Research Journal of Environmental Sciences

ISSN 1819-3412

Academic Journals Inc.

www.academicjournals.com

Research Journal of Environmental Sciences 5 (4): 386-393, 2011
ISSN 1819-3412 / DOI: 10.3923/rjes.2011.386.393

Design of Barotropic Spectral Model for Predicting of Vorticity Field

[1]P. Ghafarian, [2]M.T. Zamanian and [3]T. Haghroosta
[1]Iran Meteorological Organization, Tehran, Iran
[2]Atmospheric Science and Meteorological Research Center, Tehran, Iran
[3]Department of Oceanography, Islamic Azad University, Tehran North Branch, Tehran, Iran

Corresponding Author: Tahereh Haghroosta, Department of Oceanography, Islamic Azad University, Tehran North Branch, Tehran, Iran

ABSTRACT
In this study a global model, with the spectral method, is designed for predicting the vorticity field. To obtain the global coverage, the spherical harmonics which are the basic functions in present study, are used. Initial data is entered by bilinear interpolation on the Gaussian grid (which is an equidistant grid). The functions are expanded in terms of the wave numbers, then the triangular truncation is applied and with the aid of the Fourier and Legendre transformations, physical space is transformed to the spectral space. The vorticity field is predicted by the leapfrog scheme and then by an inverse transformation, the physical space is obtained from the spectral space, then by using the inverses of this transformation, we return to physical space. Running the model has good results, especially in mid-latitudes. The results of running this model show that the Real vorticity and the prediction vorticity are compatible in order, value and sign point of view.

Key words: Spectral space, spherical harmonics, legendre transformations, bilinear interpolation, triangular truncation

INTRODUCTION
Since, Fourier (1822), spectral method was used in analytical study of differential equations and using numerical method for solving differential equations refers to Lanczos period (1938). There are three methods of Spectral modeling: Tau, Pseudo spectral and Galerkin (Fornberg,1998). Spectral method, applies in turbulence modeling, trembling detections, nonlinear wave equations, astrology and weather prediction. At the first time, Spectral method in meteorology modeling was presented by Silberman (1954). He solved no divergent barotropic vorticity equation on a geometrical sphere. The first metrological centers that used spectral programs in their own predictions, were in Australia and Canada (1976) and next in America (1980), France (1982), Japan and European Center for Medium range Weather Forecast (1983).

In its earlier days, the spectral method was particularly suitable for low-resolution simple models. The equations of these simple models involved nonlinear terms evaluated at each time step. Evaluation of the nonlinear terms was performed using the interaction coefficient method and thus required large memory allocations, which was an undesirable proposition. However, with the introduction of the transform method, developed in dependently by Eliasen *et al.* (1970) and Orszag (1970), the method for evaluation of these nonlinear terms has changed completely. This transform method also made it feasible to include no adiabatic effects in the model equations. For the past couple of decades.

The spectral method has become an increasingly popular technique for studies of general circulation and numerical weather prediction at the operational and research centers.

Essential problem in many discretion methods is covering all the earth with harmonic networks (Haltiner and Williams, 1980). With using spectral methods based on spherical harmonics, it will be possible to globalize this model. In differential limited method, it is difficult to design this network in polar region. This problem has been solved and now it is an advantage.

In this study, global barotropic model is designed in spectral method (Galerkin) and with this model; vorticity is predicted for 12 h. In definition view, vorticity is an amount of wind rotation or every other flow in unit of area. Cyclone regions have positive vorticity and anticyclone regions have negative vorticity. In some regions, where the vorticity is positive and has a magnitude, strong cyclones will be developed.

In the atmosphere and ocean, barotropic environment is a kind of hypothetical environment. The atmosphere is heated by radiation of the earth. Everywhere, warmer air is caused by more radiation, so there is a horizontal temperature gradient and also horizontal density gradient (Holton, 1992). For easy to use, it is supposed that the environment is barotropic. With this conception, the barotropic model could be used in oceans and shallow waters. On the other hand, barotropic environment is where that the horizontal density changes (Robert, 1968).

The aim of this study is designing and testing a global model for predicting meteorological parameters in Iran country. At first step, it is supposed that the environment is barotropic and then the model is designed, which its output is just one parameter. In next studies and developing the model, it is expected that it is used by meteorological forecasters.

MATERIALS AND METHODS

In this study, the model is designed for whole of the earth as a global model, thus every location can be considered. For instance, accuracy of this model is achieved by running model in Tehran in 08.02.2005 when there were a heavy snow and the results are compared with real data.

Definition of spectral method: Spectral method is a numerical method for solving differential equations. In this method, relative variables are expanded in series which are in orthogonal functions (Fourier series are used).

Main equations are converted to a series of normal differential equations (which its coefficients depend on time). This method is used for global modeling. In this modeling, spherical harmonic equations are used, which are more complex than Fourier series. This model predicts atmospheric parameters in a spectral space and then converts them to a physical space for showing.

Galerkin method: Limited differential methods are used in solving differential equations with special points in space and time and Tailor series approximations are used for derivation these equations. Galerkin method presents dependant variables as a sum of some functions that appoint spatial structure. These function's coefficients generally depend on time. This method converts differential equations with partial derivations to a set of general differential equations. There are two useful ways in Galerkin method, spectral method and limited elements method. Spectral method uses orthogonal functions and it has been using in meteorological science for many years. Limited elements method is used in engineering and oceanography (Krishnanamurti, 1998).

Mathematical profile of spectral models: It can be shown that every flat function over a sphere, as a set of spherical harmonics:

$$\zeta(\lambda, \mu) = \sum_m \sum_n \zeta_n^m Y_n^m(\lambda, \mu) \quad (1)$$

Where:

$$Y_n^m(\mu, \lambda) = P_n^m(\mu)\, e^{im\lambda} \quad (2)$$

Spherical harmonic y^m_n (μ, λ) is in m order and n grad. Factor $e^{im\lambda}$ explains west-east changes and factor p^m_n (μ) shows North-South changes in Spherical harmonic. In fact the answer of Laplace equation on a sphere is like Eq. 2.

Global spectral model is formulized with using global spherical harmonics which are fundamental functions. Spherical harmonics consist of trigonometry functions in orbital direction and Legendre associated functions in Meridional direction.

$$P_n^m(\mu) = \frac{(1-\mu^2)^{m/2}}{2^n n!} \frac{d^{n+m}}{d\mu^{n+m}} (\mu^2 - 1)^n, \quad |\mu| \le 1 \quad (3)$$

where $\mu = \sin\phi$, m is wave number in orbital direction and n is wave number in Meridional direction.

Two transformations, Fourier and Legendre, are for moving from physical space to spherical space.

Fourier transformation: Spatial Fourier transformation is done in a latitude direction:

$$A^m(\mu) = \frac{1}{2\pi} \int_0^{2\pi} A(\lambda, \mu)\, e^{-im\lambda}\, d\lambda \quad (4)$$

Legendre transformation: Legendre transformation of Fourier elements is done as follows:

$$A_n^m = \int A^m(\mu)\, P_n^m(\mu) d\mu \quad (5)$$

With Gaussian integration, Eq. 5 is expanded as follows:

$$A_n^m = \frac{1}{2} \sum_{k=1}^{K} W(\mu_k)\, A^m(\mu_k)\, P_n^m(\mu_k) \quad (6)$$

where μ_k and w (μk) are, respectively Gaussian width and Gaussian weight.

Thus, at the first it should be noticed to spherical harmonic element A^m_n, Fourier element A A^m and network points values (λ_j, μ^k). It must be mentioned that Gaussian width and Gaussian weight are caused that Legendre conversion is correctly done.

As it is explained, every variables such as (ψ, ζ, ...) can be shown in a series of spherical harmonics figure. In fact working with sentences which are unlimited series, is impossible and it is necessary to do truncation in some points and to ignore some waves which are out of this spectral area. In

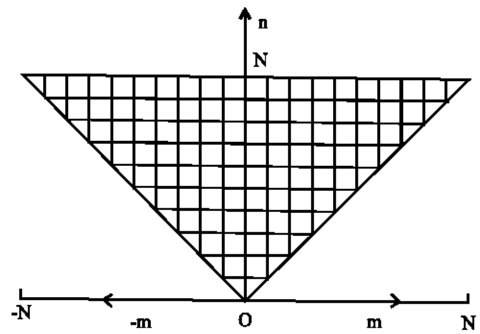

Fig. 1: Triangular truncation scheme in wave number N

designing this model, triangular truncation method has been used. This truncation is explained as follows:

$$\psi(\lambda, \mu) = \sum_{m=-N}^{N} \sum_{n=|m|}^{N} \psi_n^m Y_n^m(\lambda, \mu) \tag{7}$$

This formula is entirely shown in Fig. 1.

It is so complex that select the optimum kind of truncation in a spectral model (chu *et al*., 2000). Totally selection must be done base on standard condition which have maximum accurate and minimum time in computation (Ooyama, 2002). This selection depends on many factors, for example: the model, prediction time, the computer, programming and initial conditions.

Initial data and method: The solutions of continues nonlinear equations, which describe atmosphere movements and are used in a meteorology spectral method and climatology, need expanded approximations that are obtained by triangular truncation's different levels, which are named: T-21, T-42, T-63 and etc. Each name defines cutting type and value. If T-21 model is used, our network will be in an equal distance in 5.6 degree and if T-42 model is used, our network will be in an equal distance in 2.8 degree. In fact, the difference between these models will be the number of network's points (resolution).

It is clear that the higher resolution, the better solutions in running model. Gaussian network is operational in geographical science and meteorology. As a matter of fact, the models that are designed on a sphere need use this network. Gaussian network doesn't have any points on the polar and this network, the number of points in orbital direction is twice of the number of points in Meridional direction.

For instance, in T-42 there are 64 latitude points and 128 longitude points. The distance between points in Meridional direction is exactly the same but in orbital direction that is nearly the same. It should be mentioned that in this study initial data is in 500 hPa level. It should be noticed to data time and data limitation which have to be on the whole of earth. This data takes place on the network points with resolution level in 1 degree. Initial data for 500 hPa level is saved in binary files which are named gfs.t00z.pgrbanl and gfs.t12z.pgrbanl. t00 means time data at 00:00 UTC and t12 means time data at 12:00 and this data is real data and not prediction data because it has achieved with observations.

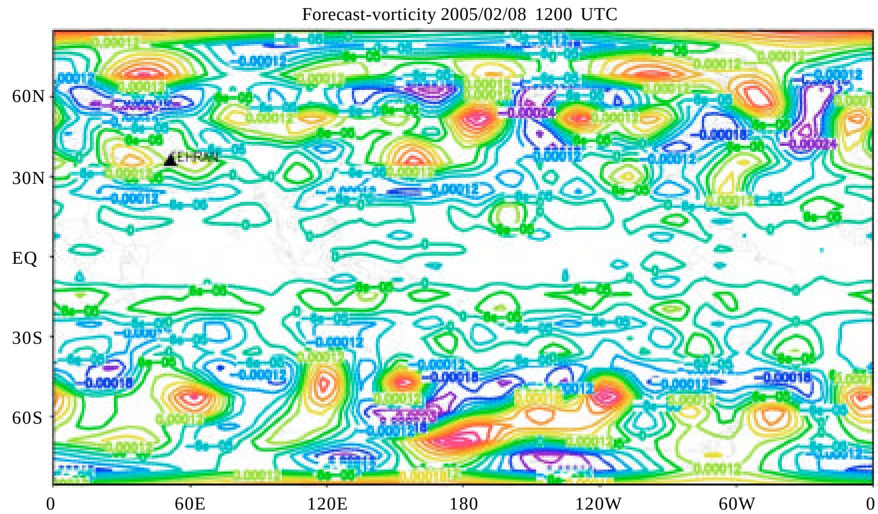

Fig. 2: Model forecast vorticity Global scheme, 500 hPa level, Tuesday (08.02.2005), 12:00 Coordinated universal time (UTC)

This data is recovered with WGRIB software and also is saved in text files. Bilinear interpolation command (Lagrange) is used for obtaining data's value on the Gaussian network points.

In this study, because of unavailability to a super computer, T-21 model (low resolution) should be designed.

In practical method, the parameter which is selected for predicting by bilinear interpolation is transferred on the Gaussian network. Then spectral coefficients are obtained and also

$$\psi_n^m(t+\Delta t)$$

values are calculated for next time step, with using:

$$\frac{d\psi_n^m}{dt}$$

and

$$\psi_n^m(t-\Delta t)$$

as follows:

$$\psi_n^m(t+\Delta t)=\psi_n^m(t-\Delta t)+2\,\Delta t\frac{d\psi_n^m}{dt} \quad (8)$$

Also in this model, Leapfrog scheme (three level schemes) is used.

As it is seen in the Eq. 8, the way that is used for time spreading is a two steps method and using this method for the first time step is impossible (Durran, 1999). Thus it must be used one step method in the first time step of prediction and for the other time steps, three level-central methods is used, thus forward-central scheme must be used. In fact prediction is done in spectral space and with using vice versa of this transform; parameter's values are achieved on the network points.

RESULTS AND DISCUSSION

In this section, we investigated the solutions of this model running and then compared them with real values. Real values of vorticity were received and were drawn from the NCEP-NCAR data center. The time was selected 08.02.2005 because in this time, Iran was affected by strong atmosphere system.

To be sure of accurate of model's operation, real vorticity and also the output of vorticity by running model were drawn by GRADS software. The model was run in 0000UTC time 08.02.2005 and the vorticity field was predicted for next 12 h. Because this model is a global model, whole of earth is shown in maps. As you see in Fig. 2, accuracy of this model is the same in northern and southern hemisphere and there are not any divergence problems in polar, which are already seen in limited differential methods. The weakness of this field in tropical regions is clear in Fig. 2. For more clearness in this figure, Tehran latitude and longitude were shown with solid triangle sign. Also the order of vorticity shows the power of atmospheric system over Iran. The vorticity value over Tehran was predicted $3.4e^{-05}$ s^{-1}, while real value over Tehran is $2.9e^{-05}$ s^{-1} that it shows that this model is very strong. Sign and order were the same and error values were caused by assumptions which were used in this model designing. The results of this study can be compared with real one (Fig. 3).

To compare real maps and model output maps, we conclude that, the environment is baroclinic, but the result is entirely satisfactory. Real vorticity and prediction vorticity affected by running model are the same in grad, value and sign view. As we work in spectral area, every kind of time step is selectable. Thus the model is also stable with selecting every optional time step. Of course changing in time step from 2400 seconds to 240 seconds makes a better result especially in polar. In this model, the problems of the boundary conditions that there are in the most numerical models,

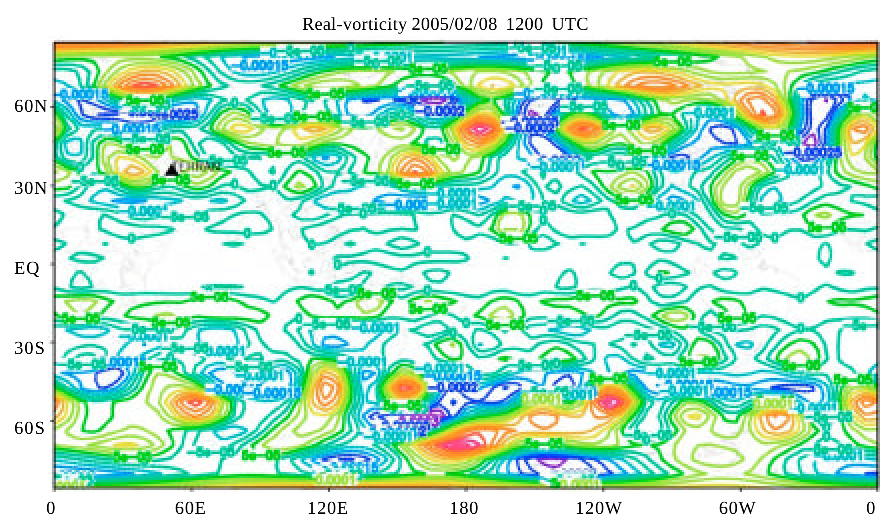

Fig. 3: Global vorticity scheme, 500-hPa level, Tuesday (08.02.2005), 12:00 Coordinated universal time (UTC)

are solved, this is similar to Haugen and Machenhauer (1993) studies. They solved the lateral boundary problem by extending the domain beyond lateral boundaries to handle the periodicity of sine and cosine basis functions (Krishnanamurti, 1998).

It must be noticed that Global model designing with high resolution, the more time will be needed. For instance, if T-21 model is replaced with T-42 model, because of the resolution is 4 times, the program running time will be four times. The retention in the spectral viscosity solution of high-wave number information allows the successful application of high-resolution post processing methods (Gelb and Gleeson, 2001).

A global model was designed by European Centre for Medium-Range Weather Forecasts (ECMWF) which was not in barotropic system and was run in high resolution mode (Palmer *et al.*, 1990). The accuracy of that model is very high and predicts many atmosphere parameters such as: pressure, temperature, precipitation and etc for different atmosphere levels.

For having progress in this model, it needs to use super computers and to have an extended work group study.

This model is just based on constancy absolute vorticity and it just predicts vorticity. In fact this model designing is the first step for running global models that all the meteorology parameters can be predicted for Iran. Also, it is suggested that according to existing quick processor system, the model should be run with higher resolution. This model can help us to design a global model for the atmosphere with attention to shallow water (Chu *et al.*, 2000). The model can be expanded for predicting cloud formation procedure, boundary layer procedure and etc. and also it will be execute in forecasting centers.

ACKNOWLEDGMENT

The authors would like to thank the Iran Meteorological Organization (IMO) for financial support.

REFERENCES

Chu, P.C., C. Xiong-Shan and F. Chenwu, 2000. Comparison between wave number truncation and horizontal diffusion in spectral models. Monthly Weather Review, pp: 77.

Durran, D.R., 1999. Numerical Methods for Wave Equation in Geophysical Fluid Dynamics. Springer, New York, pp: 465.

Eliasen, E., B. Machenhauer and E. Rasmussen, 1970. On a numerical method for integration of the hydrodynamical equations with a spectral representation of the horizontal fields. Report No. 2, Institut for Teoretisk Meteorologi, Kobenhavns Universitet, Denmark.

Fornberg, B., 1998. A Practical Guide to Pseudospectral Methods. Cambridge University Press, Cambridge, pp: 217.

Gelb, A. and J. Gleeson, 2001. Spectral viscosity for shallow water equations in spherical geometry. Monthly Weather Review, pp: 15.

Haltiner, G.J. and R.T. Williams, 1980. Numerical Prediction and Dynamic Meteorology. 2nd Edn., Wiley and Sons, New York.

Haugen, J.E. and B. Machenhauer, 1993. A spectral limited-area model formulation with time-dependent boundary conditions applied to the shallow-water equations. Monthly Weather Rev., 121: 2618-2630.

Holton, J., 1992. An Introduction to Dynamic Meteorology. Academic Press, New York, pp: 511.

Krishnanamurti, T.N., 1998. An Introduction to Global Spectral Modeling. Oxford University Press, Oxford, pp: 253.

Ooyama, K.V., 2002. The cubic splines transform method: Basic definitions and tests in a 1D single domain. Monthly Weather Rev., 130: 2392-2415.

Orszag, S.A., 1970. Transform method for the calculation of vector-coupled sums: Application to the spectral form of the vorticity equation. J. Atmospheric Sci., 27: 890-895.

Palmer, T.N., C. Brankovic, F. Molteni, S. Tibaldi and L. Ferranti *et al*., 1990. The European Centre for Medium-Range Weather Forecasts (ECMWF) program on extended range prediction. Bull. Am. Meteorol. Soc., 71: 1317-1330.

Robert, A.J., 1968. Integration of a spectral barotropic model from global 500-MB charts. Monthly Weather Rev., 96: 83-83.

Silberman, I., 1954. Planetary waves in the atmosphere. J. Meteorol., 11: 27-34.