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## Mechanical Reaction of Vegetation Canopies to Wind Flow

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**Abstract:** In this study, an index is proposed to characterize physical properties of tree species. This will allow the application of a single momentum (or energy) equation to determine overall reaction of variety of tree species in a community. The index is derived based on the resonance frequency of the first mode of vibration of trees and a fundamental relationship for the homogeneous beams. The derived indexes for four species of coniferous trees were used in a mathematical model to estimate the drag and energy coefficients as representatives for tree reaction to wind flow and were able to account for the differences due to the leaf density, shape and rigidity of the tree species.

**Key words:** Mechanics of tree, natural frequency, vegetation canopy

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### INTRODUCTION

Physical properties of tree species determine their reaction to external loads as a result of wind flow. The greater the trees ability to withstand wind load, the greater the amount of wind energy absorption, generation of wind eddies and the acceleration of turbulence in forest canopies. This will increase ventilation, exchange of mass, photosynthesis and therefore will have considerable effects on the productivity of vegetation.

Study of the mechanics of trees probably started with the analysis of a Cambridge mathematician named A.G. Greenhill for derivation of a relationship between height and diameter of trunk. McMahon in an attempt to find a governing relation between the length and diameter of trees, tested a large range of models of similar elastic cylinders and columns, clamped horizontally and vertically (Fathi-Moghadam and Kouwen, 1997). McMahon and Kronauer (1976) found further evidence supporting the conviction expressed in the literature that the branching pattern within any species is approximately stationary. This means that a tree's structure is self-similar, so that any patch of the structure is a model of the entire tree and even whole species. In a more dynamic hypothesis he suggests that, the only way the crown shape could be maintained the same during growth would be to keep the chord angles the same, which is just the condition for elastic similarity.

Physical and mechanical properties of tree species and their reaction to wind flow is an important subject when dealing with exchange of mass (gases), momentum (wind load and fencing) and energy (turbulence mixing) between atmosphere and forest canopies. The greater the exchange of vital gases and water vapor, as a result of increased turbulence mixing, improves photosynthesis and productivity of tree communities.

Tree reaction or in fact resistance to wind flow is very sensitive to the wind velocity and its own physical and mechanical condition. The resistance to flow decreases rapidly as the wind velocity increases, due to the streamlining and the resulting reduction of the frontal area of tree foliage. The fundamental physical and mechanical properties to be considered in establishing a resistance equation are leaf density, shape, flexibility and manner of defection of the tree species (Kouwen and Fathi-Moghadam, 2000; Jarvela, 2004).

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In order to develop a single mathematical model to estimate wind load or in fact reaction of all tree species in a community, an index (symbolized as  $\xi E$ ) is required in a particular model to account for the physical properties (i.e., the effects of leaf density, shape and rigidity) of individual trees. The purpose of this study is to present a practical procedure for estimation of tree index ( $\xi E$ ) and identifying the reaction of a tree species. The resistance coefficients are used in this study to measure tree reaction to wind flow. They include the drag force coefficient for momentum approach and the energy loss coefficient (or in this case the wind flow energy transfer coefficient,  $f$ ) when the energy equation is used to solve a particular problem. These coefficients can easily be derived from one-another for a given condition (Kouwen and Fathi-Moghadam, 2000). In general, these coefficients are key parameters for estimating the wind velocity profile and characterizing turbulent boundary layer flow above forest canopy. These are interesting subjects when dealing with the rate of exchange of mass, momentum and energy between atmosphere and forest canopies as were investigated by Thom (1971) and De Bruin and Verhoef (1997).

The proposed method is used to estimate tree index for four species of coniferous trees including cedar (*Thuja occidentalis*), spruce (*Picea glauca*), white pine (*Pinus strobus*) and Austrian pine (*Pinus palustris*). The calculated indexes for four species of coniferous trees were used in a mathematical model developed by Fathi-Moghadam (Fisher and Dawson, 2003) to estimate the drag and energy lost coefficient and were able to account for the differences due to the leaf density, shape and rigidity of the tree species.

### THEORY AND MEASUREMENTS

Theoretically, any beam, tree or plant stem with mass and elasticity may exhibit one or more resonance frequencies of vibration depending on damping (McMahon and Kronauer, 1976; Niklas and Moon, 1988). For small damping, these resonance frequencies are close to the natural frequencies of the beam. Natural frequencies result from the cyclic exchange of kinetic and potential energy when a structure such as a beam or plant stem is vibrated. The kinetic energy is proportional to the square of the velocity of the structural mass, while the potential energy is proportional to the square of the elastic strains. The rate of exchange between kinetic and potential energy is the natural frequency of vibration.

The resonance frequencies,  $f_j$  (with  $j = 1, 2, 3, \dots, n$ , where,  $f_1$  is the fundamental or base natural frequency and  $f_{2..n}$  are higher modes of natural frequencies) of a linear and homogeneous beam depend upon its length ( $l$ ), mass per unit length ( $m$ ), second moment of inertia ( $I$ ), modulus of elasticity ( $E$ ), as well as a dimensionless parameter ( $\lambda_j$ ) which is a function of beam geometry and the boundary conditions under which the beam is tested. The relationship between the resonance frequencies and the above variables is given by the following equation (Humar and Ruban, 2002):

$$f_j = \frac{\lambda_j^2}{2\pi} \left( \frac{EI}{ml^4} \right)^{1/2} \quad (1)$$

where,  $EI$  is flexural stiffness.

The values of ( $\lambda_j$ ) have been theoretically calculated for a variety of beam geometry (i.e., prismatic, non-prismatic and tapered beams) and methods of attachment (i.e., boundary conditions) which can be found in advanced dynamic and vibration standard texts. Karnovsky and Lebed (2001) presented tables for natural frequencies of tapered beams. The tables cover the whole range of elliptical tapering.

The published values for  $\lambda_j$ ,  $E$  and  $I$  can be used in Eq. 1 with sufficient accuracy for linear and homogeneous beams. If the geometry of beam does not resemble the geometry of beam presented in

the literature, then a model can be constructed with specified dimensions and known material properties, i.e., known EI; after measuring  $f_1$ , the value of  $(\lambda_1)$  can be computed and then used to determine flexural stiffness EI for a given beam.

Niklas and Moon (1988) measured flexural stiffness and modulus of elasticity of flower stalks Using Multiple Resonance Frequency Analysis (MRFA) of spectra. The flower stems were attached to a shaker and their vibrations were tracked by an Optron camera and analyzed by a spectrum analyzer. For small and symmetric shape vegetation elements such as flower stems, the use of published E, I and  $\lambda_1$ -values in Eq. 1 to estimate a material property may be appropriate. Obviously, such a method or methods for simple structures (like beams) are less applicable for large scale and complex structures like trees.

**Development of a Semi-Empirical Method to Index Tree Species**

In classical mechanics of materials, a vegetative biomass is classified as a non-homogeneous visco-elastic material (Niklas, 1992). For large trees, this non-homogeneity will be much greater than short grass or analysis of small plant segments. Trees have different classes of branches and significant difference in ratios of hardwood and softwood in their segments. It should be noted that the vegetal drag coefficient for the leafy trees was found to be three to seven times that of the leafless trees (Jarvela, 2004).

The complexity and large non-homogeneity of the visco-elastic materials of large trees disqualify the use of theoretical values of E, I and  $\lambda_1$  to characterize a tree species. This defines a need to derive a semi-empirical relationship based on extensive tests on various tree species for the estimation of tree indices and quantification of the physical properties of species.

In order to avoid errors resulting from the use of theoretically-based values for  $\lambda_1$ , I and E in Eq. 1 and to minimize the number of unknowns, several simplifications have been made in this study. The dimensionless ratio of  $(I l^{-4})$  in Eq. 1 together with the parameter  $[\lambda_1^2 (2\pi)^{-1}]$  can be assumed to be a single parameter, symbolized as  $\xi$  for the base mode of vibration. This parameter characterizes height, mass or leaf density and the moment of inertia of a tree. Substituting the tree's height (h) for the beam length (l) in Eq. 1 and transferring the measurable parameters to the right side, Eq. 1 for the first mode of the natural frequency ( $f_1$ ) will be:

$$\xi E = f_1^2 \left( \frac{m_s}{h} \right) \tag{2}$$

where,  $m_s = m \cdot h$ , the total mass of the tree and  $\xi E$  is called the tree index in this study. Measuring a tree's height, mass and recording its natural frequency of the first mode of vibration, the tree index can be estimated by Eq. 2. The developed tree index includes all physical and mechanical properties of a tree species for their leaf density, shape, stiffness and manner of deflection.

**Experiment Procedure and Apparatus**

Assuming self elastic similarity hypothesis of McMahon and Kronauer (1976), the heights, weights and natural frequencies of the first mode for four species of coniferous trees, 30 samples in three categories of size including small, mid-size and full size trees with average heights of 0.3, 1.2 and 3.0 m, respectively, were measured. The natural frequency of small and medium sized categories was recorded using an accelerometer, dynamic analyzer and a frequency spectra plotter. A small silicon accelerometer with a manufacturer reported bias of  $\pm 0.1\%$  was used for small and medium sized model testing. The natural frequency of the first mode of vibration was clearly defined in the spectrum by shaking models from side to side. The recordings were started when the models were released from loading and were allowed to freely vibrate.

Figure 1 shows a plot of the natural frequency spectrum for the medium sized Austrian pine tree AP3 listed in the Table 1. The average of five excitations was used to mark the first and second modes

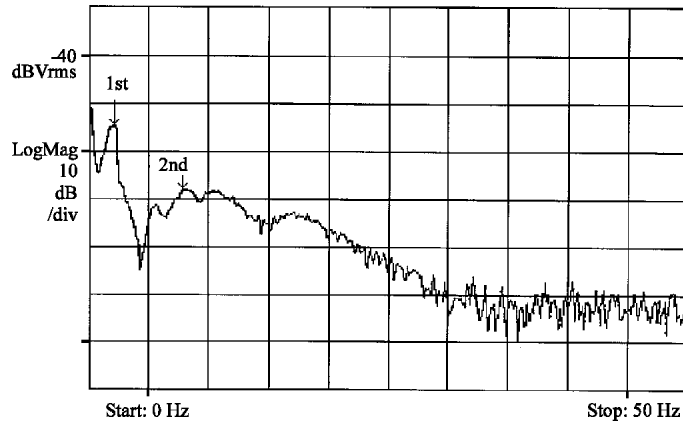


Fig. 1: Frequency spectrum of medium sized Austrian pine tree AP3 recorded by a dynamic analyzer

Table 1: Physical properties and indices ( $\zeta E$ ) of coniferous trees

General size	Tree species	Sample name	Height (h), m	Weight (W), N	Mass (m), kg	ms ( $\text{kg m}^{-1}$ )	1st mode Nf ( $\text{sec}^{-1}$ )	2nd mode Nf ( $\text{sec}^{-1}$ )	Index $\zeta E$ Nf ( $\text{sec}^{-2}$ )	Average $\zeta E$ Nf ( $\text{sec}^{-2}$ )
Small size models	Cedar	C93	0.30	1.03	0.10	0.35	3.75	15.50	4.90	4.90
	Aus. pine	AP93	0.30	1.27	0.13	0.43	4.00	19.75	6.92	6.92
Mid-size models	Cedar	C1	1.40	17.50	1.78	1.27	1.38	3.75	2.41	1.97
		C2	1.15	8.62	0.88	0.76	1.63	5.00	2.02	
		C3	0.85	4.19	0.43	0.50	1.73	5.25	1.49	
	Spruce	S1	1.55	22.02	2.25	1.45	1.65	3.38	3.94	3.25
		S2	1.25	16.82	1.72	1.37	1.60	4.00	3.51	
		S3	0.75	8.65	0.88	1.17	2.25	7.75	5.95	
	Aus. pine	AP1	1.25	25.60	2.61	2.09	1.13	4.25	2.64	3.91
AP2		1.15	18.79	1.92	1.67	1.38	4.25	3.15		
AP3		0.75	8.65	0.88	1.17	2.25	7.75	5.95		
Full size	Cedar	CW1	2.95	62.07	6.33	2.14	1.05	-	2.34	2.11
		CW2	3.30	83.40	8.50	2.58	1.02	-	2.68	
		CW3	3.10	83.87	8.55	2.76	0.92	-	2.35	
		CW4	2.85	51.93	5.29	1.86	1.08	-	2.17	
		CW5	2.50	33.78	3.44	1.38	1.12	-	1.73	
		CW6	2.20	31.78	3.24	1.47	1.07	-	1.67	
		CW7	1.90	28.59	2.91	1.53	1.09	-	1.81	
	Spruce	SW1	3.45	103.95	10.56	3.06	1.10	-	3.67	3.41
		SW2	2.35	46.30	4.72	2.01	1.43	-	4.11	
		SW3	3.15	81.36	8.29	2.63	1.13	-	3.33	
		SW4	2.65	40.22	4.10	1.55	1.24	-	2.39	
	White pine	WP5	3.85	267.81	27.30	7.09	0.71	-	3.57	2.99
		WPWI	2.80	102.42	10.44	3.73	0.86	-	2.78	
		WPW2	2.15	86.29	8.80	4.09	0.92	-	3.43	
Aus. pine	WPW3	1.90	54.58	5.56	2.93	0.98	-	2.81	5.02	
	APW1	2.95	116.44	11.87	4.02	1.09	-	4.78		
	APW2	3.20	133.30	13.59	4.25	0.94	-	3.75		
	APW3	3.10	149.83	15.27	4.93	1.05	-	5.43		
		APW4	3.25	186.80	19.04	5.86	1.02	-	6.10	

of vibration. Similar plots were recorded for all the small and medium sized samples and species which are reported in Table 1. The frequency spectrum for the model support was recorded before testing any of the models to avoid any confusion in later analysis of the spectra. In general, the second mode of the natural frequencies was damped much faster than the first mode due to its interaction with the vibration of the tree's laterals. Although the second mode of vibration was not taken into account for the analysis in this study, future work is needed to ascertain its lack of importance. Within the tested species, cedar had a better response and clearer distribution between the first and second modes of vibrations compared to the others. Spruce had the least distinct response and the highest damping.

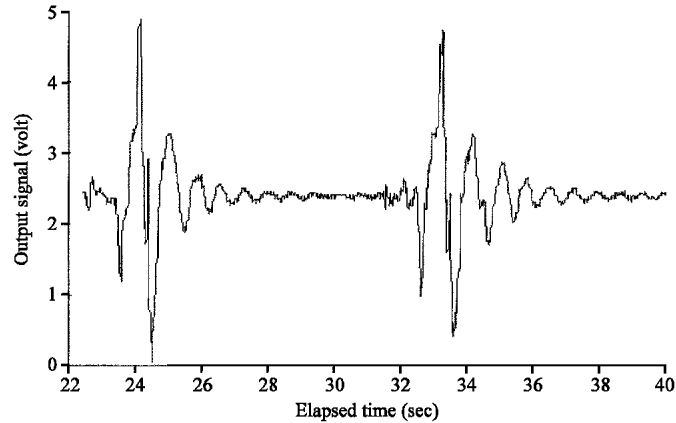


Fig. 2: Output signal of natural frequency of first mode for full sized spruce tree SW5

Full size trees with an average height of approximately 3.0 m were only tested for their first mode of vibration. A silicon micro-machined accelerometer Model 3145 with a standard range of  $\pm 2$  g and  $\pm 2$  volt output (i.e., precisely  $988 \text{ mV g}^{-1}$ ), stable up to frequency of 250 Hz and a manufacturer reported bias of  $\pm 0.2\%$  was used for full size tree measurements. The accelerometer was attached to the top half of the main stem. Trees were fixed at their base and were shaken from side to side at the top of the tree. The sinusoidal vibration of the shaken trees was converted to an output signal by the accelerometer, amplified ten times and recorded. The high frequency of data acquisition (one hundred readings per second) provided a smooth sinusoidal graph and the number of swings per second could be easily determined. Thus an estimate of the first mode of the resonant frequency was found. Figure 2 shows a plot of the natural frequency of the first mode for a full size spruce tree no SW5. The average of three excitations (as shown in Fig. 2) was used to calculate the frequency of the first mode of vibration. Similar plots were recorded for other full size trees as reported in Table 1.

## RESULTS AND DISCUSSION

The measured height, mass, natural frequency and the calculated index ( $\xi E$ ) from Eq. 2 is recorded in Table 1 for each tested sample. A simple averaging technique was applied first to the calculated indexes ( $\xi E$ ) of each species in each category of sizes and then over the size categories. The resulting representative indexes are 2.07, 3.36, 2.99 and  $4.54 \text{ N m}^{-2}$  for cedar, spruce, white pine and Austrian pine trees, respectively. In a relative sense, the representative indexes are in agreement with the reported value of modulus of elasticity ( $E$ ) in Niklas (1992). The Austrian pine and cedar have the maximum and minimum rigidity respectively, while the white pine and spruce are in the medium range of rigidity within all species of coniferous trees. It should be noted that because the small sized models of cedar and Austrian pine were low in number (Table 1), they were not used in the averaging for calculations of the representative indexes.

### Validation of the Presented Tree Indices

A mathematical model has been developed by Fathi-Moghadam (Fisher and Dawson, 2003) to estimate the resistance coefficients (coefficients of drag and energy) for flow through coniferous trees. The model is based on a dimensional analysis supported by a series of experiments. Using the mathematical model, the relationship between correlated wind energy coefficient ( $f$ ) and average wind velocity for four species of coniferous trees (cedar, spruce, white pine and Austrian pine) are shown

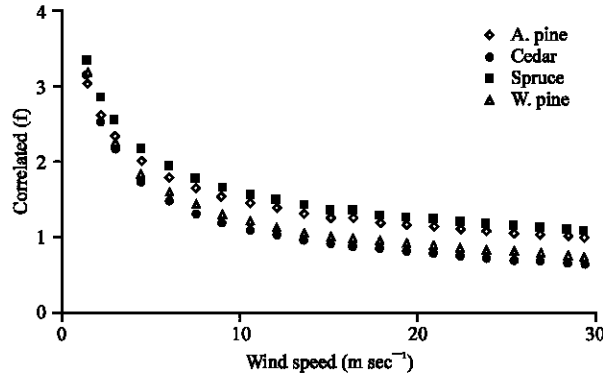


Fig. 3: Correlation of energy coefficient (f) and wind speed (V) for four species of coniferous trees

in Fig. 3. The curves in Fig. 3 are averages of testing five samples for each species of coniferous trees. A noticeable difference in wind energy coefficient (f) from one species to another in Fig. 3 may lead to criticism of the application of this single mathematical model for all species of evergreen trees. However, the difference in energy coefficient in Fig. 3 is due to variation in shape, leaf density and material properties which are entirely accounted for by the tree index ( $\xi E$ ).

To verify the representative vegetation indices ( $\xi E$ ), the average wind velocity (V) in the x-axis of Fig. 3 is normalized by the indexes for each tested tree species in the form of  $V \rho^{0.5} (\xi E)^{-0.5}$ , where  $\rho$  is mass density of fluid ( $\text{kg m}^{-3}$ ). Using unit mass density for the same fluid flow, the resulting relationships are plotted in Fig. 4 for all tree species tested in this study. As was expected, the best fit curves for each species in Fig. 4, are approximately, 50% closer together than those in Fig. 3. This will allow the curves to be combined for an average curve (coniferous index curve) that represents the physical behavior of most species of coniferous trees. The final result of a linear regression of the data for air flow ( $\rho_a = 1.23 \text{ kg m}^{-3}$ ) through coniferous trees is,

$$f = 2.98 [V \rho_a^{0.5} (\xi E)^{-0.5}]^{-0.46} \quad (3)$$

Equation 3 can be used to estimate the energy transfer coefficient (f) for wind flow through a stand of coniferous trees on forest canopies. Using principals of fluid mechanics, the drag coefficient and reacted force by trees are calculated. Similar power functions between f and V as in Eq. 3 were suggested in the literature (e.g., Freeman *et al.*, 1998) for water flow through a stand of trees on flood plains or in vegetated zones of rivers. However, no index or practical method has been proposed to account for the effect of type and variation of vegetation on the energy coefficient.

#### Relationship Between Height and Natural Frequency

In practice, Eq. 1 is not an exact relationship because damping is not considered. Damping of vibration is not negligible even for homogeneous metal materials. For a highly damped, non-homogeneous and visco-elastic tree, a relationship between the height (h) and natural frequency of first mode ( $f_1$ ) can only be empirically based on a large number of experimental data.

Figure 5 shows a graphical relationship between the height and the natural frequency of the first mode for the whole range of heights (0.3-3.85 m) and tree species tested in this study (Table 1). An overall linear logarithmic fitting between the ( $f_1$ ) and the (h) for all the data results in the following relationship:

$$f_1 = 1.8 (h)^{0.58} \quad (4)$$

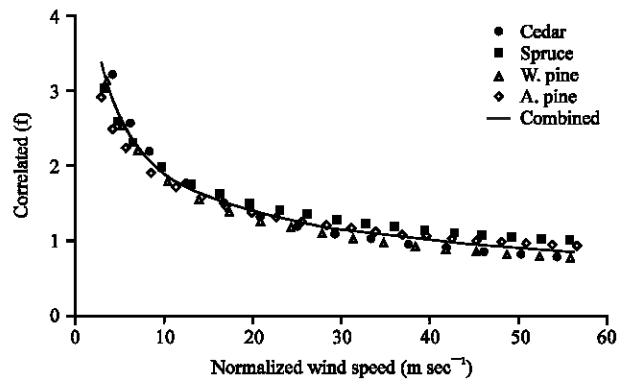


Fig. 4: Correlation of energy coefficient (f) and normalized wind speed for four species of coniferous trees

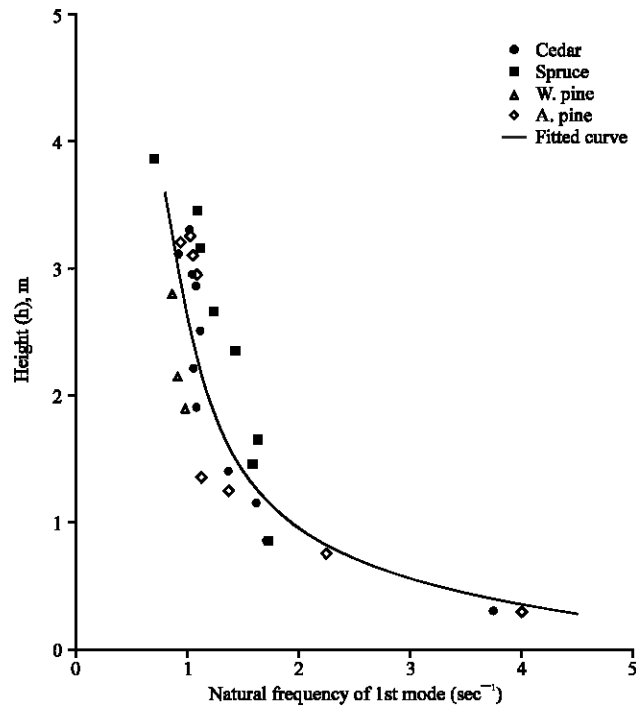


Fig. 5: Relationship between natural frequency of first mode and height of trees

Since Eq. 4 is based on the data of four species of coniferous trees, it cannot be used to estimate the ( $f_i$ ) in Eq. 2 for a particular species other than coniferous trees. As Flesch and Wilson (1999) and Moore and Maguire (2004) showed that tall trees sway at low frequencies and suggested a similar power functions as Eq. 4, many more tree species should be tested in order to develop a more general relationship between the natural frequency and tree height of different species.



## CONCLUSIONS

Physical properties of trees have significant influence on their reaction and resistance to wind flow and rate of exchange of mass, momentum and energy between atmosphere and forest canopies. In the present study, a method is developed to index physical behavior of tree species and to account for the effect of tree conditions and properties (i.e., leaf density, shape, stiffness and manner of deflection) on the drag and energy transfer coefficients. The presented trees' indices ( $\xi E$ ) were used to normalize flow velocity which enables the elimination of coefficient variations among different species of coniferous trees. A single correlation between the energy coefficient and the normalized flow velocity results in a single mathematical model for various species of coniferous trees. The trees' indices showed to be adequately capable of differentiating between tree species and condition in the mathematical model. Using the trees' indices, the model estimated energy coefficients for coniferous trees were consistent with the reported coefficients by Jarvela (2002) for willows. In the future, estimation of the new index ( $\xi E$ ) for each tree species will require extensive sampling and testing of trees having a greater variety in appearance and physical characteristics. Assuming availability of indices for different tree species, correlating the ground-based measurements and high resolution satellite data will make the analysis of large forest areas possible (Rautiainen *et al.*, 2003; Stenberg *et al.*, 2003).

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