$$

$$\sigma'_{v0} = \gamma_{sand}h'_1 + (\gamma_{sat} - \gamma_w)h_2$$

$$\sigma'_{v1} = \gamma_{sand}h'_1 + (\gamma_{sat} - \gamma_w)h'_2 \qquad (16)$$

$$\sigma'_{v1} = \gamma_{sand}(h_1 + h'_1) + (\gamma_{sat} - \gamma_w)(h_2 - h_1)$$

$$\Delta\sigma'_v = \sigma'_{v1} - \sigma'_{v0} = [\gamma_{sat} - (\gamma_{sat} - \gamma_w)]h$$

where σ_{v0} initial vertical effective stress,

σ_{v1} = Final vertical effective stress and

$\Delta\sigma_v$Estimated vertical load at top layer of clay.

In a single pumping well under operation ground water level draws down causing a hydraulic gradient that cause the well ground water flow. There are some different analyzing models for this flow such as Moench, that is a combination of Neuman and Boulton model by assumptions such as, uniform aquifer and constant pumping rate, constant physical properties.

In this situation ground water flow around a supposed single well such as Fig. 4 can be computed by following equations (Najmaei, 1990).

$$\frac{\partial^2 h}{\partial r^2} + \frac{1}{r}\frac{\partial h}{\partial r} + \frac{K_z}{K_r}\frac{\partial^2 h}{\partial z^2} = \frac{S}{bK_r}\frac{\partial h}{\partial t} \qquad (17)$$

Where the initial conditions of the equation are:

$$h(r, z, 0) = 0, h(\infty, z, t) = 0 \qquad (18)$$

$$\lim_{r \to 0} r\frac{\partial h}{\partial r} = -\frac{Q}{2\pi K_r}, \; b - 1 \le z \le b - d \qquad (19)$$

$$\frac{\partial h}{\partial z}(r, 0, t) = 0, \; z \prec b - 1; z \succ b - d \qquad (20)$$

$$\lim_{r \to 0} r\frac{\partial h}{\partial r} = 0 \qquad (21)$$
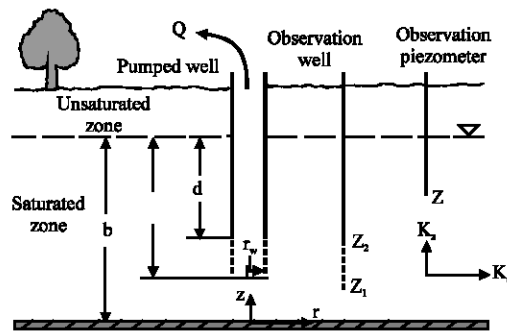
Fig. 4: Supposed single well in a uniform aquifer

Table 1: Dimensionless parameters

Dimensionless time of storage coefficient	t_{D_l}	$Tt/r^2\,S$
Dimensionless time of specific yield	t_{Dy}	$Tt/r^2\,S_y$
Dimensionless hydraulic head	h_D	$a\pi T\,(h_r\text{-}h)/Q$
Dimensionless radius of pumped well	r_D	r/b
Dimensionless depth	z_D	z/b
Dimensionless hydraulic conductivity	K_D	K_z/K_r
Dimensionless depth from initial water table to top of pumped well screen	l_D	l/b
Dimensionless depth from initial water table to bottom of pumped well screen	d_D	d/b
Dimensionless compressibility	β	$K_z\,r^2_d/K_r$
Dimensionless storage	σ	S/S
Dimensionless specific weight	γ	$\alpha_l bS_y/K_z$

In this equation is Bessel function type two and ε_n, $x_n = \left[\beta\varepsilon_n^2 + p\right]^{1/2}$ are its roots in the following form:

$$\varepsilon_n \tan(\varepsilon_n) = \frac{p}{(\sigma\beta + p/\gamma)} \tag{22}$$

and the equation for a screen well is as the following form:

$$\overline{h}_D\left(\gamma,\beta,\sigma,z_{D1},z_{D2},p\right) = \frac{1}{\left(z_{D2} - z_{D1}\right)} \int_{z_{D2}}^{z_{D2}} \overline{h}_D\left(\gamma,\beta,\sigma,z_D,p\right) d_{zD} \tag{23}$$

Equation (23) can be simplified as the following form:

$$\overline{h}_D = \sum_{n=1}^{\infty} \frac{2K_0(x_n)\left\{\sin\left[\varepsilon_n(1-1_D)\right] - \sin\left[\varepsilon_n(1-1_D)\right]\right\}\left[\sin(\varepsilon_n z_{D2}) - \sin(\varepsilon_n z_{D1})\right]}{p(1_D - d_D)\varepsilon_n\left[0.5\varepsilon_n + 0.25\sin(2\varepsilon_n)\right]} \tag{24}$$

Table 1 shows the dimensionless parameters of Eq. (23) and (24).

RESULTS

Formulation of finite element analysis for subsidence problem was discussed in previous section. A computer program was developed to predict and examine various soil behavior and conditions. In order to verify the computer model, analysis for simple behavior such as one-dimensional consolidation was performed. As an example for examination of model, properties of Sirjan aquifer in Kerman province were considered which is given in Fig. 3. It should be noted that values of E and other material properties can be varied in depth and other directions. Values of C_r C_z are functions of K, E, ν, γ and Δt and in this study Δt is supposed to be changed from 30 min to one day considering the desired accuracy. A section with height of 200 m and width of 1000 m was discretized to 160 rectangular elements with 189 nodes. For complete study two stages of analysis were performed in this research.

At first stage of this study, in order to examine the actual consolidation process that is three-dimensional in the field pumping problems, water table determined by WTAQ computer program assuming a constant pumping rate. In this situation water flows in axial and radial directions under axi-symmetric conditions.
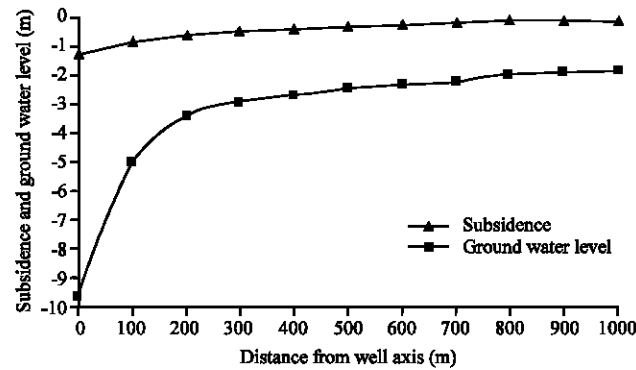
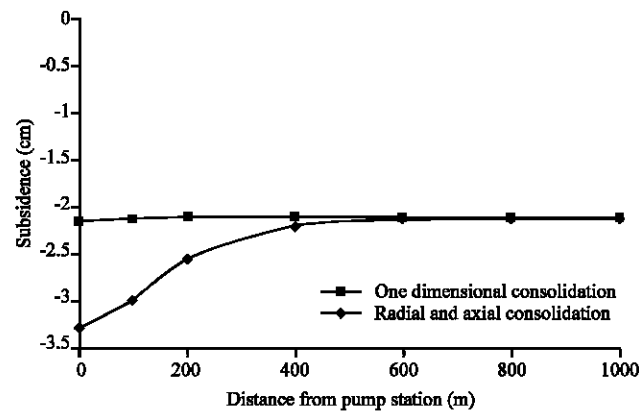Fig. 5: Subsidence and ground water level

Fig. 6: Subsidence due to 1 m water table decline after one year

This simulation is very close to actual filed condition under pumping of groundwater through wells. At this conditions water flows in three dimensions and consolidation process and water drainage occur faster than single dimension drainage. The analysis for this case is shown in Fig. 5.

It can be seen from Fig. 5 that subsidence rate around the well axis is higher than farther areas. Also there is a rough ratio of one tenth for subsidence and its related amount of water decline near the well axis and the shape of subsidence around the well is close to ground water decline cone shape, but in a much smaller scale.

Using Biot's three-dimensional equation of consolidation it is also possible to estimate differential settlement in various distances from wells. Figure 6 shows subsidence versus distance after one year for sudden water table drop of one meter. It can be seen that higher settlement occurred at areas surrounding the wells. This can be explained on the basis that at areas close to well, with higher water drop and, radial drainage causes dissipation of pore water pressure, which resulted faster settlement.

At the second stage, it is assumed that water table drops at the rate of one meter per year which is equivalent to pistachio farming area at Sirjan aquifer. One of the main advantages of developed computer program is considering the actual slow drop of water table level. In other words, the subsidence problem considered is time dependent in terms of pore water pressure condition and also time dependent in terms of load application. For this condition, the water also is time dependent in term of load application. It continuously drops at the rate of 1 m per year for five years and relation between subsidence and distance from wells for different times are shown in Fig. 7.
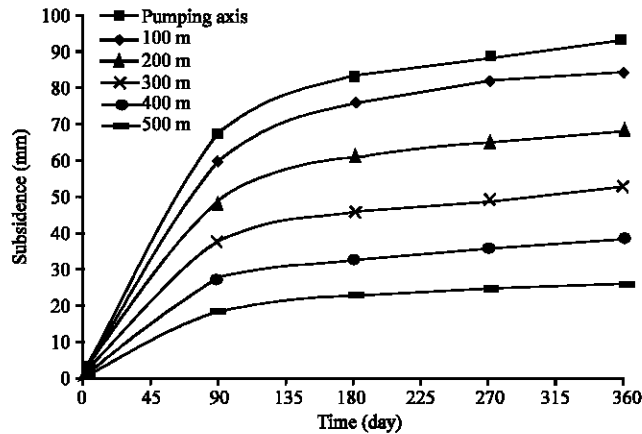
Fig. 7: Relationship between subsidence and time at various distance from pump station for equal drop
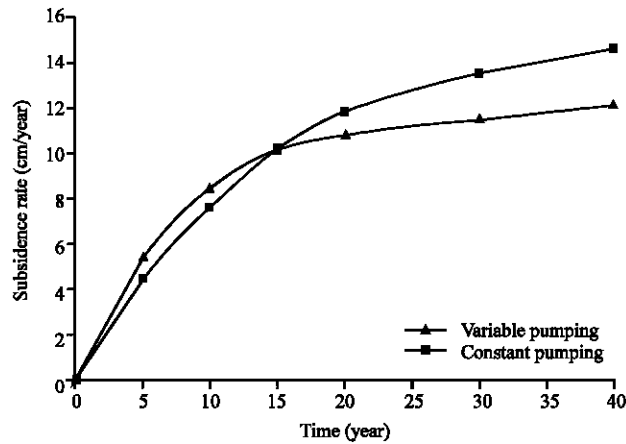

Fig. 8: Relationship between subsidence rate in constant and variable pumping

Regarding above, Fig. 5 modeled the actual and better simulation of the field pumping problems. In this case it is assumed the rate of the water drop at the pumping station is 105 cm per year and 95 cm per year at distance of 1000 m from pumping station.

From these result it can be concluded that rate of differential settlement increases with passage of time. This is because after each year, the water declination increases cumulatively and causes higher settlements and higher rates of differential settlement.

Figure 7 shows that the results of subsidence rate vs. time at different distances from the well. It can be seen that the settlement rate near the pumping station is higher than the settlement rate at farther distances. This is mainly because of faster drainage near the well. This analysis can be confirmed by the field data that after 20 years of pumping in Sirjan, the measured rate of settlement around the well is about 10 cm, which is almost equivalent to numerical analysis.

The results of subsidence rate vs. time for constant and various pumping are given in Fig. 8.

Various pumping is more probable than the constant one and it means that pumping rate is changed by the time. It can be seen that generally after pumping in constant pumping, subsidence velocity is higher than variable one, but passing the time variable pumping can produce larger amount of subsidence. This results shows 10 cm per year subsidence rate after abut 15 years of pumping in both two kind, and the related field data in Sirjan confirm this amount well.

DISCUSSION AND CONCLUSIONS

The developed computer program based on Biot's three-dimensional consolidation theory gave satisfactory results for the future subsidence. First, the proposed method was examined with classical and one dimensional consolidation theory and then it extend to more complicated case of three dimensional one which still confirmed field data using the computer program (WQAT) for water table determination. The limitation of this study is that aquifer was assumed as a confined one. This study first was developed for considering only one well but it would easily extend for groundwater withdrawal in a regional problem similar to assumption which was made in first stage of this study or consider the actual variation of water table level in the field as an input data in finite element analysis.

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