Research Journal of Environmental Sciences

ISSN 1819-3412

Academic
Journals Inc.

www.academicjournals.com

Research Journal of Environmental Sciences 4 (5): 421-432, 2010
ISSN 1819-3412

Hydrodynamic Behavior Analysis of the Saharian Aquifers with Continuous Wavelet Transform

[1]M. Chettih and [2]M. Mesbah
[1]Department of Civil Engineering, Faculty of Engineering Science,
Amar Telidji University, P.O. Box 37G, Laghouat 03000, Algeria
[2]Department of Geology, Faculty of Earth Science,
University Houari Boumediene of Science and Technology,
P.O. Box 32, El Alia, Bab Ezzouar, Algiers, Algeria

Abstract: The main objective of our study is to show non-stationarity of hydrological series by introducing the wavelet transform. The wavelet analysis allowed a study of the time-scale type of the rainfall rates and runoffs of some aquifer systems of the Saharian Atlas in Algeria and thus reveals their great temporal variability. It appears complementary to the traditional functional analysis. As a daily step, the continuous wavelet analysis of Morlet made it possible to highlight the variability of certain characteristic components and the existence of multi-annual components. The runoffs are characterized by a non-stationary behavior strongly influenced by the temporal structure of rains and by the intrinsic non-linearity of the aquifer systems. These systems are slightly sensitive to the fluctuations of rains for the short periods, but the multi-annual phenomena strongly influence them. The wavelet spectra seem to be powerful indicators of the degree of organization of the aquifer systems and consequently of their reserve.

Key words: Continuous wavelets analysis, sandstone aquifer, Saharian Atlas, multiple porosity

INTRODUCTION

The powerful Mesozoic sandy formations of the Algerian Saharian Atlas constitute the principal aquifers of the chain. They represent complex heterogeneous aquifer systems with multiple porosity where lithology, morphology and tectonics often induce a specificity of structure and function. These aquifers are drained by many springs constituting the multiple discharge system and spouting out along the tectonic undulations and on the level of stratigraphic discontinuities and regularly feed the mountain streams.

The preliminary analysis of functional approach showed that the atlasic systems have very developed networks of drainage, with however, a drowned zone of a low dynamic volume (Chettih and Mesbah, 2006). The parameters of adjustment determined on the curves of recessions translate in general the well drained character for the various systems as an important structural organization of the aquifers. In addition, the autocorrelation and spectral analysis showed that the atlasic systems have a very weak memory. Their very short impulse response, can be allotted mainly to the very developed structure of the aquifers and to their degree of organization (Chettih and Mesbah, 2006). The hydrodynamic behavior of the

---

Corresponding Author: M. Chettih, Department of Civil Engineering, Faculty of Engineering Science, Amar Telidji University, P.O. Box 37G, Laghouat 03000, Algeria

systems shows a strong dependence of the reserves with respect to precipitations whose distribution in time is very heterogeneous and discontinuous. The coherence function of the system indicates strong non-linearities. The flows are characterized by a strong variability of the variance, consequently non-stationary.

These features suggest the introduction of new techniques allowing detailed interpretations of the rainfall-runoff relationship and to highlight the characteristic scales in the functioning of hydrogeological systems. The wavelet transform allows to obtain a time-scale vision of hydrological phenomena (Labat *et al.*, 2000; Labat, 2005). It can make the transition from a representation of a signal to another as in the example of Fourier, but with a different resolution time-frequency (Smith *et al.*, 1998: Lane, 2007). The idea of wavelets is the ability to vary the width in time and frequency of a function while relocating it along the signal (Liu, 1995).

MATERIALS AND METHODS

Presentation of Study Sites

To compare between the results of the different systems and to highlight the relevance of the proposed methodology for the study of the Saharian Atlas, three systems in the Central and West Saharian Atlas have been selected for this study (Fig. 1). Seklafa system has already been the subject of a functional approach and we also have a mass of data covering five hydrological cycles sampled at daily step spanning from the first September 1975 to 31 August 1980. For Kerakda and Rhouiba systems, hydrological data are also sampled at daily step, but covering several hydrological cycles which will enable a better analysis of the systems behavior. Their observation periods spread from first September 1975 to 31 August 1990 for Kerakda system and from first September 1990 to 31 August 1999 for Rhouiba system. The main characteristics of the system are reported in Table 1.

On the geological level, the sandstone formations are aquifers with multiple porosity of interstices, cracks and channels. These geological formations constitute a multilayer system representing over 60% in aggregate thickness of atlasic geological formations. These

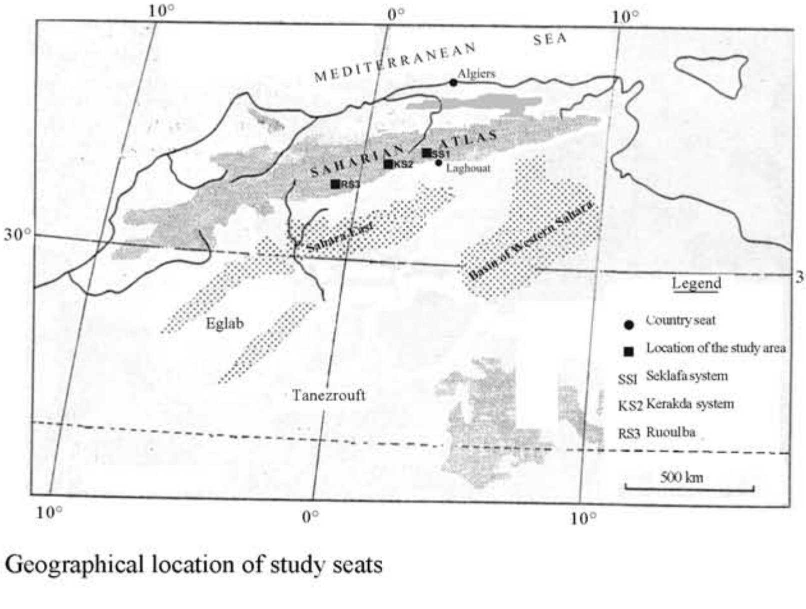

Fig. 1: Geographical location of study seats

Table 1: Characteristics of the drainage basins of the saharian atlasic systems

Hydrological systems	Seklafa system	Kerakda system	Rhouiba system
Observation period	1975-1980	1975-1990	1973-1999
Mean daily runoff	0.487 m^3 sce^{-1}	1.022 m^3 sce^{-1}	0.74 m^3 sce^{-1}
Watershed surface	783 km^2	3930 km^2	3520 km^2
Specific runoff	0.622 *l*/sec/km^2	0.260 *l*/sec/km^2	0.210 *l*/sec/km^2
Median altitude	1260 m	1120 m	1085 m

strongly fissured aquifers also have a hydraulic potential with porosity of cracks amplified by the various post-Eocene tectonic phases (Aït Ouali and Delfaud, 1995).

Continuous Wavelet Transform

In the light of these hydrological signals, in particular, the non-inclusion of temporal aspects of singularities or non-stationarities type, wavelet transform has emerged to overcome some disadvantages of the Fourier analysis.

The wavelet transform is a method of analysis whose applications are increasingly diverse. First introduced in the analysis of seismic signals by Morlet *et al.* (1982) and formalized by Grossmann and Morlet (1984) and Goupillaud *et al.* (1984), this method of analysis has since, then extended to many other application fields by Benner (1999), Higuchi *et al.* (1999), Wilson and Mordvinov (1999), Kumar (1996), Szilagyi *et al.* (1999), Smith *et al.* (1998) and Labat *et al.* (2000). Other newer applications such as: Henderson *et al.* (2009), Huang and Hiroshi (2008), Xue *et al.* (2008), Chou (2007) and Kang and Lin (2007) where it was also applied successfully.

The coefficients of the wavelet transform of a signal x (t) are given by the scalar product (Daubechies, 1992):

$$C_x(a,\tau)=\int_{-\infty}^{+\infty}x(t)\psi^*_{a,\tau}(t)dt \tag{1}$$

$$\psi_{a,\tau}(t)=\frac{1}{\sqrt{a}}\psi(\frac{t-\tau}{a}) \tag{2}$$

where, **Ψ** (t) which plays the role of a convolution kernel is named: wavelet function, which can be real or complex (*) represents the complex conjugate, the parameter a (scale factor), controls the dilatation or contraction of the function **Ψ** (t), the parameter τ is interpreted as a temporal translation factor or time frequency shift of the function **Ψ** (t).

A frequency interpretation of Eq. 1 is also possible using the Parseval theorem (Torrésani, 1995) the wavelet coefficients of the continuous time signal x (t) is also given by:

$$C_x(a,\tau)=\frac{1}{2\pi}\int_{-\infty}^{+\infty}\hat{x}(\omega)\sqrt{a}\hat{\psi}(a\omega)e^{i\omega\tau}d\omega=\frac{1}{2\pi}\int_{-\infty}^{+\infty}\hat{x}(\omega)\Psi_{a,\tau}(\omega)d\omega \tag{3}$$

This formulation indicates that the wavelet coefficients can also be interpreted by the filtering of $\hat{x}$ (Fourier transform of x (t) by a set of band-pass filters $\mathbf{\Psi}_{a,\tau}$ (**ω**) related to the Fourier transform of the wavelet function **Ψ** (t) by the relationship:

$$\Psi_{a,\tau}(\omega)=\sqrt{a}\hat{\psi}(a\omega)e^{i\omega\tau} \tag{4}$$

In this case, the parameter a can be interpreted as a dilatation or contraction factor of the filter $\hat{\psi}$ corresponding to adjacent frequency intervals while the parameter τ can be interpreted as a phase shift.

The statistical characteristics of wavelet coefficients can show the time-scale distribution of the variance of a signal or the covariance of two signals. To account for the decomposition of the total variance in time-scale, it is possible to define a continuous wavelet spectrum w (α, τ) by analogy with spectral analysis, in the form (Liu, 1995):

$$W_X(a,\tau) = C_X(a,\tau)C_X^*(a,\tau) = \left|C_X(a,\tau)\right|^2 \tag{5}$$

This wavelet spectrum can also be averaged in time or scale leading to a loss of information (Torrence and Compo, 1998). First, the average operation in time provides a breakdown of the variance of the signal throughout the scales. On the other hand, the average operation across the scales allows the temporal identification of a particular component of the signal.

By analogy with the Fourier cross-spectrum, we can define a wavelet cross-spectrum $W_{XY}(a, \tau)$ between two continuous time signals x (t) and y (t):

$$W_{XY}(a,\tau) = C_X(a,\tau)C_Y^*(a,\tau) \tag{6}$$

where C_X (a, τ) and C^*_Y (a, τ) are, respectively, the wavelet coefficients of x (t) and the conjugate wavelet coefficients of y (t). A representation of Wiener-Kintchine type is possible. Indeed, the mathematical expectation of wavelet cross-spectrum is the wavelet transform of the covariance function of the input and output signals:

$$\begin{aligned} E\left[W_{XY}(a,\tau)\right] &= E\left[\int_{-\infty}^{+\infty} x(u)\psi^*_{a,\tau}(u)du \int_{-\infty}^{+\infty} y(v)\psi_{a,\tau}(v)dv\right] \\ &= \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} E\left[x(u)y(v)\right]\psi^*_{a,\tau}(u)\psi_{a,\tau}(v)dudv \\ &= \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} Cov_{XY}(u-v)\psi^*_{a,\tau}(u)\psi_{a,\tau}(v)dudv \end{aligned} \tag{7}$$

The methods of taking averages in both time and scale applied to the wavelet spectrum are also applied in the wavelet cross-spectrum context for the distribution of variance between the signals x (t) and y (t) across the scales. Another information on the relationship between the signals of continuous-time type can be extracted from the wavelet coherence function defined by Liu (1995):

$$\Gamma(a,\tau) = \frac{W_{XY}(a,\tau)}{\sqrt{W_X(a,\tau)W_Y(a,\tau)}} = \rho_{XY}(a,\tau)\exp(i\theta_{XY}(a,\tau)) \tag{8}$$

Note that the wavelet coherence function modulus ρ_{XY} is always equal to unity by construction, whatever the scale a and the observation time τ. In order to identify temporally the time intervals where the input-output relationship is strong, the real and imaginary parts of the function Γ will be represented in addition to the phase function θ_{XY}.

RESULTS AND DISCUSSION

In general, the series temporal structure of rainfall and discharge can not be finely analyzed by classical methods which presuppose linearity and stationarity of the series

(Labat, 2005; Wang and Meng, 2007). The continuous wavelet analysis is proposed here as an alternative method (Mathevet *et al.*, 2004). Many studies have been conducted and have yielded very good results. We can cite as an example: Chen and Liu (2008) and Anctil and Tape (2004) what motivated us to apply this technique.

The application of Fourier's classical analysis to hydrological data of the Saharian Atlas has shown for the input signal as an isolated event. For the output signal, the spectrum also showed a small effect of the filter system (Chettih and Mesbah, 2006). The cross-spectral analysis expressed by the cross correlogram and the amplitude function indicates that the treated example presents an impulse response which characterizes a behavior of advanced and well drained systems.

Analysis of univariate wavelet and cross-wavelet are applied in this study for daily rainfall rates and runoff. Compared with spectral analysis, wavelet analysis leads to more accurate results in particular to highlight the temporal variability of processes.

Univariate Wavelet Analysis

The Morlet wavelet spectrum of daily rainfall rates and runoffs of Seklafa system were calculated and shown in Fig. 2. At small scales, the Morlet wavelet spectrum of rainfall rates highlight temporally processes at high frequencies, these spectra show an aliasing at small scale because the selected wavelet is probably not enough sampled at this scale. However, these structures are less visible on the spectra corresponding to the runoffs, which indicate the modulating effect of aquifer systems. For the Kerakda system (Fig. 3a-f), on a larger scale, the spectra reveal the presence of certain processes more or less localized in time. The Morlet

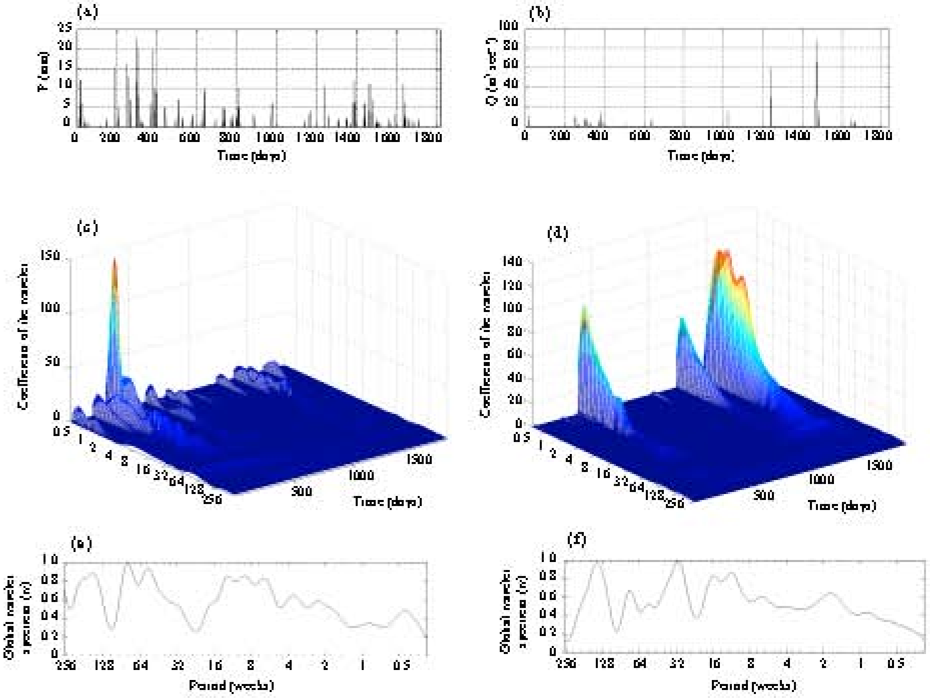

Fig. 2: (a, b) daily rainfall rates and runoff measured at the outlet of Seklafa system, (c, d) Morlet wavelet spectrum and (e, f) global Morlet wavelet spectrum of the two hydrological signals

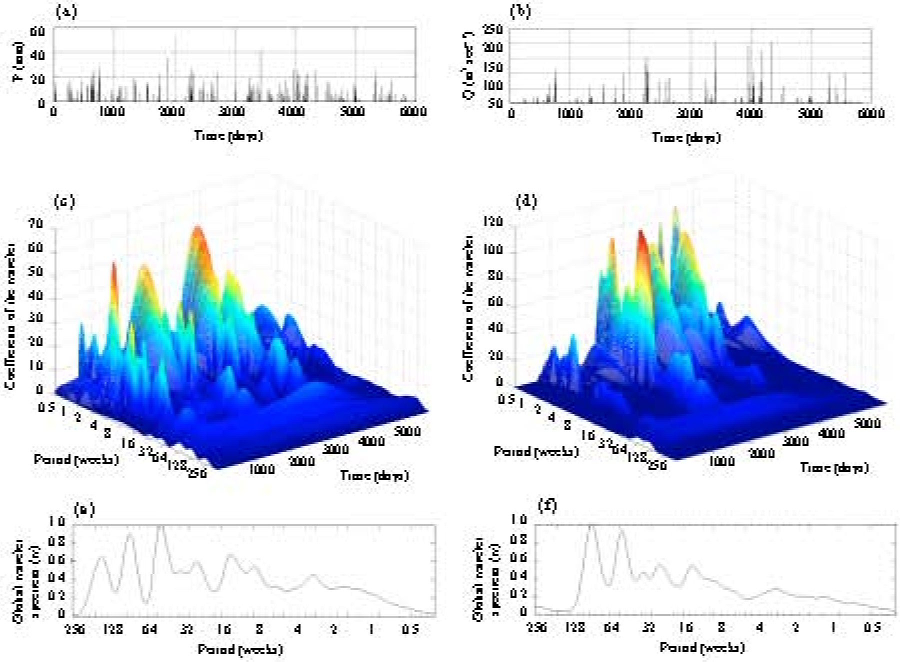

Fig. 3: (a, b) daily rainfall rates and runoff measured at the outlet of Kerakda system, (c, d) Morlet wavelet spectrum and (e, f) global Morlet wavelet spectrum of the two hydrological signals

wavelet spectrum of rainfall rates and runoff highlight large-scale components corresponding to multi-annual processes described in Mandelbrot and Wallis (1968). The Rhouiba system behavior (Fig. 4a-f) is similar to Kerakda and Seklafa.

This analysis revealed for all systems the lack of temporal structure evident in the short term and the existence of non-stationary structure in the medium and long terms.

The Morlet wavelet spectrum for runoffs clearly shows that the transition to strong wavelet coefficients resulting from a series of major floods. The influence of these floods profoundly affects the range of scales. This is a significant short-term behavior of an evolved and well drained system.

The global wavelet spectrum highlights the various components in the medium and long terms, the most spectacular are the multi-annual.

Cross-Wavelet Analysis

The Morlet wavelet cross-spectrum of the daily hydrological data systems were calculated to highlight the temporal variability of rainfall-runoff relationship.

At small scales, structures with a high coefficient are visible due to the high rainfall-runoff relationship at this scale. For the Seklafa system, the multi-annual components are not made clearly, this is linked to low internal reserves that are dependent on large-scale component of rainfall rates (Fig. 5a, b). As such, the global wavelet spectrum gives a more compact information about the different characteristic scales of the system. The amplifications of the components in the short term appear to be related to the high degree of organization of the drainage. However, the variability of the components in the medium and long terms is weakly visible and less amplified, this is linked to low ground-water resources.

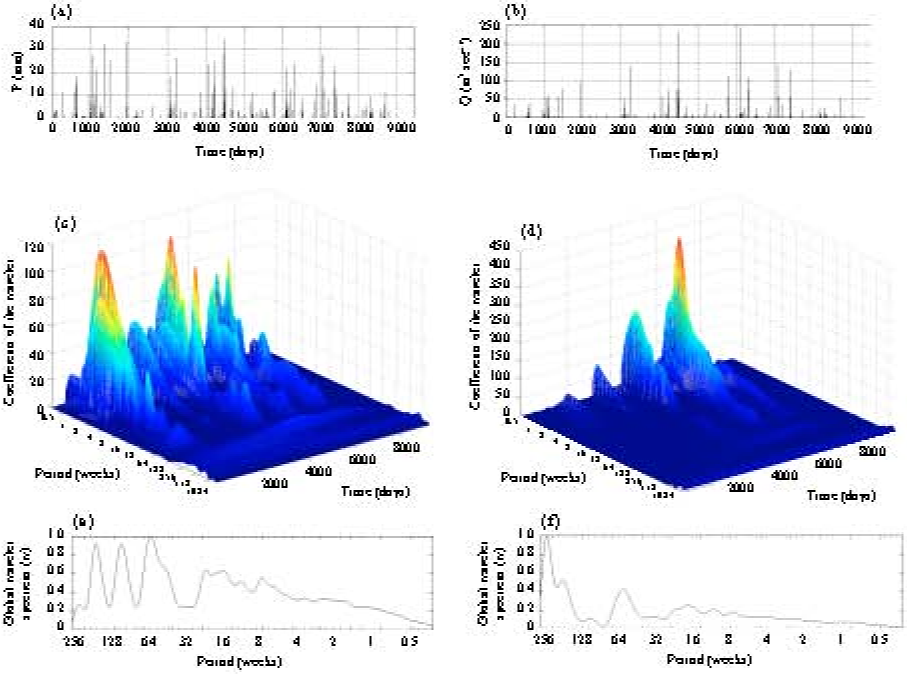

Fig. 4: (a, b), daily rainfall rates and runoff measured at the outlet of Rhouiba system, (c, d) Morlet wavelet spectrum and (e, f) global Morlet wavelet spectrum of the two hydrological signals

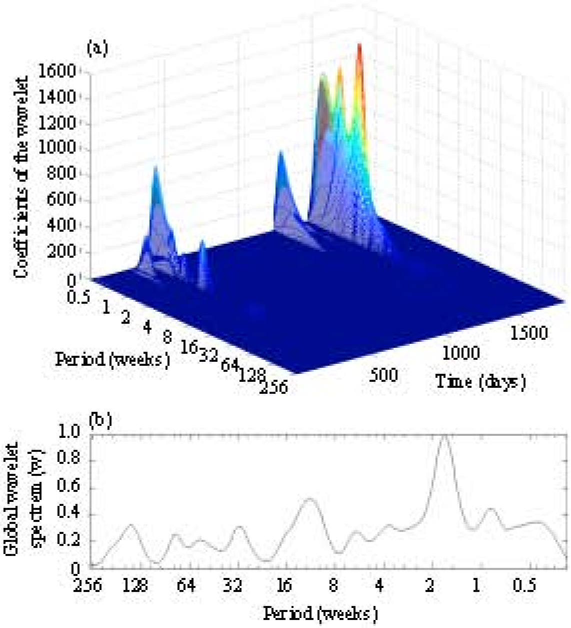

Fig. 5: (a) Morlet wavelet cross-spectrum and (b) Global Morlet wavelet cross-spectrum (Seklafa system)

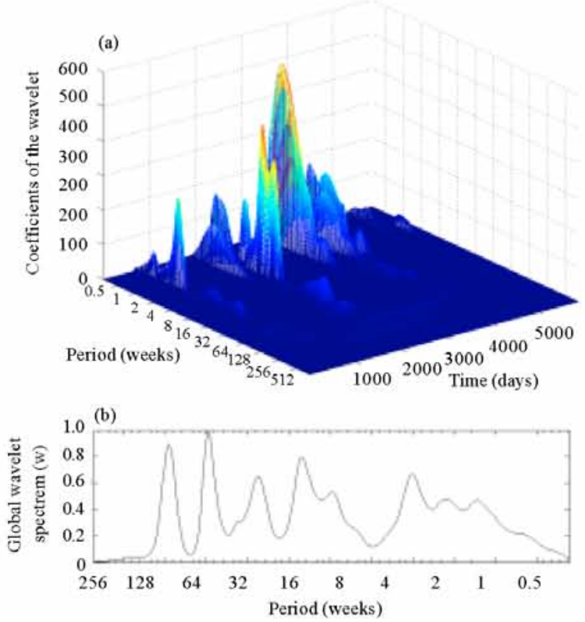

Fig. 6: (a) Morlet wavelet cross-spectrum and (b) Global Morlet wavelet cross-spectrum (Kerakda system)

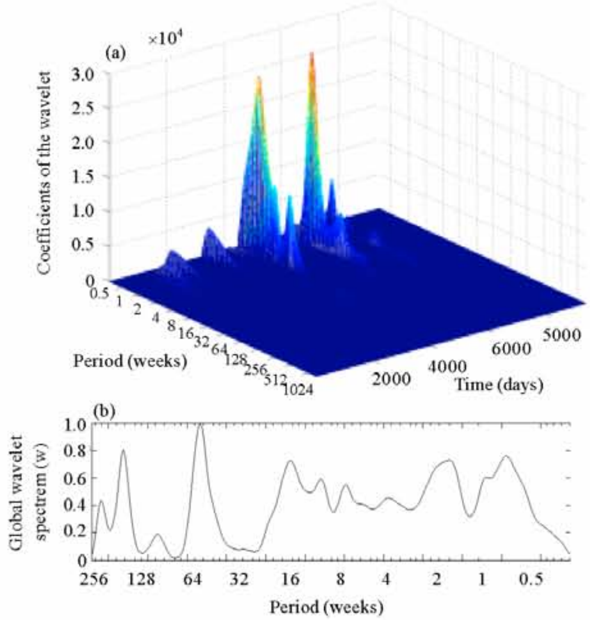

Fig. 7: (a) Morlet wavelet cross-spectrum and (b) Global Morlet wavelet cross-spectrum (Rhouiba system)

For Kerakda (Fig. 6a, b) and Rhouiba systems (Fig. 7a, b), at small-scale, their behavior is quite comparable to that of Seklafa, it is also, noted the high rainfall-runoff relationship at this scale. On the other hand, on a larger scale, the global wavelet spectrum highlights a dramatically several multi-annual components.

This analysis shows the low sensitivity of the Saharian Atlas systems to the rainfall rates of short periods, evidenced by the high rainfall-runoff relationship and the low modulation of the input signal. It also shows that the internal structure of aquifers permits to modulate the input signal and this appears as multi-annual phenomena that strongly influence the system.

The components, the most characteristic of the global wavelet cross-spectrum have been isolated to be studied using the concept of wavelet coherence. The real part to the square of the wavelet coherence and the phase of the coherence function are shown in Fig. 8, 9 and 10.

The component of 12 days was chosen for the short term process for the Seklafa system (Fig. 8a-c). We chose the component of 128 weeks for the Kerakda system (Fig. 9a-c), for Rhouiba system, we have isolated the component to 4 years for a long term (Fig. 10a-c).

For the different components, there is usually a bad coherence, since, the real part of coherence is very weak, while the phase is different from zero. This confirms the non-linearity of the rainfall-runoff relationship for our systems. The highest values of coherence correspond to consecutive rainy passages; this also shows the speed of the systems response. During periods of rain, a good agreement is found, while during dry periods no agreement is highlighted.

It appears, for the long-term that coherence component values are significant enough to the relatively wet periods and relatively low values of the phase function. It also appears, for the dry period a very low coherence but a relatively stronger phase.

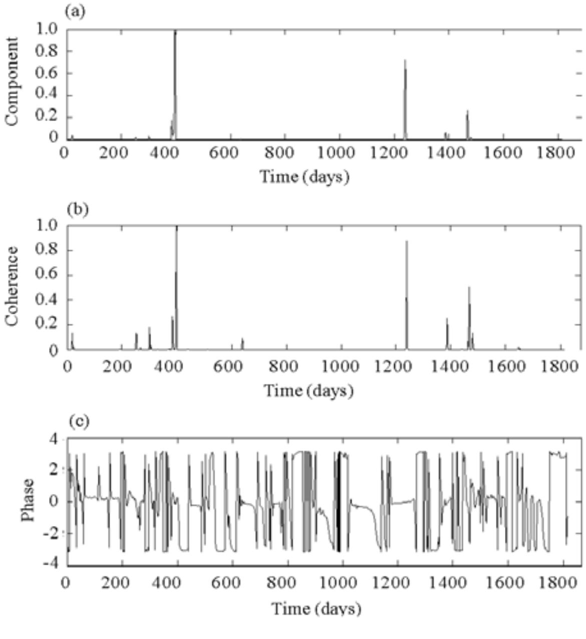

Fig. 8: (a) Component at 12 days, (b) coherence square real part and (c) phase function (Seklafa system)

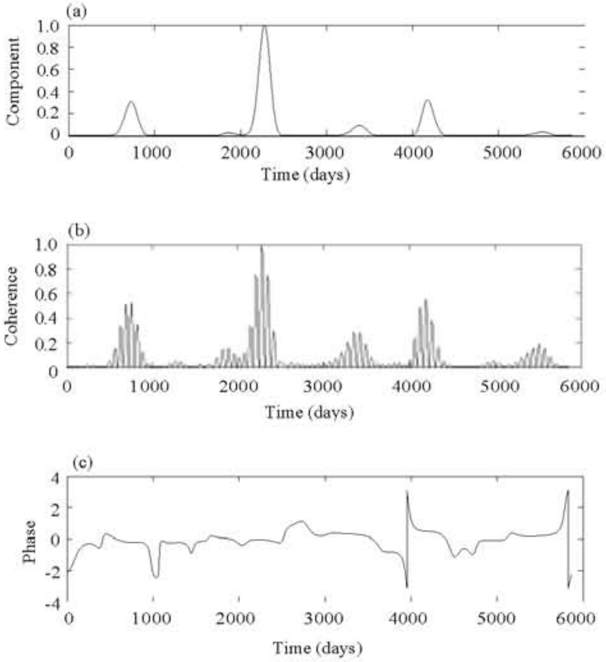

Fig. 9: (a) Component at 128 weeks, (b) coherence square real part and (c) phase function (Kerakda system)

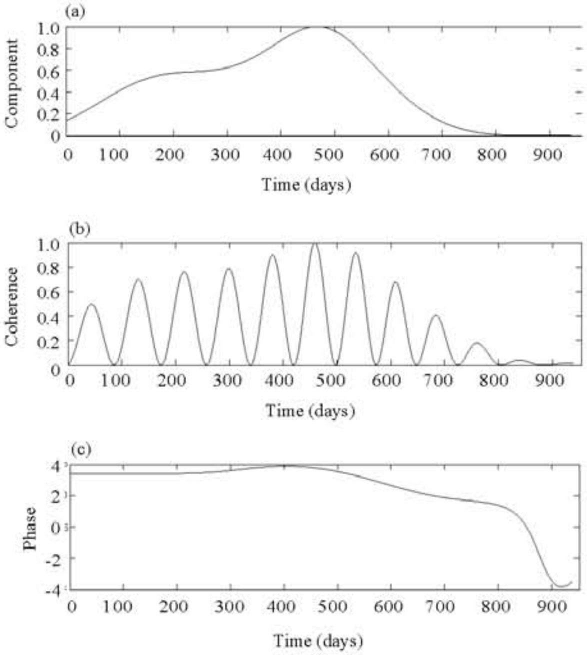

Fig. 10: (a) Component to 4 years, (b) coherence square real part and (c) phase function (Rhouiba system)

CONCLUSIONS

The continuous wavelet analysis has allowed a study of time-scale type of the rainfall rates and runoff of some aquifer systems of the Algerian Saharian Atlas and reveals their high temporal variability. It appears as a very powerful tool for the study of complex hydrological systems and complementary to the traditional functional analysis.

At daily step, the continuous wavelet analysis have revealed the variability of several characteristics components at different scales.

Runoffs are characterized by a non-stationary behavior strongly influenced by the temporal structure of rainfall rates and the lithological and structural heterogeneity of aquifers.

The Morlet wavelet cross spectrum appears as a good indicator of the degree of organization of the drainage of the aquifer systems and therefore reserves of groundwater. The poor coherence of all the studied systems confirms the non-linearity of the rainfall-runoff relationship already identified by the spectral analysis, but the highest values of coherence showed the rapid response of the systems therefore the strong drainage during the periods of rains.

Finally, Saharian atlasic aquifer systems highly heterogeneous are slightly sensitive to fluctuations in rainfall for short periods, but the multi-annual phenomena strongly influence these systems and ensure the storage of groundwater resources.

This descriptive analysis of behavior also highlights the difficulty in modeling such systems, it may, however, guide future studies to a non-linear model appropriate to this type of complex heterogeneous systems.

ACKNOWLEDGMENTS

The authors thank David Labat and Jean-Louis Dandurand for their aid and for their helpful comments. The authors also thank the National Agency of Water Resources have made available the necessary data. The principle wavelets analysis routines were provided by C. Torrence and G.P. Compo and are available at http://paos.colorado.edu/research/wavelets/.

REFERENCES

Aït Ouali, R. and J. Delfaud, 1995. Les modalités douverture du bassin des ksours au lias dans le cadre du rifting jurassique au Maghreb. C. R. Acad. Sci. Paris Série Iia., 320: 773-778.

Anctil, F. and D.G. Tape, 2004. An exploration of artificial neural network rainfall-runoff forecasting combined with wavelet decomposition. J. Environ. Eng. Sci., 3: S121-S128.

Benner, T., 1999. Central england temperatures : Long-term variability and teleconnections. Int. J. Clim., 19: 391-403.

Chen, X. and D. Liu, 2008. Wavelet analysis on inter-annual variation of precipitation in Guangdong, China. IAHS, 319: 3-9.

Chettih, M. and M. Mesbah, 2006. Use of the correlation and spectral analysis to infer the structure and the hydrodynamic behavior of the saharian atlas aquifers. Bull. Serv. Geol. Nat., 17: 145-159.

Chou, C.M., 2007. Applying multi-resolution analysis to differential hydrological grey models with dual series. J. Hydr., 332: 174-186.

Daubechies, I., 1992. Ten Lectures on Wavelets. Society for Industrial and Applied Mathematics, Philadelphia, PA., ISBN: 0-89871-274-2, pp: 354.

Goupillaud, P., A. Grossman and J. Morlet, 1984. Cycles-octaves and related transforms in seismic signal analysis. Geoexploration, 23: 85-102.

Grossmann, A. and J. Morlet, 1984. Decomposition of hardy functions into square integrable wavelets of constant shape. SIAM. J. Math. Anal., 15: 723-736.

Henderson, R.D., F.D. Day-Lewis and C.F. Harvey, 2009. Investigation of aquifer-estuary interaction using wavelet analysis of fiber-optic temperature data. Geophys. Res. Lett., 36: 1-6.

Higuchi, K., J. Huang and A. Shabbar, 1999. A wavelet characterization of the North Atlantic Oscillation variation and its relationship to the north atlantic sea surface temperature. Int. J. Clim., 19: 1119-1129.

Huang, Z. and M. Hiroshi, 2008. Wavelet based fractal analysis of El Nino/La Nina episodes 70-74. Hydr. Res. Lett., 2: 70-74.

Kang, S. and H. Lin, 2007. Wavelet analysis of hydrological and water quality signals in an agricultural watershed. J. Hydr., 338: 1-14.

Kumar, P., 1996. Role of coherent structure in the stochastic dynamic variability of precipitation. J. Geophys. Res., 101: 393-404.

Labat, D., R. Ababou and A. Mangin, 2000. Rainfall-runoff relation for karstic springs. Part II: Continuous wavelet and discrete orthogonal multiresolution analyses. J. Hydrol., 238: 149-178.

Labat, D., 2005. Recent advances in wavelet analyses: Part 1. A review of concepts. J. Hydr., 314: 275-288.

Lane, S.N., 2007. Assessment of rainfall-runoff models based upon wavelet analysis. Hydr. Proc., 21: 586-607.

Liu, P., 1995. Wavelet Spectrum Analysis and Ocean Wind Waves. In: Wavelets in Geophysics, Foufoula-Georgiou, E. and P. Kumar (Eds.). Academic Press, New York, pp: 151-166.

Mandelbrot, B. and J. Wallis, 1968. Noah, joseph and operational hydrology. Water Resour. Res., 4: 909-918.

Mathevet, T., M. Lepiller and A. Mangin, 2004. Application of time-series analyses to the hydrological functioning of an alpine karstic system: The case of Bange-LEau-Morte. Hydrol. Earth Syst. Sci., 8: 1051-1064.

Morlet, J., G. Arens, E. Fourgeau and D. Girard, 1982. Wave propagation and sampling theory 1: Complex signal and scattering in multilayered media. Geophysics, 47: 203-221.

Smith, L.C., D.L. Turcotte and B.L. Isacks, 1998. Stream flow characterization and feature detection using a discrete wavelet transform. Hydr. Proc., 12: 233-249.

Szilagyi, J., M.B. Parlange, G.G. Katul and J.D. Albertson, 1999. An objective method for determining principal time scales of coherent eddy structures using orthogonal wavelets. Adv. Water Resour., 22: 561-566.

Torrence, C. and G. Compo, 1998. A practical guide to wavelet analysis. Bull. Am. Meteorol. Soc., 79: 61-78.

Torrésani, B., 1995. Analyse Continue Par Ondelettes. CNRS., Canada, UK.

Wang, J. and J. Meng, 2007. Research on runoff variations based on wavelet analysis and wavelet neural network model: A case study of the Heihe River drainage basin (1944-2005). J. Geographical Sci., 17: 327-338.

Wilson, R.C. and A.V. Mordvinov, 1999. Time-frequency analysis of total solar irradiance variations. Geophys. Res. Lett., 26: 3613-3616.

Xue, X.J., S. Huang, Q. Huang and Y. Lei, 2008. The application of wavelet analysis in hydrological sequence trend analysis. Proceedings of the 2nd International Conference on Bioinformatics Biomedical Engineering, May 16-18, Shanghai, pp: 3495-3498.