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Research Article
New Explicit Form of Green and Ampt Model for Cumulative Infiltration Estimation

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Abstract

Background and Objective: The precise knowledge of the wet front during groundwater infiltration is necessary in order to estimate infiltration problems in irrigation, in natural or artificial recharge of aquifers as well as in water resources management. The model of Green and Ampt (GA) is a simplified natural form of the one-dimensional vertical infiltration problem and it is used systematically for the estimation of the water front in general. It is proved that the model of Green and Ampt is exact for a medium in which the moisture diffusivity is a Dirac function (delta-function medium). The purpose of this study was to present a new explicit approximate solution of the GA model, that can be used for small as well as for large values of time and cumulative infiltration. **Materials and Methods:** Because of the implicit form of the GA model, many approximate solutions have been developed, with various degrees of precision. In the present paper, a new explicit approximate solution of the GA model is presented, based on models of Philip and Valiantzas, which are improved by introducing new parameters and by using the conjugate gradient method. **Results:** The comparison of the new model with other approximate models, with the GA model, as well as with experimental data, shows that it confronts very well to practical problems of Hydraulics. **Conclusion:** The new model gives the possibility to be used not only in border irrigation problems but also in problems of artificial or natural recharge of aquifers and-more generally-water resources management in respect to other models.

Key words: Infiltration, large times, explicit equation, experimental results, Green and Ampt models

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INTRODUCTION

Infiltration is one common physical phenomenon describing the process of water entering the soil through the soil surface and towards the interior of earth. The prediction of the infiltration process constitutes a determining factor in the design of irrigation networks, in the artificial and natural recharge of aquifers and in the management of water resources of a region. Subsequently, it is of great interest to many researchers and for this reason Green and Ampt proposed their well-known model to deal with the problem[1]. This model presupposes constant adsorption in the wet front, constant hydraulic conductivity behind the front, uniform moisture content and a ponded soil with an initial ponding head at the surface. As far as the movement of water into unsaturated soil is concerned, Richards[2] proposed the equation that holds his name. Philip[3,4], proposed various solutions for the Richards equation, as well as the term sorptivity of the soil, considering it as the natural property of the soil to absorb water under the influence of capillary forces. Philip[4], also solved the linearized Richards equation and proposed analytical solutions for the cumulative infiltration, which are valid for small and high values of time.

The exact knowledge of the water front position in the soil is useful for the understanding of the one-dimensional process of infiltration in border irrigation, which is widely used in many countries like China[5]. It is to be noted, that in the case of border irrigation the root zone is assumed <100 cm deep during crop growth and the dimensionless cumulative infiltration ranged[6] from 0-20. So, all the calculations are in accordance with these limitations, while for larger values of time and cumulative infiltration the accuracy of their model diminishes. The water front position is also important in problems of artificial or natural recharge of aquifers and water resources management. Nevertheless, direct measurement of the water front is difficult. Many studies developed models for the estimation of the front, with empirical and natural methods[7,8]. The empirical models are usually based on experimental data in the field or in the laboratory, or in a simple equation, under restricted conditions[7,8]. Consequently, there is no empirical method available that fully describes the development of the process of infiltration of the water front in a general manner. The natural models describe in detail the infiltration process and the Richards model, as well as the Green and Ampt model[1], are widely used for the simulation of the infiltration process. The solution of the Richards model involves an implicit approximate numerical process with small-scale discretization in space and time, which makes this model somewhat unwieldy for calculations[9,10]. Philip[11] showed that the Green and Ampt model, would hold exactly for soils in which the diffusivity coefficient can be represented as a Delta function. In these cases, the soil is called a delta function soil.

The Green and Ampt model[1] supposes that a sharp wetting front separates the water front to a superior saturated zone and to an underlying unsaturated one. It is essentially an equation consisted of an algebraic and a logarithmic term, in which these terms are used for an implicit representation of the cumulative infiltration volume[7,12]. The solution of this equation is achieved by iterative processes and this is a disadvantage because it does not provide immediate solution.

To avoid the iterative process, many explicit equations of approaching the Green and Ampt model[1] have been developed. Some of these models are very complicated and improper for practical use and others provide reduced accuracy in estimating the cumulative infiltration. Philip[4] proposed a simple model of cumulative infiltration as a function of time with 2 parameters A and B. Youngs proposed for parameter A the sorptivity S and for parameter B the value ($1/3<K_S<2/3$), where, K_S is the hydraulic saturated conductivity[13]. Parlange proposed a model using a delta-function for the problem[14]. Barry *et al*.[15] departed from the Richards equation and after simplifications and with the help of the Lambert W function, they concluded to an equation similar to that of Green and Ampt, without considering sharp wetting front as in the case of the Green and Ampt equation. It is to be noted that their solution is valid for the condition of an arbitrary moisture tension imposed at the soil surface. Stone *et al*.[12]. expanded the Green and Ampt equation in a Taylor series and use the first two terms of the series. They suppose that the steady infiltration rate is equal to the saturated hydraulic conductivity, which is in contradiction with actual conditions.

More recently, Serrano provided an explicit solution of the Green and Ampt model based on the application of the Adomian method[16]. He also presented a comparison with the exact implicit solution as well as with the solution involving the Lambert W function, with the cumulative infiltration function, as well as with infiltration rate. Serrano model is very complicated and infiltration rate presents certain errors of 3%. The use of Lambert W function also allowed Barry *et al*.[17] to create explicit analytical expressions of the Green and Ampt equation. The solution they presented consists of algorithms with which approximations of the GA formula are achieved, with a certain precision each time. The W function has an asymptotic expansion whose coefficients are defined in terms of Stirling cycle numbers. This makes the solution very complicated. Mailapalli *et al*.[18] presented an explicit iterative algorithm that provides for high precision in the solution of the GA equation. They exposed a nonstandard explicit integration algorithm, which is iterative and complicated for practical use and their solution presents precision after a certain number of iterations. Valiantzas[9] proposed a 2 parameter equation for the estimation of the water front and compared it with experimental results for 10 different soils with satisfactory results. His solution is precise for small times, but its accuracy diminishes for large times. Almedeij and Esen[19] presented a new model with an explicit equation for the GA model. This model modifies the equation of Mein-Larson, considering that the former underestimates the soil ponding time[8]. The new model provides satisfactory precision for relatively low times and can be used for practical problems in irrigation problems due to its simplicity. Its precision diminishes and reaches errors up to 1.54% for dimensionless times of T = 3. Nie *et al*.[5]. proposed an approximate explicit solution in accordance with the Green and Ampt model. Their model is satisfactory for small times and for irrigation problems but is problematic for large times. Ali and Islam[20] gave an explicit approximation to the GA model, using two-step curve fitting technique. Although the accuracy of their model is very good, the model is complicated and improper for practical use.

Some of the proposed models are very complicated and improper for practical use and some others provide reduced accuracy in estimating the cumulative infiltration. Additionally, other models are accurate for small times and improper for large times. For that reason, the aim of this study was to provide a new explicit equation for the approximation of the Green and Ampt model for the development in time of the depth of the water front in the case of vertical one-dimensional infiltration. This new explicit model estimates the cumulative infiltration, is very simple, accurate and approaches the GA model very close in comparison to other models. The novelty is that it can be used for small as well as for large times and values of cumulative infiltration. This quality provides the possibility for the model to be used not only in border irrigation problems but also in problems of artificial or natural recharge of aquifers and water resources management.

MATERIALS AND METHODS

The present study was carried out at the Aristotle University of Thessaloniki, between January and May, 2019.

Existing mathematical models: The Green and Ampt model[1] assumes that the soil surface is constantly covered during the process of infiltration with a ponding head h_0 and that the saturated region is separated by the unwetted one with a sharp wetting front (Fig. 1). The infiltration rate is thus given by:

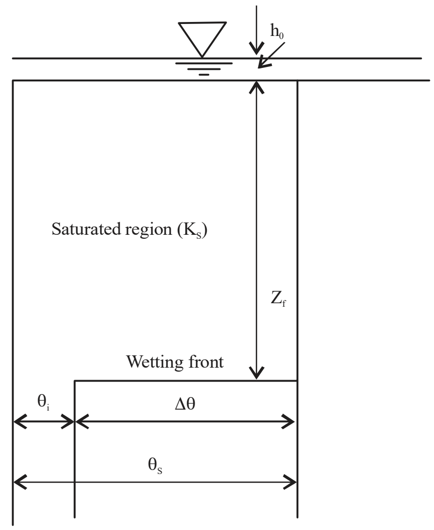

Fig. 1: Wetting front of the Green and Ampt model
Source: https://swat.tamu.edu/media/77464/i31-kuwajima.pdf

$$i = \frac{dI}{dt} = K_S + \frac{K_S(h_0 + \psi_m)\Delta\theta}{I} \quad (1)$$

Where:
K_S = Hydraulic conductivity
$\Delta\theta$ = θ_S-θ_r
θ_S = Soil volumetric moisture content in saturated conditions
θ_r = Residual volumetric moisture content
$I = Z_f\Delta\theta$ = Cumulative infiltration
Z_f = Depth of vertical infiltration front

Integrating Eq. 1, the Green and Ampt model[1] is achieved:

$$I = K_S t + \Delta\theta(h_0 + \psi_m)\ln\left(1 + \frac{I}{(h_0 + \psi_m)\Delta\theta}\right) \quad (2)$$

By introducing sorptivity[3]:

$$S = \sqrt{2K_S(h_0 + \psi_m)\Delta\theta\phi}$$

The above equation becomes:

$$I = K_S t + \frac{S^2}{2K_S}\ln\left(1 + \frac{2K_S I}{S^2}\right) \quad (3)$$

Using the variables of dimensionless time T* and dimensionless cumulative infiltration I*:

$$T^* = \frac{2K_S^2 t}{S^2},\ I^* = \frac{2K_S I}{S^2} \quad (3a)$$

The Green and Ampt model becomes:

$$I^* = T^* + \ln(1 + I^*) \quad (4)$$

Philip equation: Philip[3] showed that the Green and Ampt model[1] would be exact for soils of which diffusivity coefficient D can be represented by a delta function:

$$\begin{aligned} &D = S^2(\theta_s - \theta)\delta(\theta'), \theta_s > \theta \geq \theta_s - \varepsilon, \\ &D = 0, \theta_s - \varepsilon > \theta > \theta_s, \\ &\theta' = \theta_s - \theta \end{aligned} \quad (5)$$

where, δ is the Dirac delta function (Delta function soil). Philip[4] also proved that for the solution of the linearized Richards equation, the diffusivity coefficient is equal to:

$$D = \pi S_2/4(\theta_s - \theta_i) \quad (6)$$

And provided a solution for the cumulative infiltration of the linearized equation of Richards in a dimensionless form:

$$I^* = 0.5\left[\sqrt{\frac{T^*}{2\pi^2}}\exp\left(-\frac{T^*}{2\pi}\right) + 0.5\,\mathrm{erf}\left(\sqrt{\frac{T^*}{2\pi}}\right) - \frac{T^*}{2\pi}\mathrm{erfc}\left(\sqrt{\frac{T^*}{2\pi}}\right)\right] \quad (7)$$

For small time values Philip provided the following form:

$$I_1 = \sqrt{\frac{T_1}{\pi}} - \frac{T_1}{2} + \frac{T_1\sqrt{T_1}}{3\sqrt{\pi}} + O\left(T_1^{5/2}\right) \quad (8)$$

Where:

$$T_1 = \frac{K_S^2 t}{\pi S^2},\ I_1 = \frac{K_S I}{\pi S^2} - \frac{K_S^2 t}{\pi S^2}$$

The above Eq. 8, on the base of relations (Eq. 3a) becomes:

$$I^* = 0.5T^* + \sqrt{2T^*}\left(1 + \frac{T}{6\pi}\right) + O\left(T^{*5/2}\right) \quad (9)$$

For relatively high time values, Philip suggests:

$$I^* = T^* + \frac{\pi}{2} \quad (10)$$

The two-parameter Parlange[14] model, by replacing the logarithmic function of the GA model with an exponential one, becomes:

$$I = K_S t - \frac{S^2}{2K_S}\left\{\exp\left(-\frac{2K_S I}{S^2}\right) - 1\right\} \quad (11)$$

This model, in dimensionless variables is given by:

$$I^* = T^* - \exp(-I^*) + 1 \quad (12)$$

For large time values, the slope of the curve becomes equal to the slope of the curve of the Philip model (Eq. 10).

The Stone model[12] is given by:

$$I = K_S t + S\sqrt{t} - 0.2978\frac{S^2}{2K_S}\left(\frac{2K_S^2 t}{S^2}\right)^{0.7913} \quad (13)$$

Or, by using dimensionless variables:

$$I^* = T^* + \sqrt{2T^*} - 0.2978\left(T^*\right)^{0.7913} \quad (14)$$

The Valiantzas[9] model, is as follows:

$$I = 0.5K_S t + S\sqrt{t}\left[1 + \frac{2K_S^2 t}{S^2}/8\right]^{0.5} \quad (15)$$

Or in dimensionless variables:

$$I^* = 0.5\,T^* + \sqrt{2\,T^*}\left(1 + \frac{T^*}{8}\right)^{1/2} \quad (16)$$

The Valiantzas[9] model (Fig. 2) approaches the equation for the cumulative infiltration of the linearized Philip model (Eq. 7) very well and the mean squared error between the two equations equals 1.17×10^{-4} for dimensionless times T* up to 40. This equation, by developing the power ½ takes the following form, which is related to the Philip[4] Eq. 9:

$$I^* = 0.5T^* + \sqrt{2T^*}\left(1 + \frac{T^*}{16}\right) + O\left(T^{*5/2}\right) \quad (17)$$

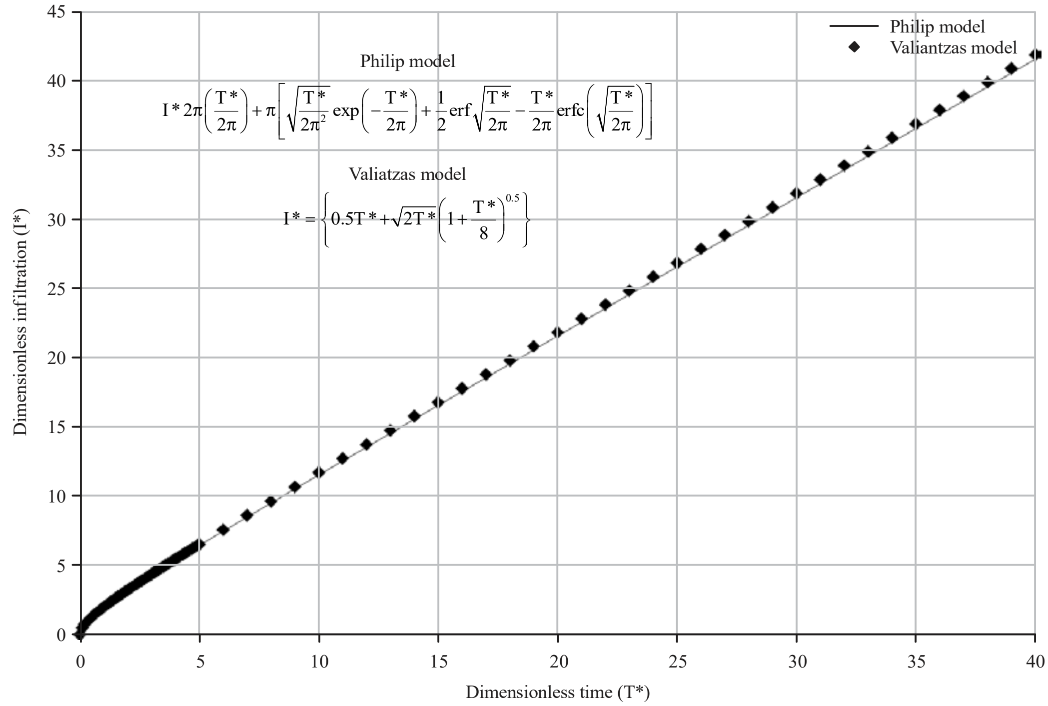

Fig. 2: Philip model compared to Valiantzas[9] model
I*: Dimensionless infiltration, T*: Dimensionless time (Eq. 3a)

The Almedeij and Esen[19] model is as follows:

$$I=\frac{1}{2}\left(K_S t+\sqrt{\frac{K_S^2 t^2}{S^2}+4K_S t}\right)+0.15K_S t \quad (18)$$

Or in non-dimensional variables:

$$I^*=\frac{1}{2}\left(T^*+\sqrt{(T^*)^2+8T^*}\right)+0.15T^*$$

And in its final form:

$$I^*=0.65T^*+\sqrt{0.25(T^*)^2+2T^*)} \quad (19)$$

Where:

$$T^*=\frac{K_S t}{(h_0+\psi_m)\Delta\theta}=\frac{2K_S^2 t}{S^2}$$

Almedeij and Esen[19] arrived in Eq. 19 from the following equation of Li *et al.*[21]:

$$I^*=\frac{1}{2}\left(T^*+\sqrt{(T^*)^2+8T^*}\right) \quad (20)$$

To which a term of 0.15T* was added. This term was derived from the difference between this equation and the exact equation of Green and Ampt (4). Although the Almedeij and Esen[19] model presents satisfactory precision for relatively low time values, it can be used satisfactorily only in practical irrigation problems, but not in water management problems, since its accuracy diminishes for large time values as shown in Fig. 3.

Mailapalli *et al.*[18] model: Mailapalli *et al.*[18] applied a nonstandard explicit iterative integration algorithm (EIA) in order to solve the Green and Ampt infiltration equation. The EIA is a nonlinear one-step method to solve initial-value problems and yields second-order accuracy. Their algorithm is:

$$I_{n+1}=I_n+\frac{hK_s\left(1+\frac{\psi_f\Delta\theta}{I_n}\right)}{2-h\left(-\frac{0.5K_s\psi_\theta\Delta\theta}{I_n^2}\right)} \quad (21)$$

The accuracy of the proposed method depends not only on the step size h, but also on the iteration number n. Accordingly this method is complicated and improper for practical use.

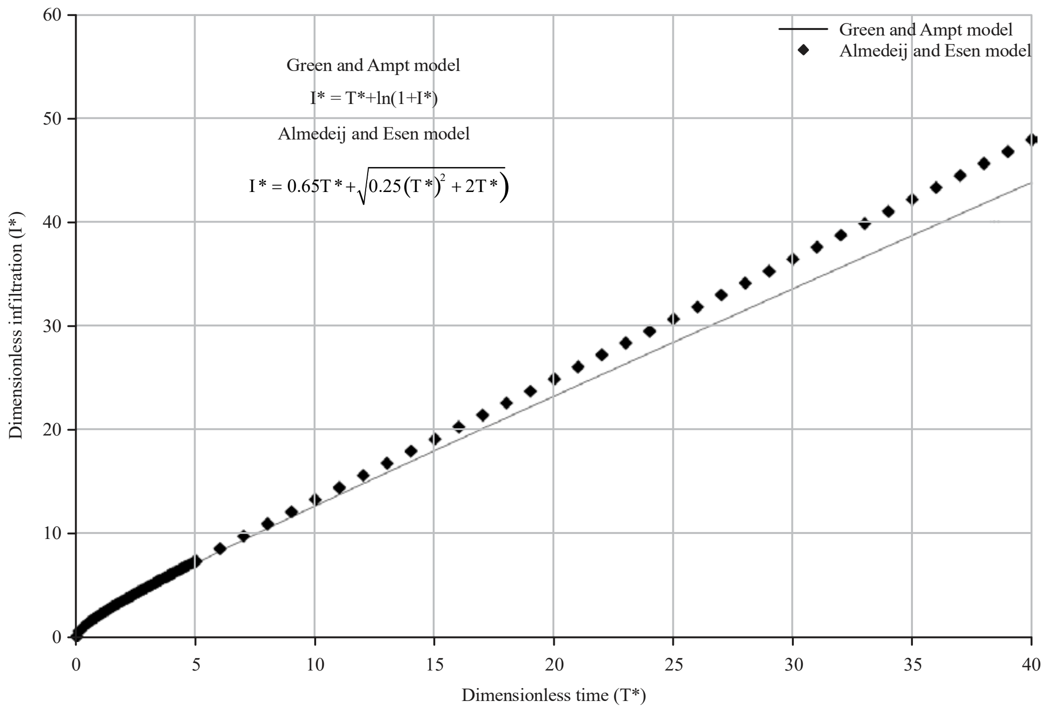

Fig. 3: Comparison of Almedeij and Esen model with Green and Ampt model

The Nie *et al.*[5] model, is expressed as:

$$I = 0.5K_s t + S\sqrt{t}\left(\sqrt{1+\frac{2K_s^2 t}{S^2}} / 8\right) + 0.1461\frac{S^2}{2K_s}\left(\frac{2K_s^2 t}{S^2}\right)^{0.788} \quad (22)$$

Or by using dimensionless variables:

$$I^* = 0.5T^* + \sqrt{2T^*}\left(1+\frac{T^*}{8}\right)^{0.5} + 0.1461\left(T^*\right)^{0.788} \quad (23)$$

Nie *et al.*[5] observed that the Valiantzas' model (Eq. 13) was missing a quantity є in comparison with the Green and Ampt model. So, they used an adaptation technique with the least squares method and added the above-mentioned term $\varepsilon = 0.1461\ (T^*)^{0.788}$.

Ali and Islam[20] model: Ali and Islam[20] gave an explicit approximation to the GA model, using 2 step curve fitting technique. The Marquardt's algorithm is employed for least-squares estimation of nonlinear parameters. Their second approximation is:

$$I^* = T^* + 2.5009\ln\left(1+0.5833\sqrt{T^*}\right)\left\{\begin{array}{l} 0.9723 + 0.0117\left[1-\exp\left(-27.36T^*\right)\right] + \\ 0.0162\left[1-\exp\left(-2.516T^*\right)\right] \end{array}\right\} \quad (24)$$

The relative error between the proposed explicit model and GA model has a maximum bound of 0.146%. Although the accuracy is very good, their model is complicated and improper for practical use.

RESULTS AND DISCUSSION

Proposed model: The afore-mentioned equations of Philip[3]:

$$I^* = 0.5T^* + \sqrt{2T^*}\left(1+\frac{T^*}{6\pi}\right) + O\left(T^{*5/2}\right)$$

And of Valiantzas[9]:

$$I^* = 0.5T^* + \sqrt{2T^*}\left(1+\frac{T^*}{16}\right) + O\left(T^{*5/2}\right)$$

are simple in their form but provide good results comparing to the Green and Ampt model[1] only for small values of time. Thus, an effort was made to use an analogous equation with new coefficients, adapted to the exact form of the Green and Ampt model. The new proposed model is of the form:

$$I^* = 0.5T^* + \sqrt{2T^*}\left(a+\frac{T^*}{b}\right)^c \quad (25)$$

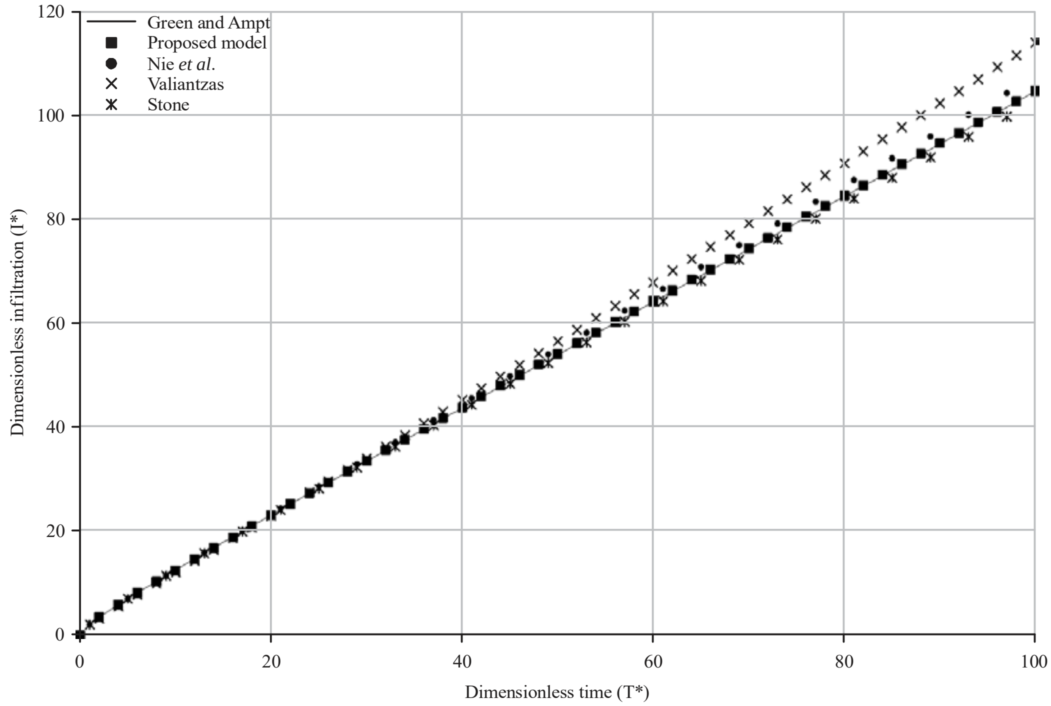

Fig. 4: Dimensionless infiltration I^* versus dimensionless time T*, as derived from the proposed model, as well as from the Green and Ampt, Nie *et al.*[5], Valiantzas and Stone models

Table 1: Reduced mean square error for all cases of the mode is mentioned above

a/a	Models	Small times	Large times
1	Nie *et al.*	2.86 E-04	3.76 E-04
2	Valiantzas	4.61 E-02	2.11 E-03
3	Stone *et al.*	1.56 E-05	1.34 E-04
4	Proposed model	4.66 E-05	1.30 E-05

where, the values of the parameters a, b, c are: a = 1.27, b = 4.85, c = 0.44. This equation is an improvement of previous equations and approaches the Green and Ampt model[1] for small as well as for large depths. The coefficients a, b, c came out of minimization of the sum of squares of the difference between the values given from the Green and Ampt model[1] and the proposed model with the help of the conjugate gradient method. For very small times T* (<0.05) these numbers take the following values: a = 0.49, b = 0.90, c = 1.01. Figure 4 presents the dimensionless infiltration (I*) versus dimensionless time (T*) derived from the proposed model as well as the models of GA, Nie *et al.*[5], Valiantzas[9] and Stone *et al.*[12]. The proposed model provides the best fit to the Green and Ampt model[1].

Divergence between the GA model and the other models: Figure 5 shows the dimensionless relative error between each model and the Green and Ampt model[1]. For dimensionless times higher than 40, the model of Stone has divergence higher than 1%, while the model of Nie *et al.*[5] presents the same divergence for T*>20. For even larger times the divergence of these models attains values of 3%. The Valiantzas model presents divergence values up to 8.5% for small times and diminishes gradually until the value of 3%. The proposed model presents lower divergence than the other ones, which takes a maximum value of 5.7% for T* = 6 and subsequently falls under 2% for all values of time. Table 1 presents the reduced mean squared error for all cases of the models mentioned above. This error for the present model is of the order of 4.66×10^{-5} for small times and of 1.3×10^{-5} for large times and is much lower than the one of the other equations.

APPLICATION OF THE PROPOSED MODEL TO EXPERIMENTAL DATA

Experimental data of Angelaki[22]: The proposed equation was applied to 2 series of experimental data of Angelaki[22]. In the first of these data series, with small infiltration times (t<16 min), the Parlange equation underestimates infiltration, while the other equations adapt well to the data, each one in a different time span (Fig. 6). In the second data series, with large infiltration times (Fig. 7), the proposed equation, underestimates infiltration, in contrast with the other equations, which overestimate it. Nevertheless, its adjustment is better.

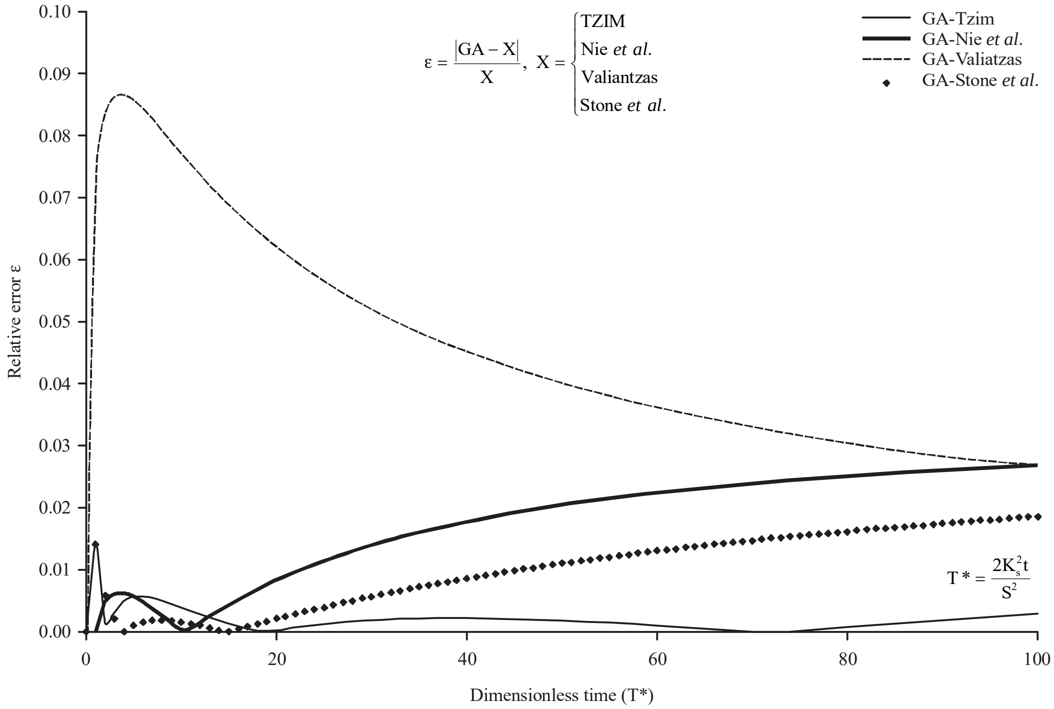

Fig. 5: Dimensionless relative error ε versus dimensionless time T*

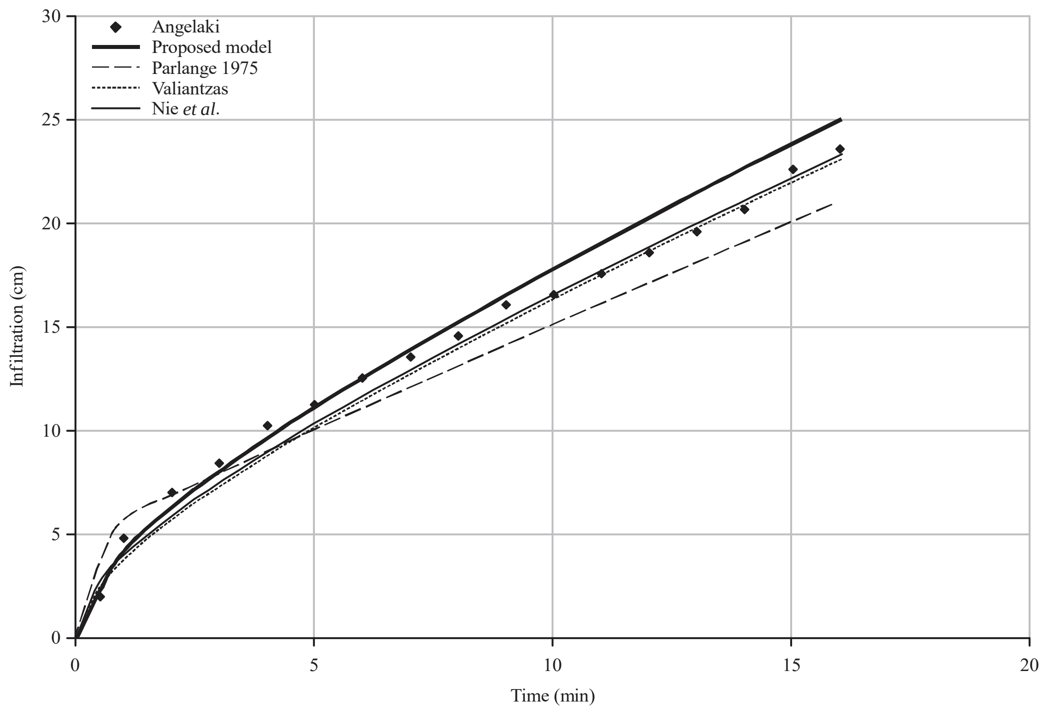

Fig. 6: Infiltration versus time derived from the proposed model as well as from other models compared to experimental data of Angelaki[22] (small times, $t \leq 16$ min)

Experimental data of Tsaoussis[23]: The proposed equation was also applied to another experimental data series from Tsaoussis[23], with very encouraging results (Fig. 8). The proposed equation fits very well the experimental results up to time values of 25 min.

In its dimensionless form, the proposed model as well as the Nie *et al.*[5] model fit the experimental data very well (Fig. 9).

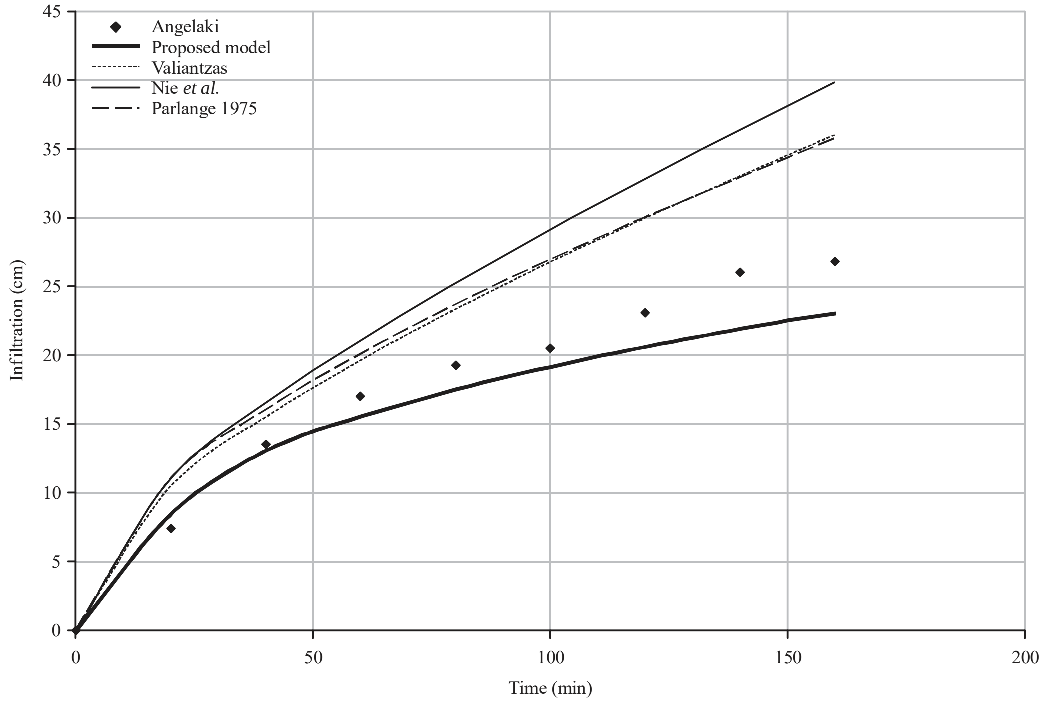

Fig. 7: Infiltration versus time derived from the proposed model as well as from other models compared to experimental data of Angelaki[22] (large times, t≤160 min)

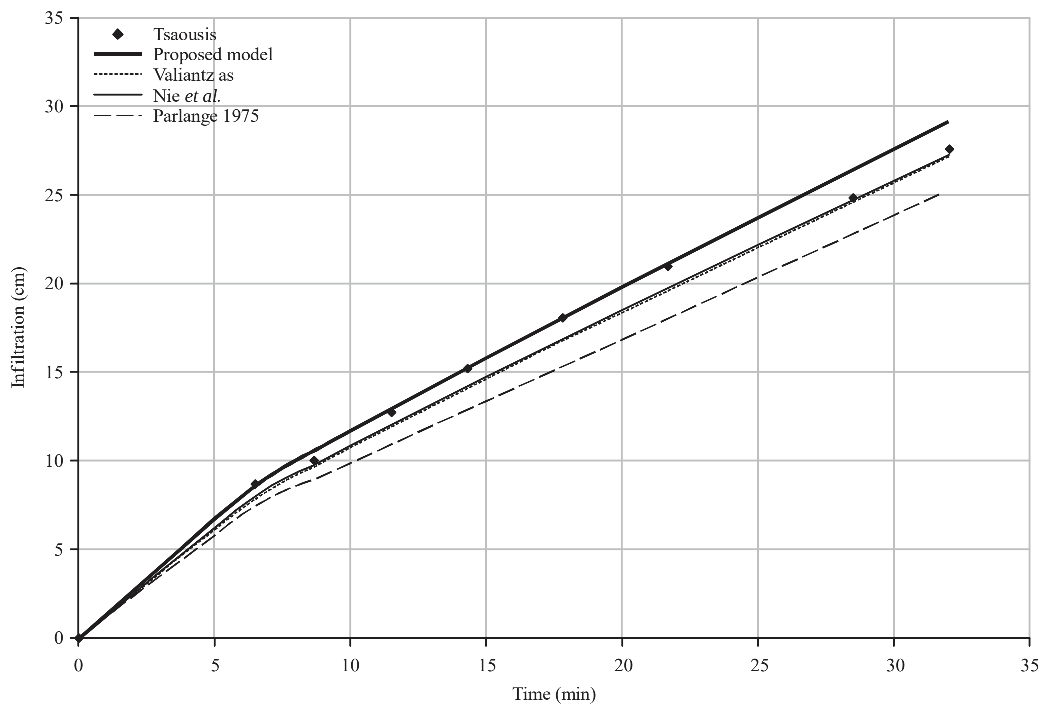

Fig. 8: Infiltration versus time derived from the proposed model as well as from other models compared to experimental data of Tsaoussis[23] (small times, t≤35 min)

Experimental data of Nie *et al.*[5]: The proposed model was applied to 2 experimental data series of Nie *et al.*[5] (series L1 and L2) as shown in Fig. 10 and 11. In the first case (L1, Fig. 10), the value of sorptivity S is equal to S = 0.318 cm min$^{-1/2}$ and both dimensionless times T* varied between 0.001 and 0.021.

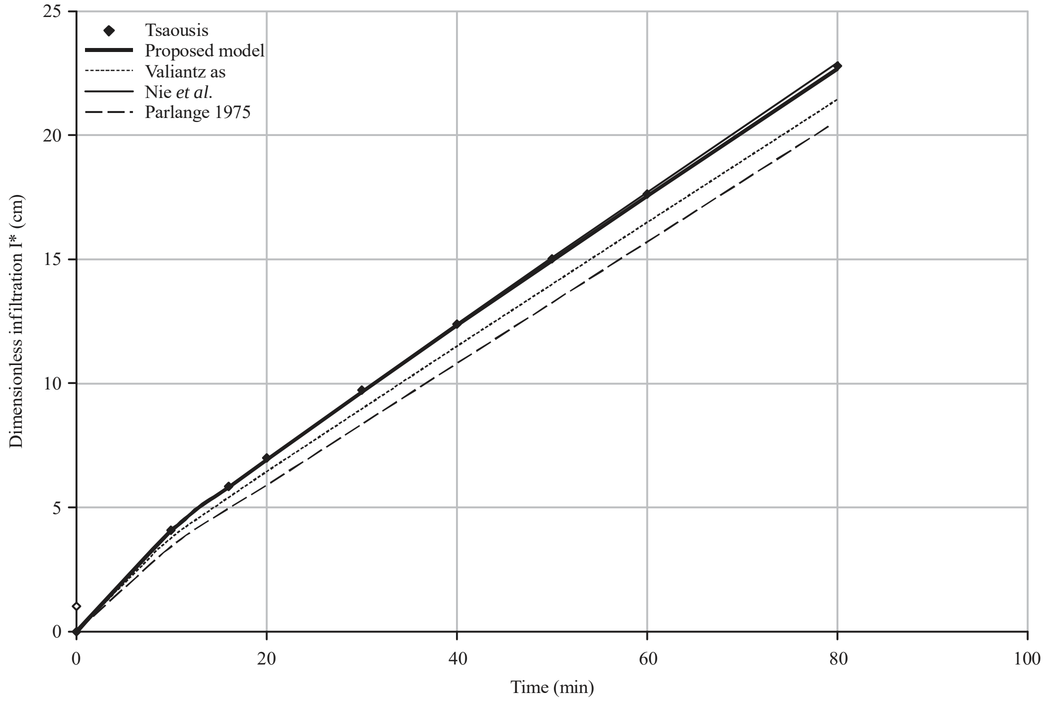

Fig. 9: Dimensionless infiltration I* versus time derived from the proposed model as well as from other models compared to experimental data of Tsaoussis[23] (large times, t≤80 min)

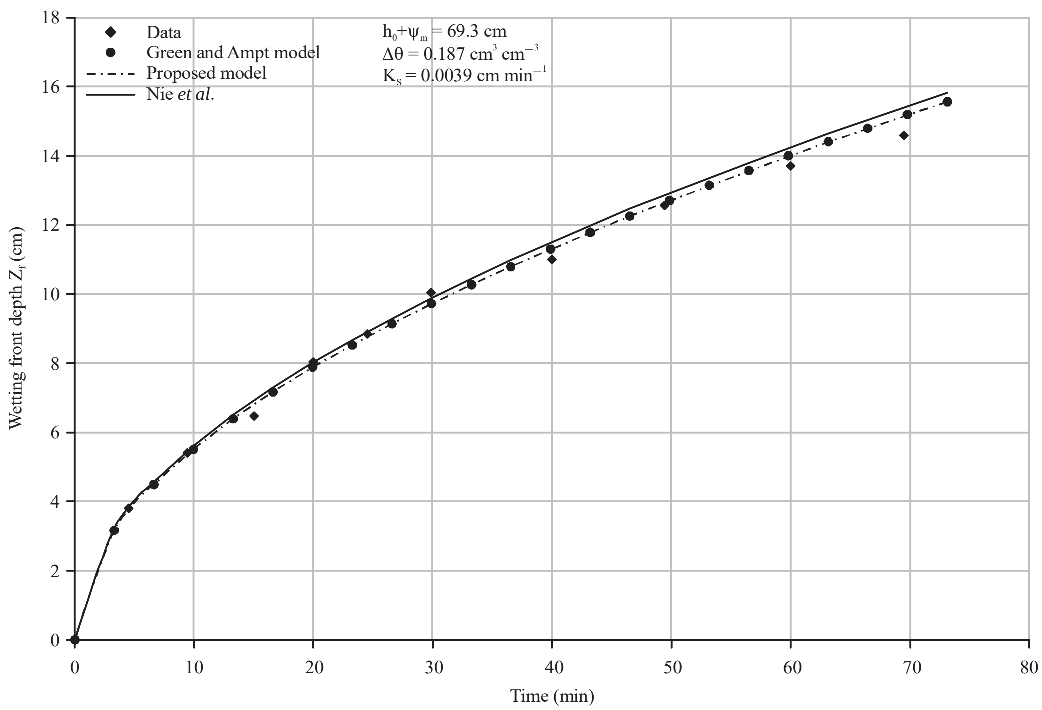

Fig. 10: Depth of the wetting front versus time, as derived from the proposed model and other models as well as the experimental data of Nie *et al.*[5] (experiment No. L1 of Nie *et al.*[5], t<80 min)

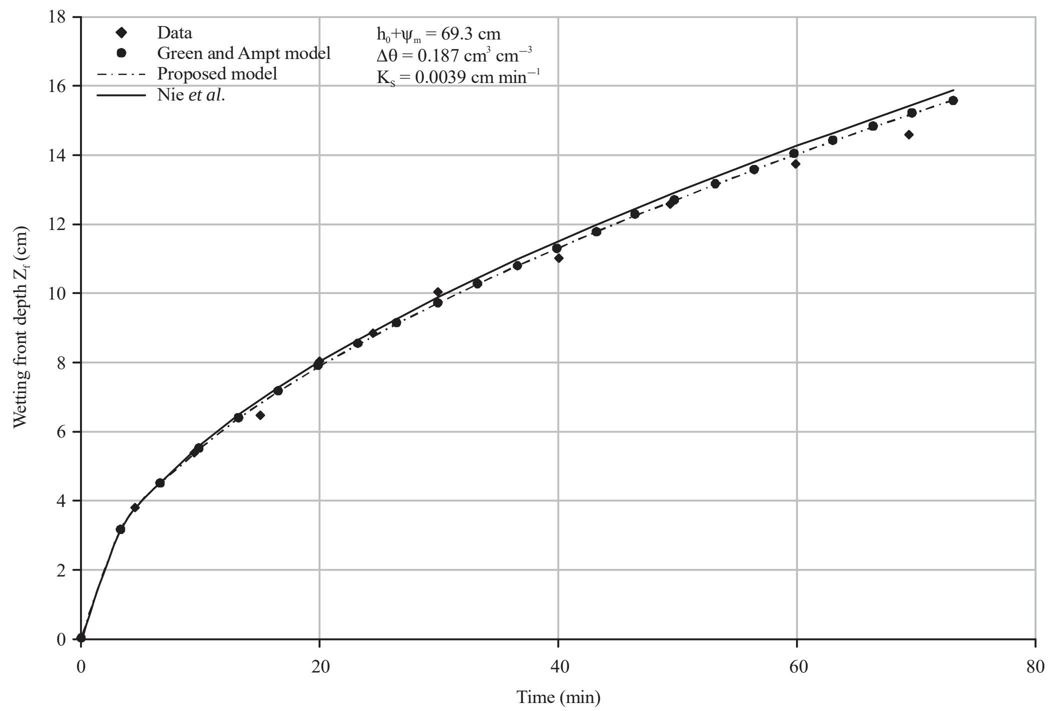

Fig. 11: Depth of the wetting front versus time, as derived from the proposed model and other models as well as the experimental data of Nie *et al.*[5] (experiment No. L2 of Nie *et al.*[5], t<90 min)

In the second case (L2, Fig. 11) the value of sorptivity S is equal to S = 0.460 cm min$^{-1/2}$ and the dimension less times T* varied between 0.001 and 0.025, that is, very small dimensionless times. The fitting of the proposed model to the GA model[1] is excellent, with mean divergence equal to 4×10^{-4} and it is also very good as far as the experimental data is concerned. The adaptation of the model of Nie *et al.*[5] is also very good.

A proposed new simple model for the infiltration is compared with the Green and Ampt model[1], as well as with three other existing models (Nie *et al.*[5], Valiantzas[9], Parlange[14]). The proposed model shows a very good fit to the Green and Ampt model compared to the other models, as shown in Fig. 5 and Table 1.

The proposed model shows a very good adaptation to the 2 series of experimental results that it was compared to. It shows very good adaptation in both small and large times compared to the other three models examined. Due to all the above, the proposed model can be utilized for irrigation problems, aquifer recharge problems and in general water resources management applications.

CONCLUSION

In the present study, a new simple model was proposed for the vertical infiltration of water into soil. This model was mainly based on the equations of Philip and Valiantzas and its parameters were derived from the minimization of the difference of the Green and Ampt model from these equations. This was achieved by using the conjugate gradient method.

This new equation converges with the Green and Ampt model with very high accuracy, in small flow depths, as well as in large ones. The mean reduced squared error of the difference between the new model and the Green and Ampt model is 4.66×10^{-5} for small time values and 1.3×10^{-5} for large time values. The model was compared to experimental data and it has shown a very good agreement.

This model consists of a simple explicit equation and thus, it is easy to use and can be of interest to researches of hydraulic problems, such as irrigation problems, aquifers' recharge problems, or even water resources management problems. The novelty is that it can be used for small and also for large times and cumulative infiltration.

SIGNIFICANCE STATEMENT

The objective of this study was to propose an approximate explicit Green and Ampt model to estimate the cumulative infiltration. Numerous papers have proposed a variety of different explicit extensions of the basic Green and Ampt model. Generally, many models have their own advantages and applicable scopes for estimating the cumulative infiltration and many other present disadvantages. This new explicit model estimates cumulative infiltration and is simple, accurate and approaches Green and Ampt model very closely. It was based on reducing the mean squared error between the new model and the Green and Ampt model by using the conjugated gradient method. This method gave a new explicit model very close to the GA model in comparison to other models. The novelty is that it can be used for a wider span of time and cumulative infiltration values with very good precision. Consequently, it can be efficiently applied not only in border irrigation problems, but in problems of artificial or natural recharge of aquifers and in water resources management problems as well.

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