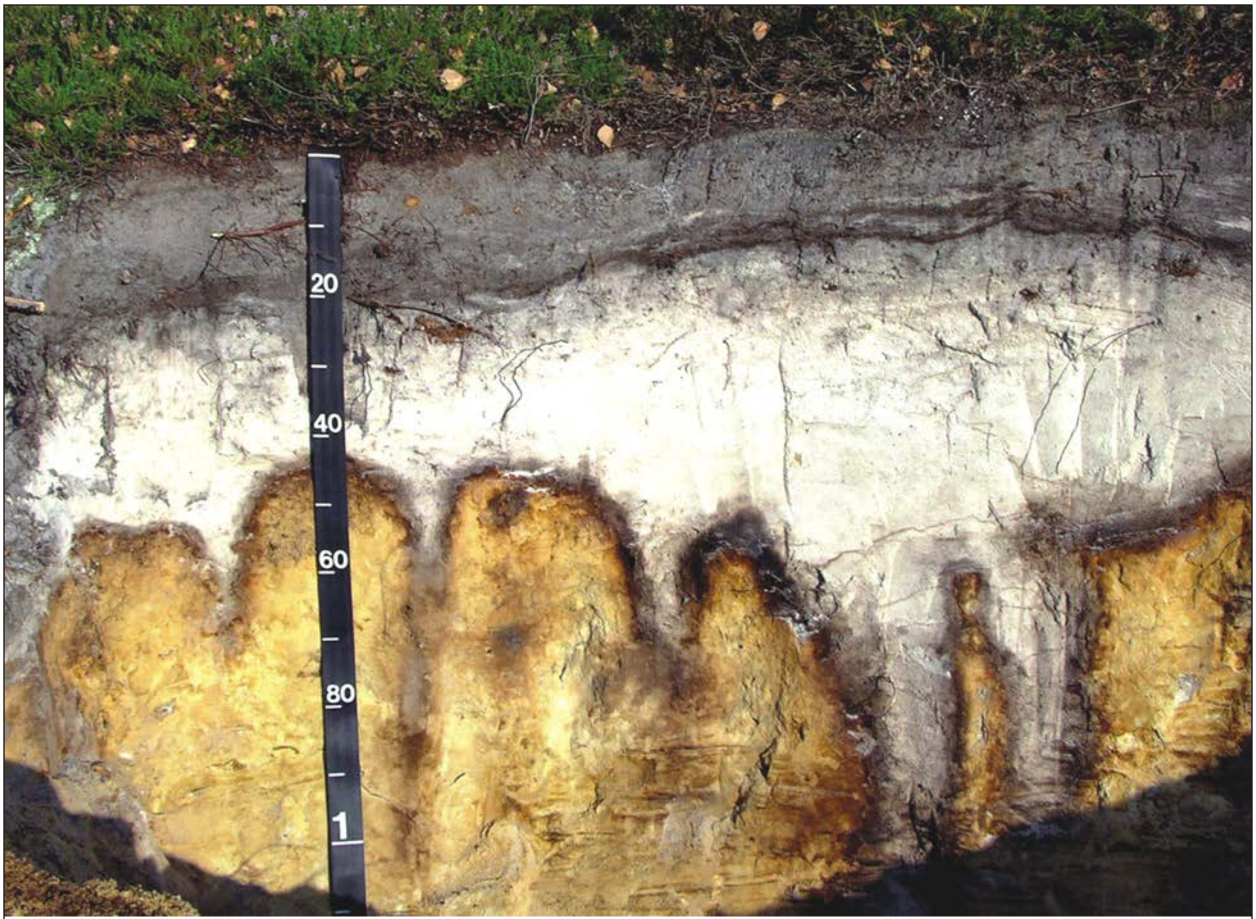

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Research Article
Comparison of Constitutive Soil Models in Predicting Movements Caused by an Underground Excavation

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Abstract

Background and Objective: The design of high-rise buildings often necessitates ground excavation, where buildings are in proximity to the construction, thus there is a potential for damage to these structures. As a result, various researchers have studied different aspects of excavations both in design and construction methods. In present study, it is aimed to evaluate the performance of constitutive soil models in a case study of anchored wall by using a finite difference analysis software. **Materials and Methods:** For this purpose, implications of the use of some advanced soil models to simulate the behavior of in-situ soil on the overall response of excavated wall have been studied and compared with respect to the most prevalent used Mohr-Coulomb (MC) soil model. The analyses concentrated on predicting three major behavior of tieback walls, i.e., excavation basal heave, horizontal displacement and deflection pattern of the wall. **Results:** The results indicated that Duncan-Chang (DC) model can reasonably predict the movements induced by excavation projects, while the modified Mohr-Coulomb (MMC) model also can be an alternative in estimating base heave and horizontal displacement at top of the wall when insufficient data is available. **Conclusion:** The findings of this research could be of interest to the engineers in order to achieve an optimum compromise between model accuracy and geotechnical survey efforts employed in deformation analysis of excavation problems.

Key words: Constitutive models, Numerical simulation, underground excavations, tieback walls, ground movements

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Data Availability: All relevant data are within the paper and its supporting information files.

INTRODUCTION

The performance of tieback walls is considerably affected due to the complex interaction between the soil, the reinforcement (i.e., anchors or strands), the prestressing force and the facing. Consequently, in practice, to study the complex soil-structure interaction and to assess the performance of excavated walls, often numerical simulations are performed using rigorous computational codes based on numerical techniques such as finite element method[1-3], discrete element method[4] and finite difference method[5]. It is well understood that the accuracy of numerical simulations depends remarkably on the applied constitutive soil model[6] and the selection of the proper corresponding model parameters[7].

Different researchers have studied the accuracy of different soil constitutive models in excavation and tunneling problems considering existing soil models in literature[8-10]. Most of these studies were performed using first order Mohr-Coulomb model (MC) which suffers from simplifying assumptions applied in numerical modeling of excavation problems[11-13]. However, few researchers have used advanced soil models in the numerical analyses of anchored walls in which more geotechnical parameters are required compared to simple models[14-16]. Thus, it is desirable to improve simple models by modifying employed assumptions and/or to reduce geotechnical data essential for advanced models by simplifying them to meet more reasonable results in different aspects of the problem.

In the present study, four soil constitutive models namely, modified Mohr-Coulomb soil model (MMC), Duncan-Chang soil model (DC), simplified Duncan-Chang model (SDC) and cap yield soil model (Cysoil) are benchmarked with respect to the most prevalent used MC model for the numerical simulations of a famous well-documented excavated wall.

METHODOLOGY AND MATERIAL PARAMETERS

Texas A and M University case study: Numerical analyses were carried out on an existing well-known full-scale instrumented tieback wall in literature at the National Geotechnical Experimentation Site on the Riverside Campus of Texas A and M University which was sponsored by The Federal Highway Administration and Schnabel Foundation in 1991[15]. This wall (Fig. 1) is 60 m in length and 7.5 m in height. A steel H pile and two-row anchor was used to stabilize the excavated wall. The grouted anchors were inclined 30° with the horizontal and located at 1.8 and 4.8 m below the top of the wall, they were 89 mm in diameter, 12.35 m in length with a 7.3 m tendon bonded length. The steel tendon itself was 25 mm in diameter. The wall was instrumented to obtain bending moment profiles, horizontal deflection profiles and anchor forces. The numerical values of the parameters used in the simulation of the case history are listed in Table 1.

Numerical modeling: Numerical simulations of this wall are performed via two dimensional finite difference Fast Lagrangian Analysis of Continua (FLAC2D) V7.0 software considering five simple and advanced constitutive models and observations are made regarding vertical displacements of the excavation base, horizontal displacement of top of the wall and the lateral deformation pattern of the wall facing after the last construction stage. In the following subsections a brief description about the various constitutive models used in current study is presented.

MC model: MC is an elastic perfectly plastic model, which combines Hooke's law and the Coulomb's failure criterion. It is a first order model for soils which requires the five basic input parameters namely Young's modulus, E and Poisson's

Table 1: Parameters used for FDM simulation[15]

Data	Parameters	Values
Soil	Initial tangent modulus factor K	300
	Initial tangent modulus exponent n	0.85
	Strength raio, R_f	0.93
	Friction angle Φ	32°
	Cohesion c	0
	Unloading-reloading modulus number K_{ur}	1200
	Bulk modulus number K_B	272
	Bulk modulus exponent n_B	0.5
	Unit weight γ_s	20 kN.m^3
	At-rest earth pressure coefficient K_o	0.65
Anchor	Tendon unbonded length	5.05 m
	Tendon bonded length	7.3 m
	Lock-off load-Row 1	182.35 kN
	Lock-off load-Row 2	160.0 kN
	Tendon stiffness-Row 1	19846 kN.m
	Tendon stiffness-Row 2	19479 kN.m
	Angle of inclination β	30°
Soldier pile	Length of soldier pile	9.15 m
	Embedment	1.65 m
	Diameter of pipe pile	0.25 m
	Thickness of pipe pile	0.00896 m
	Horizontal spacing of piles	2.44 m
	Elastic modulus of steel pipe pile	2.1×10^8 kN.m^2
	Flexural stiffness EI	11620 kN.m^2
	Axial stiffness AE	1.47×10^6 kN

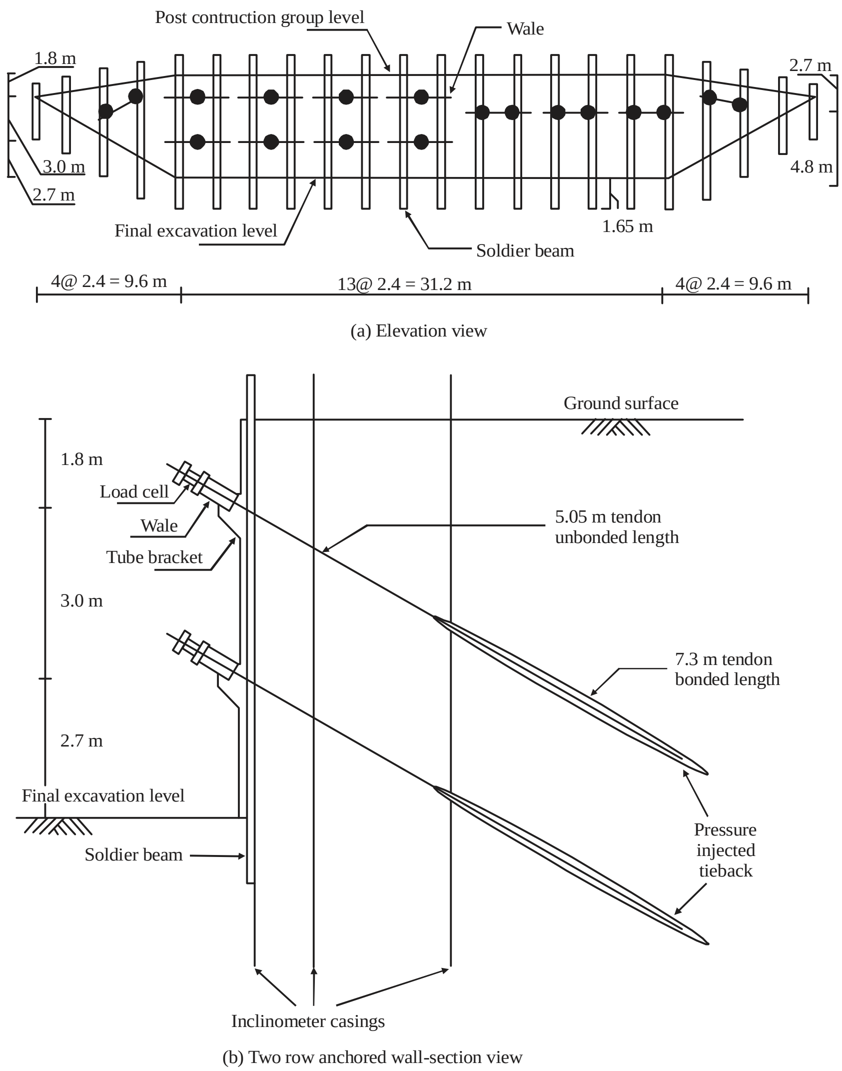

Fig. 1: Elevation and section view of texas A and M University Tieback wall[15]

ratio, ν, for soil elasticity, soil friction angle Φ and soil cohesion, C, for soil plasticity and the dilatancy angle, ψ. In FLAC2D, E and ν are replaced with bulk modulus, K and shear modulus, G, which can be determined by Eq. 1. Since the prefailure stiffness behavior is assumed to be linear elastic, the model has a limitation in terms of predicting the deformation behavior before failure[17]:

$$K = \frac{E}{3(1-2\upsilon)},\ G = \frac{E}{2(1+\upsilon)} \tag{1}$$

MMC model: One of the drawbacks of MC model is that it assumes equal soil stiffness both in loading and unloading conditions. Since soil stiffness in unloading is greater than its correspondence value in loading condition, the results of a tieback wall analyzed by MC model is greatly influenced by this fact. Another shortcoming of this model is the ignorance of soil stiffness and confining pressure correlation, which was studied by several authors and reported in literature[18].

In order to solve the former disadvantage of MC model, one common task in practical design of geotechnical problems, specifically in tunneling projects, is to divide the model into two parts: Loading and unloading portions. According to Imam and Hoseini[19] the loading and unloading regions can be divided with respect to the variations in vertical stresses induced in the soil behind and beneath the excavated wall[19]. They proposed a line with slope equal to 15 degrees relative to horizontal axis. In this method, the loading and unloading regions was specified at the last stage of constructions, while in current study the aforementioned regions are determined in each

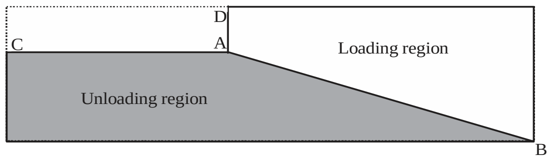

Fig. 2: Loading and unloading regions in an excavation model

individual excavation steps. This was done by writing a pertinent code in FISH (i.e., short for FLAC-ish) language of FLAC software.

Thus, the only additional parameter required for this modified method is to assign unload-reload modulus, E_{ur}, for the unloading region (for current study: $E_{ur} = 4E_t$). In current study, the line which separates these two regions was assumed approximately to be originated at the excavation corner (i.e., point A in Fig. 2) and ended in the model right and bottom corner (i.e., point B in Fig. 2) in each phase of the construction.

DC model: This soil model is a modified nonlinear hyperbolic model that includes the influence of the stress level on stiffness, strength and volume change characteristics of the soil[20,21]. With this model it is possible to simulate the hysteresis behavior of the soil. Equation (2) gives tangent Young's modulus, E_t, for the hyperbolic model:

$$E_t = \left[1 - \frac{R_f\,(1-\sin\phi)(\sigma_1-\sigma_3)}{2(c\,\cos\phi + \sigma_3\sin\phi)}\right]^2 Kp_a\left(\frac{\sigma_3}{p_a}\right) \quad (2)$$

where, σ_1 and σ_3 have initial values of gz and K_0gz (z = depth), respectively and are updated as the loading and unloading takes place in increments and p_a = atmospheric pressure. The unload-reload E_{ur} modulus is given by Eq. 3:

$$E_{ur} = K_{ur}p_a\left(\frac{\sigma_3}{p_a}\right)^n \quad (3)$$

At the point of unloading on the stress-strain curve, the modulus changes from E_t from Eq. 2 to E_{ur} from Eq. 3. To decide whether an element is on the loading or unloading path, a stress state (SS) coefficient is calculated at each step using Eq. 4[15]:

$$SS = \frac{\sigma_1-\sigma_3}{(\sigma_1-\sigma_3)_f}\sqrt[4]{\frac{\sigma_3}{p_a}} \quad (4)$$

If the current value of SS is larger or equal to the highest past value of SS (SS max-past) then E_t is used. If SS <SSmax, the unloading modulus is then used. This hyperbolic model was coded in FLAC2D V7.0 using FISH language. Parameters used for this constitutive model are presented in Table 1.

SDC model: As mentioned earlier, DC model accounts for correlation between stiffness modulus and confining pressure. This correlation requires some additional soil parameters, which may be difficult to obtain in common geotechnical surveys. Moreover, the model has a particular rule for identifying the loading and unloading soil elements, which is a positive point with respect to simple soil models (i.e., MC and MMC models). Here, in order to reduce the required parameters and to evaluate the performance of model, the relation between stiffness modulus and confining pressures is ignored by applying changes to previously provided FISH codes.

Cysoil model: As described in the work of Do *et al.*[10], the Cysoil model is a strain-hardening constitutive model that is characterized by a frictional MC shear envelope (zero cohesion) and an elliptic volumetric cap in the (q, p') plane, in which q is the deviatoric stress and p' is the effective mean stress[10]. Apart from the cap hardening law and the compaction/dilation law, which allow the volumetric power law behavior observed in isotropic compaction tests and the irrecoverable volumetric strain that occurs as a result of soil shearing to be captured, the friction hardening law in the Cysoil model offers the possibility of alternatively expressing the hyperbolic behavior. In the Cysoil model, the stiffness is adopted as a function of the effective confinement and it leads to a higher value for unloading-reloading stiffness.

Friction hardening Eq. 5 is characterized by a hyperbolic relation between the sine of mobilized friction angle (sin ϕ) and plastic shear strain (γ^p)[22]. The law is given in implicit form, by:

$$\gamma^p = \frac{p_{ref}}{G^e_{ref}}\left(\frac{p'_o}{p_{ref}}\right)^{1-m}\frac{\sin\phi_f}{R_f}\left[\frac{1}{1-\frac{\sin\phi}{\sin\phi_f}R_f} - 1\right] \quad (5)$$

where, γ^p is the second deviatoric plastic strain invariant, p'_o is the initial effective pressure, G^e_{ref} is the elastic tangent shear modulus at reference (effective) pressure p_{ref}, m is a constant ($m \le 1$) and ϕ_f is the ultimate friction angle. The failure ratio R_f is a constant used to assign a lower bound for the plastic shear modulus. Note that for values of m other than 1, the hardening law Eq. 5 is a function of initial effective pressure, p'_o and thus, in general, a function of depth. This law has 5 parameters: G^{e}_{ref}, p_{ref}, R_f, ϕ_f and m. The cap hardening law relates cap pressure, p_c, to (minus) cap plastic volumetric strain, e^p. A power law behavior is specified according to Eq. 6[23]:

$$\frac{dp_c}{de^p} = \frac{1+R}{R} K^{iso}_{ref} \left(\frac{p_c}{p_{ref}}\right)^m \quad (6)$$

where, K^{iso}_{ref} is the slope of the laboratory curve for p_c versus (minus) volumetric strain, e, at reference pressure, p_{ref}, in an isotropic compression test and m is a constant ($m \leq 1$). In the logic of the Cysoil model, it is assumed that the current tangent elastic bulk modulus, K^e, is equal to a constant, R, times the current value of hardening modulus, dp_c/de^p. Thus, Eq. 7 can be derived according to Eq. 6[23]:

$$K^e = (1+R) K^{iso}_{ref} \left(\frac{p_c}{p_{ref}}\right)^m \quad (7)$$

Also, the ratio of current elastic shear and bulk modulus, G^e/K^e, is assumed to remain constant and equal to the ratio, G/K, of upper bound (input) values of shear and bulk modulus. Hence, G^e can be determined by Eq. 8[23]:

$$G^e = \frac{G}{K}(1+R) K^{iso}_{ref} \left(\frac{p_c}{p_{ref}}\right)^m \quad (8)$$

The law Eq. 6 has 4 parameters: K^{iso}_{ref}, p_{ref}, m and R. However, not all parameters are independent: Since, by definition, G^{e}_{ref} is the elastic tangent shear modulus at reference (effective) pressure, p_{ref} and the following parameter consistency condition holds Eq. 9[23]:

$$R = \frac{K}{G}\frac{G^{e}_{ref}}{K^{iso}_{ref}} - 1 \quad (9)$$

The initial cap pressure in the model is calculated assuming normal consolidation on the cap. The initial effective stress is used to calculate the initial values of p' and q and the initial value of p_c is then derived from the cap yield function in Eq. 10[23]:

$$p_c = \sqrt{\left(\frac{q^2}{\alpha^2} + p'^2\right)} \quad (10)$$

The hardening behavior applied with the Cysoil model (as described above) offers some resemblance to the Plaxis hardening-soil model as described in the Plaxis Material Models Manual[19]. As such, the following connection as shown in Table 2, is proposed between hardening Cysoil properties and Plaxis hardening-soil properties. Note, however, that among other things, differences exist in the hardening and dilatancy laws. Thus, the model responses should not be expected to be identical. Table 3 shows the Cysoil parameters derived from the correlations presented earlier in Table 2.

Table 2: Relation between Cysoil and Plaxis hardening-soil properties[24]

Plaxis hardening-soil	Hardening cysoil
E^{ref}_{50}	-
E^{ref}_{ur}	$G^{e}_{ref} = \frac{E^{ref}_{ur}}{2(1+\upsilon_{ur})}$
E^{ref}_{oed}	$K^{iso}_{ref} = E^{ref}_{oed}$
-	$R = \frac{E^{ref}_{ur}}{3(1-2\upsilon_{ur})E^{ref}_{oed}} - 1$
Cohesion, C	zero
Friction angle, ϕ	ϕ_f
Dilation angle, ψ	ψ_f
Poisson's ratio, ν_{ur}	ν_{ur}
Power, m	idem
K^{nc}_0 (using cap)	idem
Tensile strength	Zero
Failure ratio, R_f	idem

Table 3: Hardening Cysoil properties in current study

Parameters	Unit	Values
Density	Kg m^{-3}	2000
Cap-yield surface parameter, α	-	1.00
Ultimate friction angle, ϕ	Degrees	32.00
Ultimate dilation angle, ψ	Degrees	0.00
Multiplier, R	-	1.22
G^{ref}_{e}	MPa	13.30
K^{ref}_{iso}	MPa	8.00
Reference pressure, p_{ref}	MPa	0.10
Poisson's ratio, ν_{ur}	-	0.20
Cohesion, C	MPa	0.00
Power, m	-	0.85
Knc	-	0.52
Failure ratio, R_f	-	0.93

RESULTS AND DISCUSSION

For the tieback wall introduced earlier, some important aspects such as the base heave of excavation and lateral displacement of the wall (both the magnitude and pattern) are studied. It is worth mentioning that for the same finite difference mesh density and geometric parameters, the overall calculation time for the numerical simulation of the wall increased to about 7 times by the use of Cysoil model compared to the other models.

Base heave: Figure 3 shows upward heave of the excavated soil with construction stages for the tieback wall simulated

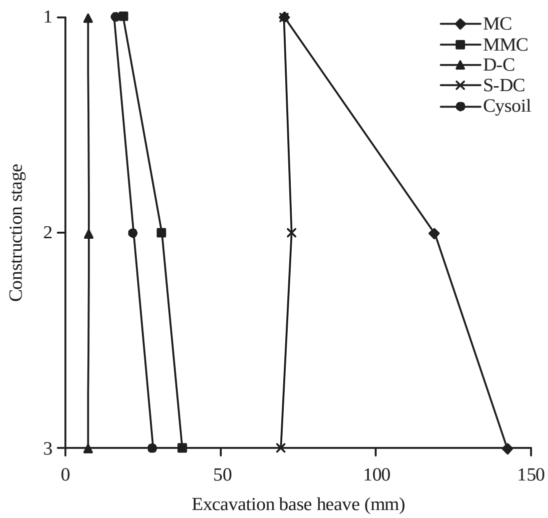

Fig. 3: Excavation base heave with construction

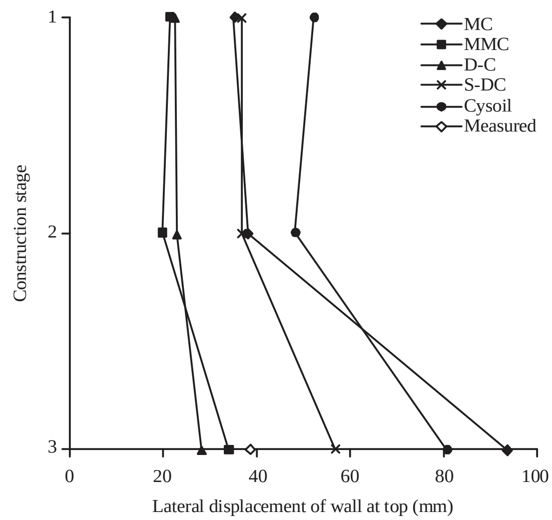

Fig. 4: Lateral displacement of tieback wall with construction stage

using five different material models introduced earlier. It should be noted that no measured values were obtained for base heave in Texas A and M University and the following comparison is made just between the soil models.

For each construction stage, base heave shown in this figure represents the maximum value of the vertical upward displacement of excavation base AC (Fig. 2). From Fig. 3, it is evident that MC model considerably over-estimates the heave of the excavation base as that predicted by other models. This result may be due to (1) Considering linear elastic pre-failure soil behavior assumed in MC model formulation, (2) Assuming equal soil stiffness for both loading and unloading regions and (3) Constant stiffness in soil layers of the model. This observation regarding over-prediction of base heave is in good agreement with literature[9,17].

DC model predicted the least heave at the base, which seems more realistic than the others that is identical to the findings presented by previous studies on constitutive soil models[15,16]. However, Cysoil and MMC models also exhibit acceptable base heaves. Since MMC needs less geotechnical parameters than the others do, so it can be considered as an appropriate model in predicting the heave of an excavated ground. As can be seen from Fig. 3, the SDC model also did not predict the heave properly which may be attributed to removing the relation between stiffness and confining pressure in the original DC code, which indicates the importance of considering this feature in the analysis.

Lateral displacement: Figure 4 shows the maximum lateral displacement of the tieback wall with construction stages for five different material models employed in this research. For each construction stage, lateral displacement shown in Fig. 4 is the value of the horizontal displacement at point D, top of the wall (Fig. 2). It may be observed from Fig. 4 that again the DC and MMC models are in good agreement with each other and with the measured lateral displacement of the wall at the last stage of construction. The horizontal displacement obtained in this study by means of DC model in a finite difference software supports the results obtained by Briaud and Lim[15] in a separate study on the same case study performed using a finite element code. Moreover, the MMC results in current research concerning the horizontal measurement of point D are consistent with the results obtained by Imam and Hoseini[19] on excavations in clayey sands. The MC and Cysoil models presented a poor prediction for horizontal displacement of the wall. The incorrect estimation of lateral displacements by MC model is reported by several authors[9,11,17], however, there is few researches which presented reasonable performance of MC model in cemented soils[12]. It is clear that with Cysoil model, the attention must be given to parameter selection, while the low bulk and shear modulus computed for unloading portions of the model is unrealistic. The improper results obtained for MC model is because of the large heave calculated by this soil model, which may affect the horizontal displacements of the wall. SDC model presented moderate values compared to the others due to (1) Simplifications applied in the original DC model (i.e., negative effect), (2) The irrational heave obtained in the analysis (i.e., negative effect) and (3) Considering different stiffness for loading and unloading regions (i.e., positive effect).

Deflection pattern: Figure 5 presents the deflection pattern of the wall predicted by all five models. As can be observed from this graph, Cysoil, MC and MMC models predicted more deflection in the lower part of the wall, which is incorrect as compared with the measured values that exhibited a cantilever type of deflection. These observations may be attributed to the following two reasons: (1) That with the increasing construction stages in Cysoil model cumulative plastic strains in the model increase and thereby reduce the stiffness in the retained soil mass excessively and (2) Assumption of the linear elastic pre-failure behavior of the soil in MC and MMC models. It is worth mentioning that the deflection pattern in studies performed by Muntohar and Liao[1] and Nasekhian[12], respectively in alluvial silty and cemented soils were in good agreement with measured values. It is a clear indication that MC model cannot capture the deflection pattern of excavated walls truly in sands. Besides, DC model again exhibited good compatibility with the measured values, which supports the findings obtained by Briaud and Lim[15] whom utilized DC in a finite element code for the analysis.

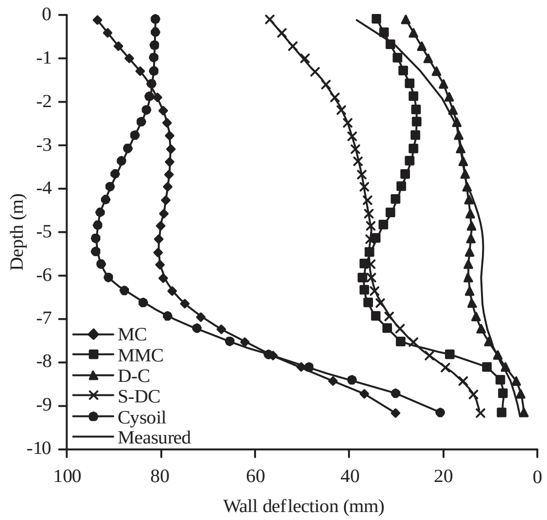

Fig. 5: Lateral displacement pattern of tieback wall

In SDC model as it was mentioned before, the relation between soil stiffness and confining pressures was ignored due to simplicity. As a result, it is obvious that the trend of the displacements is nearly identical to DC model but since the bulk and shear modulus do not increase in depth, so the calculated displacements are less than the DC or measured values.

The good prediction of DC model may be attributed to way of specifying the loading and unloading regions. As can be observed from Fig. 6, this model estimates unloading condition in the regions adjacent to the anchors, so providing greater values for bulk and shear modulus in these locations. Thus, the lateral displacements are better restricted in lower part of the wall and the predictions moves closer to the real values. It may be recalled that in MMC model these two regions were separated approximately and no unloading condition was assumed in soil behind the wall, so the deflection pattern of the wall was not captured realistically in this modified model.

As it was mentioned earlier, because of considering the relation between soil stiffness and cap pressure in Cysoil model and also the negligible variation of cap pressure in the

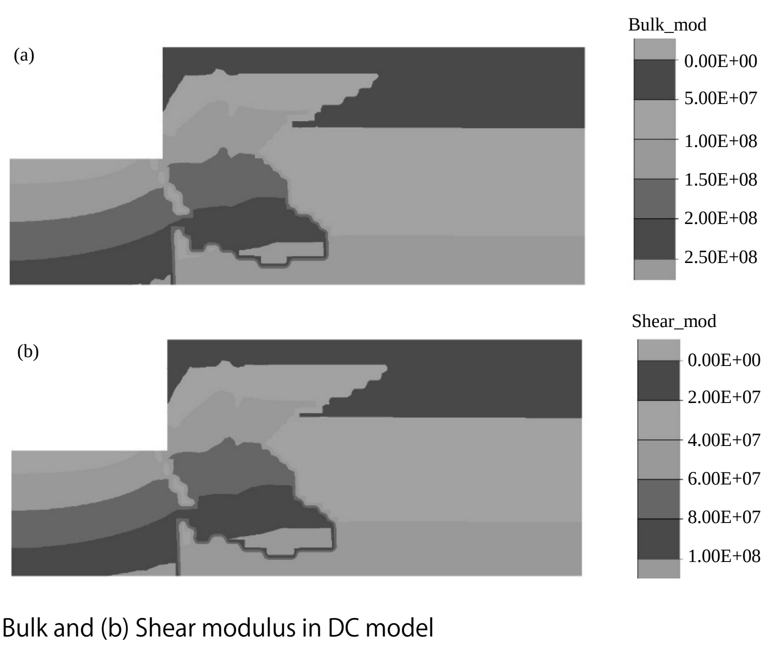

Fig. 6(a-b): Variation of (a) Bulk and (b) Shear modulus in DC model

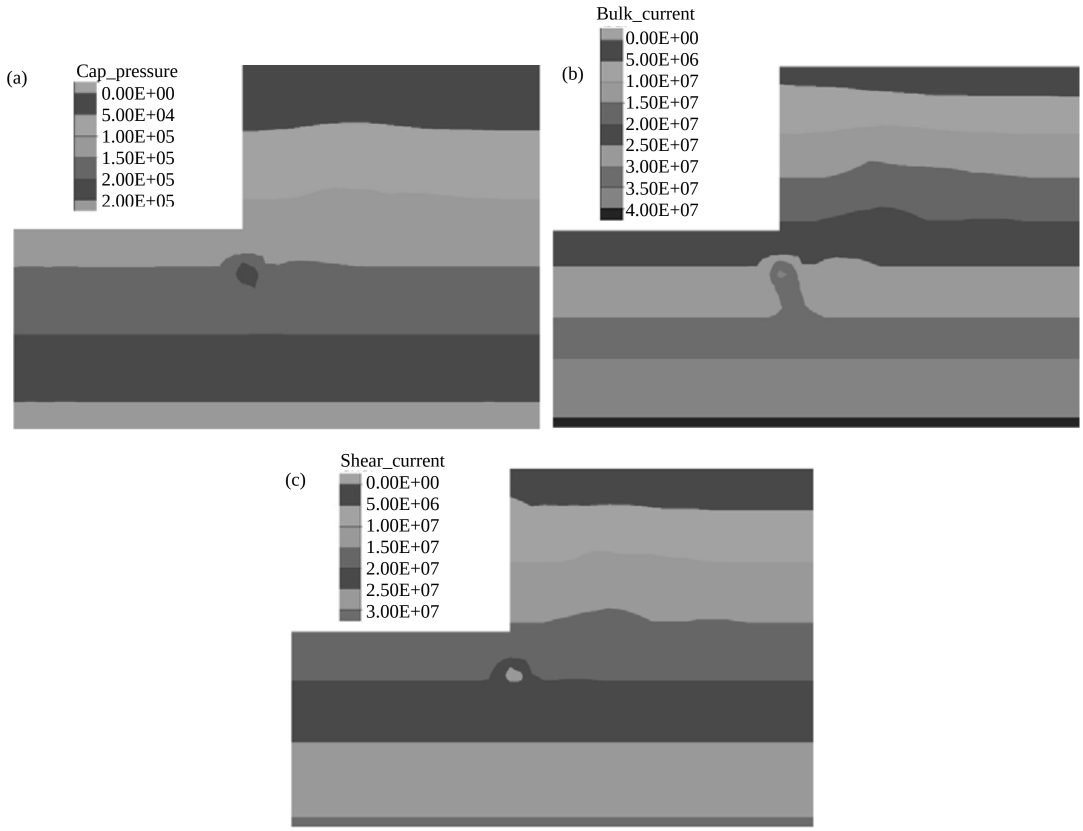

Fig. 7(a-b): Variation of (a) Cap pressure, (b) Bulk modulus and (c) Shear modulus predicted by Cysoil model

model, the bulk and shear modulus values behind the wall and beneath the excavation base were much lower compared to DC model (Fig. 7). Therefore, both the values and the pattern of the wall displacement were predicted unrealistic by Cysoil.

In order to compare the results in a mathematically way, the Root-Mean-Square Error (RMSE) value was used. The Root-Mean-Square Deviation (RMSD) or RMSE is a frequently used measure of the differences between values (sample and population values) predicted by a model or an estimator and the values actually observed. The RMSD represents the sample standard deviation of the differences between predicted values and observed values. RMSD is a good measure of accuracy but only to compare forecasting errors of different models for a particular variable and not between variables as it is scale-dependent[25].

Normalizing the RMSD facilitates the comparison between datasets or models with different scales. Though there is no consistent means of normalization in the literature, common choices are the range (defined as the maximum value minus the minimum value) of the measured data. Thus, NRMSE can be evaluated by Eq. 11:

$$\text{NRMSE} = \frac{\text{RMSE}}{y_{max} - y_{min}} \tag{11}$$

This value is commonly referred to as the normalized RMSD or RMSE (i.e., NRMSD or NRMSE, respectively) and often expressed as a percentage, where lower values indicate less residual variance. NRMSE determined for MC, MMC, DC, SDC and Cysoil models were equal to 168, 42, 8, 59 and 183%, respectively. It is obvious that, DC predicted the pattern of displacements more reasonable (i.e., NRMSE less than 10%) than the other constitutive models, while the MC and Cysoil models predicted unrealistic results.

As a result, it seems clear that MC model cannot predict soil behavior in excavation problems reasonably. However, there is exceptions reported in literature regarding proper performance of MC in cemented and silty soils which requires further researches. Moreover, MMC model presented good results both in heave and lateral displacement in top of the wall but employing this model in predicting the deflection pattern of the wall is not recommended. The DC model, however, presented the best predictions in all three aspects of the research and therefore it can be considered as a reliable model in estimating movements induced by an excavation, when more geotechnical parameters are available. The simplification applied in DC model, made the results unrealistic compared to original DC, which moved the results somewhere between the best and worst predictions. Although, Cysoil evaluated the base heave reasonably but this model exhibited some shortcomings in predicting lateral displacement of the wall. It must be emphasized that the performance of Cysoil in excavation projects still needs to be studied and the authors recommend more researches in this field.

SIGNIFICANCE STATEMENTS

This study discovers the possible application of different constitutive soil models in estimating various aspects of movements induced by excavation practices. This study will assist the engineers to achieve an optimum compromise between model accuracy and geotechnical survey efforts required in deformation analysis of excavation problems.

CONCLUSION

When sufficient data is available, DC soil model predicts all deformation aspects of tieback walls better than the others do. For the purpose of estimating the basal heave and deflection pattern of an excavated ground, respectively, MMC and SDC models also perform satisfactorily in case of limited geotechnical data. These findings become more important when the amount of required geotechnical parameters which affect the project costs plays a significant role in the project.

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