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Experimental and Theoretical Study of Determination of Effective Thermal Diffusivity of Some Fruits With Temperature

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Abstract: Transient method is used to determine effective thermal diffusivity of vegetables/fruits in temperature range of 0 to 45°C. The effective thermal diffusivity is obtained by solution of one dimensional (1D) Fourier heat conduction equation applied to a cylinder. The temperature is recorded at a number of points along the radius with respect to time and no approximation to surface convective heat transfer is required. The fruits taken are cucurbits, *Musa acuminate* L. (ripe and raw) and *Malus domestica* L. Effective thermal diffusivity in 0 to 45°C temperature varies from 1.38×10^{-9} to 16.6×10^{-9} m^2 sec^{-1} *Cucumis sativus* L., 14.2×10^{-9} to 98.7×10^{-9} m^2 sec^{-1} *Luffa acutangula* L. and 2.27×10^{-9} to 49.7×10^{-9} *Lagenaria siceraria* L. theoretically. The similar trend has also been found for other samples. The theoretical and experimental results are in good agreement.

Key words: Thermal diffusivity, transient technique, temperature, data acquisition module, water content and fruits

INTRODUCTION

The knowledge of thermo-physical properties is essential to designers and researchers in the field of food engineering. It is also essential for drying, heating or cooling in storage of fruits etc. The thermal processing mechanism of food materials involve unsteady state techniques where the material is subjected to a spatially and temporally variant temperature field therefore, effective thermal diffusivity is convenient and useful. The mathematical basis of these techniques is well established (Ozisik, 1980) however, relative thermo-physical data is needed for the design and optimization of the system.

Effective thermal diffusivity which indicates rate of heat propagation through a sample has been reported as a function of shape, size and thermo-physical properties by Baucour *et al*. (2003), Stephanopoulos (1984), Holdsworth (1997) and Glavina *et al*. (2006). The thermal diffusivity of foods is influenced by its compositions and temperature. Therefore, for heterogeneous materials it cannot be predicted with simple additive resister methods. Hence, accurate experimental data is also needed for appropriate model of thermal diffusivity.

The apparent diffusivities were directly measured by using a specific treatment of moisture and salt content profiles by Broyart *et al*. (2007) and compared with values calculated from a computing method with no assumption about the nature of mathematical relationship between diffusivity and gel composition. Yang *et al*. (2002) calculated thermal diffusivity of borage seeds with the ratio of thermal conductivity and volumetric specific heat. He found the values between 2.32×10^{-7} to 3.18×10^{-7} m^2 sec^{-1}. Kee *et al*. (2002) obtained thermal diffusivity of corned beef and mashed potato by the log method which considers exponential temperature change in cylindrical sample. However, a comparison with literature values shows a large deviation in diffusivity value for corned beef as well

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as for potato. Martens *et al*. (1980) have given regression equation to determine diffusivity as a function of temperature and water content. The measurements for α by Kee (2002) also show a noticeable deviation from α predicted by correlation equation by Martens (1980).

Sakiyama *et al*. (1999) obtained effective thermal diffusivity for porous food materials. In the modeling of effective thermal diffusivity they considered heat transport by conduction as well as latent heat transport. The thermal diffusivity value predicted is at average temperature $\left(\frac{T_{initial}+T_{surface}}{2}\right)$ which may cause a large prediction error at large temperature range.

Bairi *et al*. (2007) proposed a one dimensional (1D) fourier cylindrical solution to determine thermal diffusivity with the assumption that the surface convective heat transfer was very large. Similar method has also been given by Jain and Pathare (2007) with the assumption of long time sample exposure. However, these assumptions are not required in the present method.

In the present study effective thermal diffusivity of sample in a temperature range of 0 to 45°C is determined. The transient analysis of heat conduction equation gives spatial and temporal temperature variation. In the present analysis any two simultaneous temperature measurements at r_1 and r_2 and rise in temperature at any r gives effective thermal diffusivity. Therefore, approximation to surface heat convection and temperature at the exact center as by Tavman *et al*. (1997) are not needed.

Theory

It is assumed that heat flow is along direction of radius. Then the spatial and temporal temperature distribution is given by solution of one dimension (1D) heat conduction equation in cylindrical coordinates (Ozisik, 1980).

$$\frac{\partial^2 T}{\partial r^2}+\frac{1}{r}\frac{\partial T}{\partial r}=\frac{1}{\alpha}\frac{\partial T}{\partial t} \quad \text{in } 0<r<a, t>0 \tag{1}$$

The boundary conditions are:

$$-k\frac{\partial T}{\partial r}\Big|_{r=a}=H\left(T|_{r=a}-Ta\right) \quad \text{at } r=a \tag{2}$$

$$T(r,t)|_{r=a}=Ta \quad t>0, r=a \tag{3}$$

$$T(r,t)|_{t=0}=Ts \quad t=0, \text{in all region of } r \tag{4}$$

where,

Ta = Temperature of surrounding of the sample,
Ts = Initial temperature of sample and
a = Radius of sample.

Defining dimensionless space, temperature, time and heat transfer variables as:

$$R=\frac{r}{a}, \theta=\frac{T-Ts}{Ts-Ta}, \tau=\frac{\alpha t}{a^2}, Bi\equiv\frac{Ha}{K}$$

the solution of Eq. 1 for function θ (R, τ) is obtained by separation of a space and time dependent functions in the form

$$\theta(R, \tau) = \theta_0(R)\Gamma(\tau) \quad (5)$$

And the complete solution (Ozisik, 1980) is

$$\theta(\theta_0, \Gamma) = \sum_{m=1}^{\infty} \frac{1}{N(\beta_m)} e^{-\beta_m^2 \tau} \theta_0(\beta_m, R) \int_0^1 R'\theta_0(\beta_m, R')dR' \quad (6)$$

Using the eigen function θ_0 (β_m, R), the Norm N (β_m) and the eigen values β_m (Ozisik, 1980)

$$\theta = \sum_{m=1}^{\infty} e^{-\beta_m^2 \tau} \frac{2}{J_0^2(\beta_m)} \frac{\beta_m^2}{(Bi^2 + \beta_m^2)} J_0(\beta_m, R) \int_0^1 R'\theta_0(\beta_m, R')dR' \quad (7)$$

where, β_m are the +ve roots of

$$\beta_m J_1(\beta_m) = Bi J_0(\beta_m) \quad (8)$$

Thus,

$$\theta = 2Bi \sum_{m=1}^{\infty} e^{-\beta_m^2 \tau} \frac{J_0(\beta_m, R)}{(\beta_m^2 + Bi^2) J_0(\beta_m)} \quad (9)$$

and for m = 1 Eq. 9 reduces to

$$\theta = 2Bie^{-\beta_1^2 \tau} \frac{J_0(\beta_1, R)}{(\beta_1^2 + Bi^2) J_0(\beta_1)} \quad (10)$$

Thus, β_1 is determined from $\theta(R, \tau)$ at R = 0 and R = 1 and $\theta(R, \tau)$ at extremum gives

$$\alpha = \frac{a^2}{\beta_1^2(t_2 - t_1)} \ln\left(\frac{T_1 - Ta}{T_2 - Ta}\right) \quad (11)$$

where, T_1 and T_2 are the temperatures at times t_1 and t_2 and at the same position.

MATERIALS AND METHODS

The uniformly cooled or heated sample is placed in a double walled and insulated chamber which is maintained at constant temperature by flowing a fluid in the outer. The transient temperature distribution inside the sample is measured by an array of eight thermocouples. The thermal diffusivity measuring apparatus which is shown in Fig. 1 consists of computer, ADAM data acquisition module, temperature probe, constant temperature bath and sample container.

The sample is kept in a double walled cylindrical copper vessel of 15.5 cm length and inner diameter of 7.3 cm. The fruit sample, which is cooled and has constant temperature throughout, is placed in constant temperature chamber. The constant temperature is maintained by flowing fluid in the outside of chamber. The fluid at constant temperature with variation $< \pm 1°C$ is circulated from a constant temperature bath (Julabo F-32). The chamber is insulated by foam rubber.

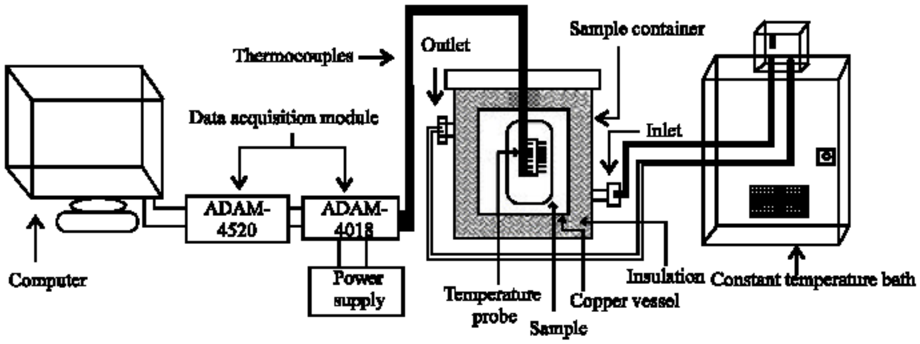

Fig. 1: Experimental arrangement of thermal diffusivity

The temperature probe is made of eight copper-constantan thermocouples. These thermocouples are arranged on a teflon sheet with the help of needles of diameter 1.6 mm. Each thermocouple is placed in a needle with insulating powder and packed with araldite. The temperature probe is inserted across the diameter of the sample. All the thermocouples are connected to an ADAM data acquisition module which is setup to simultaneously read and store the temperature in computer after every one second interval with a precision of 0.001°C. The GeniDaq software is used for storing the data. Each experimental run takes about 3 to 4 h. Each experiment is iterated 3 times to reduce the effect of various factors (beyond the control of the experiment) such as small fluctuations in the temperature, random electric effects in the electric equipment etc and average values are reported. The effective thermal diffusivity is calculated by the following expression by spline interpolation from temperature-time data at extrema.

$$\alpha\,(T) = \frac{\dfrac{\partial T}{\partial t}}{\dfrac{\partial^2 T}{\partial r^2}}\bigg|_{\text{extremum}}$$

RESULTS AND DISCUSSION

The effective thermal diffusivity of cucurbits i.e., *Cucumis sativus* L., *Luffa acutangula* (L.) and *Lagenaria siceraria* (L.) and *Malus domestica* (L.) and *Musa acuminate* (L.) (ripe and raw) is determined in the temperature range of 0 to 45°C. The initial temperature of the sample is 0°C, whereas the constant temperature bath is maintained at 60±0.1°C during the measurements. Since, the rise in temperature at the points along the diameter is continuously recorded; therefore, thermal diffusivity of the sample at any temperature may be determined by applying cubic spline numerical technique and using the Eq. 12. The calculations for effective thermal diffusivity in temperature spread of 1°C have been made and for a given temperature the value is grouped in ±2°C at temperatures for cucurbits.

In the Fig. 2-4 a comparison in experimental effective thermal diffusivity values as a function of temperature and values obtained by present model is shown for *Cucumis sativus* (L.), *Luffa acutangula* (L.) and *Lagenaria siceraria* (L.). The experimental diffusivity for *Cucumis sativus* (L.) changes from 2.69×10^{-9} (0°C) to 19.6×10^{-9} m^2 sec^{-1} (45°C) whereas the respective theoretical values are 1.38×10^{-9} (0°C) to 16.6×10^{-9} m^2 sec^{-1}(45°C). The maximum deviation being 11.6%.

For *Luffa acutangula* (L.) effective thermal diffusivity (Fig. 3) in 0 to 45°C varies between 12×10^{-9} to 108×10^{-9} m^2 sec^{-1} (experimentally) and 14.2×10^{-9} to 98.7×10^{-9} m^2 sec^{-1} (theoretically). Here also the maximum deviation is 9.40%.

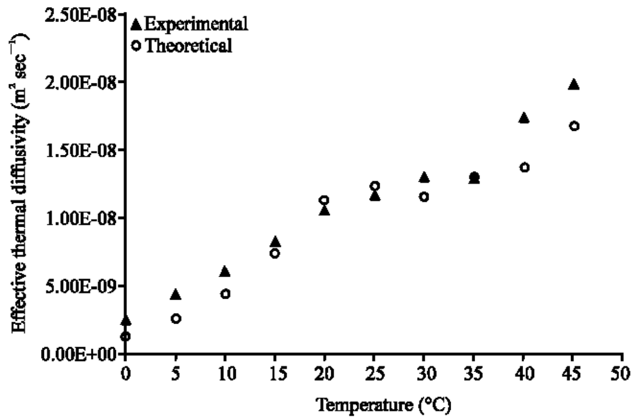

Fig. 2: Variation of effective thermal diffusivity of *Cucumis sativus* (L.) with temperature

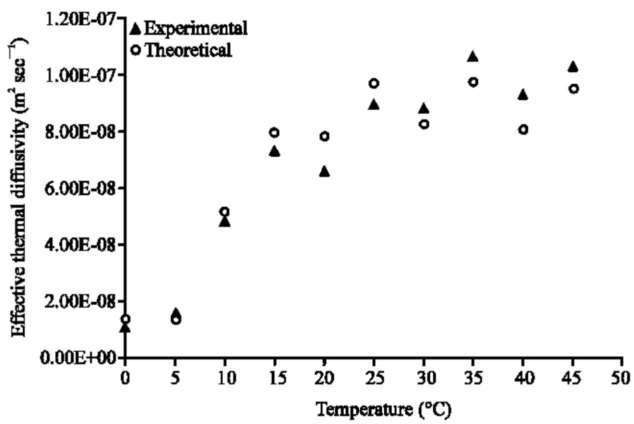

Fig. 3: Variation of effective thermal diffusivity of *Luffa acutangula* (L.) with temperature

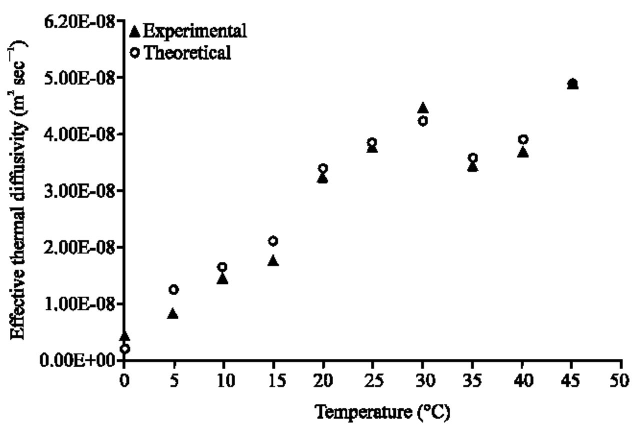

Fig. 4: Variation of effective thermal diffusivity of *Lagenaria siceraria* (L.) with temperature

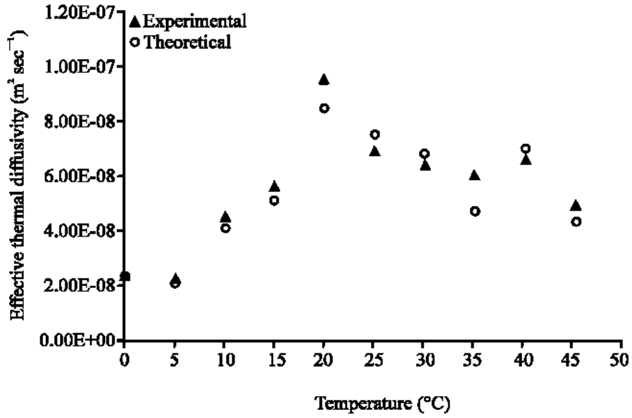

Fig. 5: Variation of effective thermal diffusivity of *Musa acuminate* (L.) (ripe) with temperature

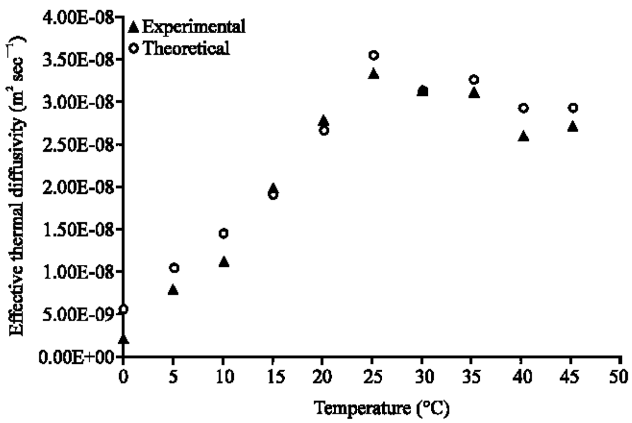

Fig. 6: Variation of effective thermal diffusivity of *Musa acuminate* (L.) (raw) with temperature

The experimental and theoretical value of effective thermal diffusivity lies between 4.23×10^{-9} to 49.8×10^{-9} m^2 sec^{-1} and 2.27×10^{-9} to 49.7×10^{-9} m^2 sec^{-1}, respectively (Fig. 4). The maximum percentage deviation in these values is 12.1. It may be noticed that effective thermal diffusivity is an increasing function of temperature.

The effective thermal diffusivity of ripe fruit varies from 22.5×10^{-9} to 95.8×10^{-9} m^2 sec^{-1} (Fig. 5) whereas for raw fruit its values are 2.39×10^{-9} to 33.5×10^{-9} m^2 sec^{-1} (Fig. 6) in the temperature range of 0 to 45°C. The continuous chemical reactions increase water content. Since the thermal diffusivity of water is 129.7×10^{-9} m^2 sec^{-1} therefore ripe fruit diffusivity increases with increasing time.

The experimental values are 5.89×10^{-9} to 28.9×10^{-9} m^2 sec^{-1} in 0 to 45°C and theoretical diffusivity values are 5.98×10^{-9} to 30.6×10^{-9} m^2 sec^{-1} (Fig. 7). It is seen that effective thermal diffusivity shows continuous increase with increase in temperature. Sakiyama *et al*. (1999) also found the effective thermal diffusivity of foods increased with temperature significantly. The comparison of theoretical and experimental values is also shown in Table 1.

The numerical results can be considered as reference for the evaluation of the reliability of the analytical method. Both of the experimental and transient method gave qualitatively the same trends,

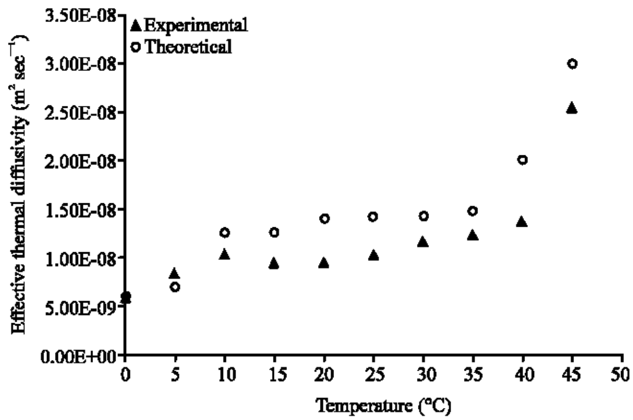

Fig. 7: Variation of effective thermal diffusivity of *Malus domestica* (L.) with temperature

Table 1: Comparison of effective thermal diffusivity

		Effective thermal diffusivity (m^2 sec^{-1})	
Sample name	Temperature (°C)	Theoretical ($\times10^{-9}$)	Experimental ($\times10^{-9}$)
Cucumis sativus (L.)	0	1.383	2.699
	5	2.730	4.440
	10	4.494	6.109
	15	7.547	8.317
	20	11.195	10.497
	25	12.253	11.679
	30	11.519	12.903
	35	12.992	12.944
	40	13.641	17.200
	45	16.654	19.654
Luffa acutangula (L.)	0	14.163	12.047
	5	15.173	16.578
	10	51.755	49.719
	15	80.584	73.720
	20	79.412	66.978
	25	98.315	90.254
	30	83.677	89.287
	35	98.704	107.792
	40	81.838	94.205
	45	96.167	104.249
Lagenaria siceraria (L.)	0	2.274	4.231
	5	12.731	8.337
	10	16.748	14.911
	15	21.550	17.856
	20	33.992	32.968
	25	38.908	38.448
	30	43.063	45.364
	35	36.099	35.005
	40	39.449	37.305
	45	49.746	49.767
Musa acuminate (L.) (ripe)	0	23.906	24.241
	5	21.269	22.479
	10	42.287	44.511
	15	51.621	56.860
	20	85.684	95.787
	25	75.548	69.575
	30	68.650	64.825
	35	47.752	60.818
	40	70.935	66.682
	45	43.854	49.786

Table 1: Continued

		Effective thermal diffusivity (m^2 sec^{-1})	
Sample name	Temperature (°C)	Theoretical ($\times 10^{-9}$)	Experimental ($\times 10^{-9}$)
Musa cuminate (L.) (raw)	0	5.848	2.390
	5	10.591	8.074
	10	14.664	11.355
	15	19.433	19.796
	20	27.316	28.062
	25	35.799	33.515
	30	31.577	31.715
	35	32.743	31.390
	40	29.522	26.181
	45	29.598	27.365
Malus domestica (L.)	0	5.985	5.890
	5	7.171	8.323
	10	12.479	10.601
	15	12.765	9.648
	20	14.227	9.706
	25	14.545	10.469
	30	14.523	11.796
	35	15.041	12.468
	40	20.521	13.959
	45	30.586	25.895

the effective thermal diffusivity have been found to increase with temperature for all the samples we considered. The maximum deviation in effective thermal diffusivity values, for all the samples is not more than 12.1%.

It may be noted that both methods were based on the 1D Fourier heat conduction equation and so handled only the thickness reduction of the sample. They did not account for the overall surface area reduction. Only a 3D diffusivity model would provide a basis for exact processing of the experimental data.

CONCLUSIONS

The effective thermal diffusivity of vegetables/fruits is determined by transient analysis and a comparison with experimental values made. The experimental results are obtained by numerical solutions of Fourier diffusion equation using cubic spline interpolation whereas transient analytical solutions of heat conduction equation using boundary value equations give theoretical values. The two methods gave qualitatively the same trends. The effective thermal diffusivity increases with increasing temperature. Also, the effective thermal diffusivity of ripe fruit varies from 22.5×10^{-9} to 95.8×10^{-9} m^2 sec^{-1} whereas for raw fruit its values are 2.39×10^{-9} to 33.5×10^{-9} m^2 sec^{-1} in the temperature range of 0 to 45°C for *Musa acuminate* (L.).

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NOMENCLATURE

Bi	Biot number
J_0, J_1	Bessel function of order 0 and 1

R	Dimensionless radial variable
θ_0 (β_m, R)	Eigen function
H	Heat transfer coefficient (W m^{-2} $°C^{-1}$)
N(β)	Norm
α	Radius of the sample (m)
r	Radial variable (m)
T	Temperature (°C)
T_1, T_2	Particular values of T (°C)
Ta	Temperature of surrounding of the sample (°C)
Ts	Initial temperature of sample (°C)
K	Thermal conductivity (W m^{-2} $°C^{-1}$)
t	Time (sec)
t_1, t_2	Particular values of time (sec)
Greek	
⊖	Dimensionless temperature
τ	Dimensionless time
α	Effective thermal diffusivity (m^2 sec^{-1})
β_m	Positive root

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