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Aircraft Response to Atmospheric Turbulence at Various Types of the Input Excitation

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Abstract: Based on statistical and power spectral technique, methodology to predict the responses of aircraft in the atmospheric turbulence is presented. Modified longitudinal aircraft equations of motion to reflect gust inputs are solved. The responses of aircraft were tested under two categories of turbulence excitation, including the discrete and random disturbance atmosphere. Five new linear dynamics models of increasing gust excitation complexity are developed to describe the normal load factor throughout an aircraft due to vertical gust. Numerical model constructed for a Convair CV-M880 jet transport is solved to illustrate the results. Models 1 and 5 exhibit higher frequency contents and give a rapid estimation of normal load factor in case complete data are not readily available. These models predict the load factor with (2-3.5%) error compared with model 2 which considered all gust penetration effects. Finally, the results show a good agreement with the published work in load factor determination, at different probabilities not exceeding this value (load factor estimation) when encountering a turbulent vertical gust.

Key words: Aircraft response, stochastic dynamics, gust load determination, power spectral technique

INTRODUCTION

The effects of atmospheric turbulence on many of the modern sophisticated transport systems have become an important desigu parameter from both structural and performance aspects. Aircraft encounters with turbulence represent a serious safety threat for airlines. The problem of gusty winds proved itself to be a major obstacle to successful flight. The history of aviation abounds with incidents and accidents in which the variability of the wind in space or time played a decisive role. Loss of control of the altitude or the flight path and even the crashes of jet aircraft were not uncommon. Aside from the human catastrophes, annnal injuries to passengers and flight crew cost airlines millions in lost work time and medical expenses (Prince and Robinson, 2001). Turbulence refers to an irregular or disturbed flow in atmosphere that produces gusts and eddies. The most economic and practical method to explore innovative concepts and to investigate configuration options at an early stage is to first conduct an analytical and/or simulation study using an appropriate engineering mathematical model of the relevant physics (Buck and Newman, 2005). Often in an aircraft model simulation development, the gust effects of the atmosphere are neglected for various reasons and removed in the final form of the equations. Here, gust effects are the key excitation of interest. The models of the wind have to accommodate both events that are perceived as discrete (usnally described as gusts), as well as the phenomenon described as continuous turbulence. Discrete events are isolated encounters with steep gradients (horizontal or vertical) in horizontal or vertical speed of air. The discrete gust has evolved over the years from the isolated sharp-edged step function used in the airworthiness requirements to

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the currently favored one-minus-cosine. Static gust loadings are still determined by one-minus-cosine vertical gust velocity shape with the aircraft motion constrained to the plunge mode only. Haddadpour and Shams (2005) showed that the linear model analysis technique and linear qusi-steady aerodynamic are still used for structure modeling and aerodynamic modeling, respectively. Random turbulence is a chaotic motion of air that is described by its statistical properties (Kim *et al*., 1999). The main statistical features that need to be considered are: stationary, homogeneity, probability distribution and correlations and spectra. The power spectral approach offers a more realistic representation of the continuous nature of atmospheric turbulent and it allows more rational consideration of design and operational variations such as configuration changes, mission changes and airplane degrees of freedom. The main object of this paper is to analyze the response of aircraft under excitation of various types of turbnlence atmosphere, based on statistical and spectral technique. Five new linear dynamics models are developed to describe the normal acceleration throughout an aircraft due to vertical gust effect. To the best of authors knowledge, no attempts have been made to investigate the effect of atmospheric turbulence on aircraft response with these various models for the base line Convair CV-880 jet transport aircraft model.

MATERIALS AND METHODS

Aerodynamic and Stability Derivatives Model

The Aircraft selected as a model in this research work is the Convair CV-M880 jet transport operating at Mach = 0.86 and altitudes of 7005 m (23000 ft) and 10661 m (35000 ft).The airframe fixed coordinate system, dimensional aerodynamic and stability derivatives influence coefficients of aircraft are access to flight data test from the model original in accord with NASA convention in USA, 1973 (Schmidt, 1998). The Convair CV-M880 jet transport layout can be shown in Fig. 1. Flight conditions and stability derivatives of jet transport were illustrated in Table 1.

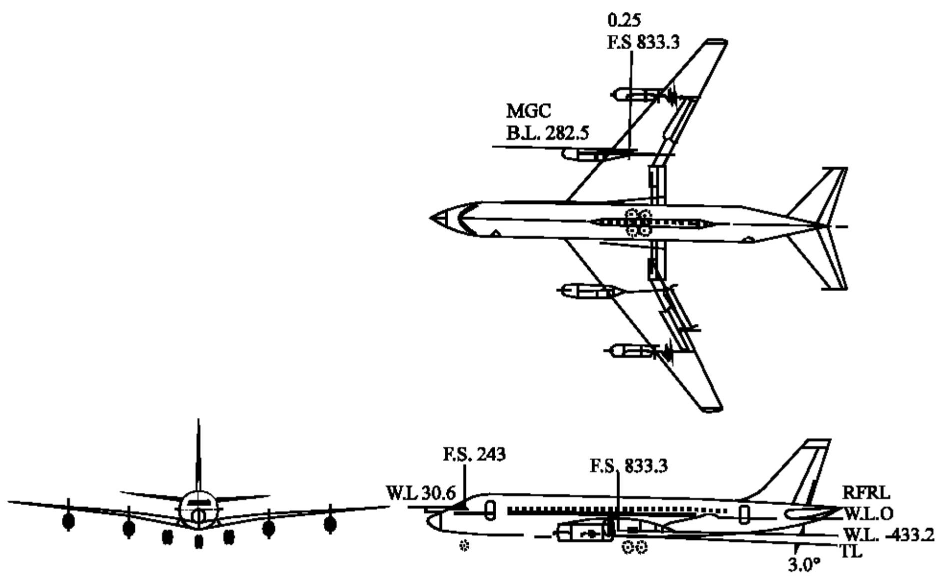

Fig. 1: Convair CV-880M jet transport layout

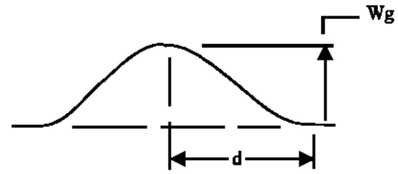

Fig. 2: One-minus-cosine discrete gust

Table 1: Flight conditions and stability derivatives of Convair CV-M880 jet transport flying at Mach = 0.86 in both conditions

Height (m)	$C_{L\alpha}$ (rad^{-1})	M_α (sec^{-1})	$M_{\dot{\alpha}}$ (sec^{-1})	Z_α/V (sec^{-1})	M_q (sec^{-1})
7005	4.410	-3.827	-0.335	-0.9267	-0.850
10661	4.900	-2.885	-0.203	-0.6311	-0.530

Aircraft Response Model to Discrete Gust

The idealized sharp-edged gust is a very severe type of a velocity profile that seldom occurs in nature. Instead, a discrete gust may be modeled more practically by a ramp input that reaches a peak value in a distance known as the gradient distance. The one- minus-cosine model (Fig. 2) is more frequently used in the determining gust-induced load factors rather than a ramp rising to a steady peak gust. The aircraft response when interring one-minus-cosine gust is in the vertical (plunging) degree of freedom mode only. The load factor for aircraft constrained to the plunging mode can be obtained from Eq. 1. The full derivation can be found in (Schmidt, 1998):

$$\Delta n_z(t) = \frac{w_g}{2g}\left[\omega\sin\omega t + \frac{1}{1+(\omega\lambda)^{-2}}\left(\frac{1}{\lambda}e^{-t/\lambda} - \frac{1}{\lambda}\cos\omega t - \omega\sin\omega t\right)\right] \tag{1}$$

Where,
Δn_z (t) = Local load factor,
λ = Time constant (in seconds)

$$\lambda = \frac{(W/S)}{C_{L\alpha}}\frac{2}{\rho V g}, \quad \omega = \pi V/d \quad (\text{in rad sec}^{-1})$$

Where,
d = Gradient distance,
W = Aircraft weight,
S = Wing span,
W_g = Vertical gust velocity and
$C_{l\alpha}$ = Lift curve slope.

The maximum load factor will occur near to the time for peak gust value.

Modified Aircraft Equations of Motion to Reflect the Gust Input

The use of the short period dynamic model will provide an insight as to import of increasing the airframe degrees of freedom when representing the airframe dynamics. The simplified set of short period equations of motion can be expressed as (Schmidt, 1998):

$$\begin{aligned}(V - Z_{\dot{\alpha}})\dot{\alpha} &= Z_\alpha\alpha + (V + Z_q)q + Z_\delta\delta \\ -M_{\dot{\alpha}}\dot{\alpha} + \dot{q} &= M_\alpha\alpha + M_q q + M_\delta\delta\end{aligned} \tag{2}$$

Where,

$Z_{\dot{\alpha}}$ = Normal force due to angle of attack rate,
Z_α = Normal force due to angle of attack,
V = Aircraft velocity,
q = Pitch rate,
Z_q = Normal force due to pitch rate,
Z_δ = Normal force due to elevator,
$M_{\dot{\alpha}}$ = Pitching moment due to angle of attack rate,
M_u = Longitudinal stability derivative,
M_q = Pitch damping,
δ = Control input and
M_δ = Pitching moment due to elevator.

The two coupled linear Eq. 2 will be restated as functions solely of α and q by the use of algebraic substitutions, i.e.,

$$\dot{\alpha} = \frac{Z_\alpha}{(V - Z_{\dot{\alpha}})}\alpha + \frac{V + Z_q}{(V - Z_{\dot{\alpha}})}q + \frac{Z_\delta}{(V - Z_{\dot{\alpha}})}\delta$$

$$\dot{q} = \left[M_\alpha + \frac{M_{\dot{\alpha}} Z_\alpha}{(V - Z_{\dot{\alpha}})}\right]\alpha + \left[M_q + \frac{(V + Z_q)M_{\dot{\alpha}}}{(V - Z_{\dot{\alpha}})}\right]q + \left[M_\delta + \frac{M_{\dot{\alpha}} Z_\delta}{(V - Z_{\dot{\alpha}})}\right]\delta$$

A further simplification can be made by recognizing that both $Z_{\dot{\alpha}}$ and Z_q are nearly zero in magnitude and most assuredly are negligible when compared to the free stream velocity in the preceding equations. The short period approximation in a commonly used becomes

$$\begin{Bmatrix} \dot{\alpha} \\ \dot{q} \end{Bmatrix} = \begin{bmatrix} Z_\alpha / V & 1 \\ M_\alpha + \dfrac{M_{\dot{\alpha}} Z_\alpha}{V} & (M_q + M_{\dot{\alpha}}) \end{bmatrix} \begin{Bmatrix} \alpha \\ q \end{Bmatrix} + \begin{Bmatrix} Z_\delta / V \\ M_\delta + \dfrac{M_{\dot{\alpha}} Z_\delta}{V} \end{Bmatrix} \delta \tag{3}$$

In present study the longitudinal model (short period response) modified to reflect gust inputs of α_g (t) and q_g (t) in place of control inputs. The longitudinal equations of motion can be written in state space form as follow:

$$\begin{Bmatrix} \dot{\alpha} \\ \dot{q} \end{Bmatrix} = \begin{bmatrix} Z_\alpha / V & 1 \\ M'_\alpha & M'_q \end{bmatrix} \begin{Bmatrix} \alpha \\ q \end{Bmatrix} + \begin{Bmatrix} Z_\alpha / V \\ M'_\alpha \end{Bmatrix} \alpha_g + \begin{Bmatrix} 0 \\ M_q \end{Bmatrix} q_g \tag{4}$$

Where, $M'_\alpha = M_\alpha + M_{\dot{\alpha}} \dfrac{Z_\alpha}{V}$, $M'_\alpha = M_\alpha + M_{\dot{\alpha}} \dfrac{Z_\alpha}{V}$, $M'_q = M_q + M_{\dot{\alpha}}$

The normal acceleration output is given by

$$\Delta n_Z = -Z_\alpha (\alpha + \alpha_g)/g \tag{5}$$

Power Spectral Technique

The power spectrum represents a frequency viewpoint for describing the square of random variable that is originally considered in time domain. The original time-varying random signal or function x (t), shown in Fig. 3a, is processed (or filtered) through a unit rectangular filter, shown in Fig. 3b, to yield a truncated signal $x_T(t)$ that is zero when $|t| > T$ as shown in Fig. 3c and this signal is

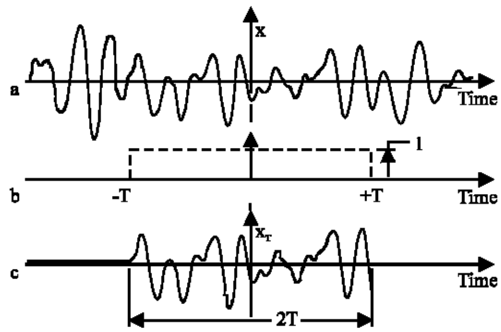

Fig. 3: Truncation of a random signal

absolutely integrable because is finite and the function is assumed to be bounded variation (McLean, 1990). Hence

$$\int_{-\infty}^{\infty} |x_T|\, dt = \int_{-T}^{T} |x_T|\, dt = \text{exists}$$

Consequently, that a Fourier transformation of $X_T(t)$ exists may be expressed as:

$$\begin{aligned} x_T(t) &= \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} X_T(\omega) e^{i\omega t} d\omega \\ X_T(\omega) &= \frac{1}{\sqrt{2\pi}} \int_{-T}^{T} x(t) e^{-i\omega t} dt \end{aligned} \tag{6}$$

Since $X_T(\omega)$ in Eq. 6 is a complex quantity whereas $X_T(t)$ is a real quantity.
From Parseval's theorem, which can, which can be described in the preceding notation as:

$$\int_{-T}^{T} x_T^2(t)\, dt = \int_{-\infty}^{\infty} |X_T(\omega)|^2\, d\omega \tag{7}$$

The mean square expectation can be defined as:

$$E(x^2) = \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} x^2(t)\, dt \tag{8}$$

The development of Power Spectral Density (PSD) follows from applying Parseval's theorem to Eq. 8 to obtain an alternate form for the mean square that involves frequency -dependent function, i.e.,

$$E(x^2) = \lim_{T\to\infty} \int_{-\infty}^{\infty} \frac{|X_T(\omega)|^2}{2T} d\omega = \lim_{T\to\infty} \int_{0}^{\infty} \frac{|X_T(\omega)|^2}{T} d\omega \tag{9}$$

The limiting action on the integr and in the preceding expression leads to the definition of the power spectral density,

$$\lim_{T\to\infty} \frac{|X_T(\omega)|^2}{T} d\omega = \phi(\omega)$$

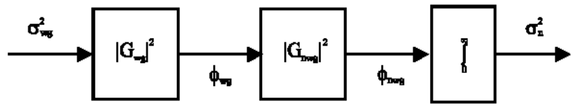

Fig. 4: Load factor response to a vertical gust input

Therefore the expectation for mean square may be described statistically in term of frequency content by

$$E(x^2) = \int_0^\infty \phi(\omega)d\omega \tag{10}$$

Aircraft Response Models to Random Gust

The aircraft normal load factor, in response to a turbulent vertical gust may be found by the series application by Dryden vertical gust model's transfer function (squared) to the aircraft transfer function (squared) of normal load factor to vertical gust input. This statement can be shown in Fig. 4. The expectation of the normal load factor response is obtained by integrating the power spectral density. The Dryden vertical gust model may be expressed in a transfer function format as (Schmidt, 1998)

$$\phi_{wg}(\omega) = |G_{wg}(\omega)|^2 \sigma^2_w \text{ and } G_{wg}(\omega) = G_{wg}(s)|_{s=i\omega} \tag{11}$$

σ_w = Root mean square (rms) of stochastic gust.

The transfer function Gw_g (S) can be expressed in terms of the Laplace transform variable as follows,

$$G_{w_g}(s) = K\frac{(s+\theta)}{(s+\lambda)^2} \tag{12}$$

Where, $K = (3V/\pi L_2)^{1/2}$, $\theta = V/\sqrt{3}L_2$ and $\lambda = V/L_2$

L_2 is the scale of vertical turbulence gust.

The aircraft longitudinal response is based on short period approximation where it is noted that α = w/V and α_g=w_g/V. The state variable form,

$$\begin{aligned} \dot{x} &= Ax + Bw_g \text{ and} \\ a_n &= Cx + Dw_g \end{aligned} \tag{13}$$

α_n = Normal acceleration, x = State variables

Where [A], [B], [C] and [D] are in accord with equation 4 whereas {X} = $[wq]^T$.
The transfer function of $G_{nwg(S)}$ becomes

$$G_{nwg(S)} = C[sI\text{-}A]^{-1} B+D \tag{14}$$

Which leads to PSD as:

$$\phi_{nwg}(\omega) = |\phi_{nwg}(s)|^2{}_{s=i\omega} \tag{15}$$

Table 2: Model assumption

Short period P.S concept Model (1)	Short period P.S concept Model (2)	Short period P.S concept Model (3)	Lyapunov's approach Model (4)	Lyapunov's approach Model (5)
$M_{\dot{\alpha}} = 0.0$	$M_{\dot{\alpha}} \neq 0.0$	$M_{\dot{\alpha}} \neq 0.0$	$M_{\dot{\alpha}} \neq 0.0$	$M_{\dot{\alpha}} = 0.0$
$q_g = 0.0$	$q_g \neq 0.0$	$q_g = 0.0$	$q_g = 0.0$	$q_g = 0.0$
$\theta_o = 0.0$	$\theta_o = 0.0$	$\theta_o = 0.0$	$\theta_o = 0.0$	$\theta_o = 0.0$

PS: Power Spectral, θ_o: Pitch angle, (rad), q_g: Pitch rate gust, body axis, (rad sec^{-1})

and the output PSD as:

$$\phi_n(\omega) = \phi_{wg}(\omega)\,\phi_{nwg}(\omega) \tag{16}$$

Finally, the normal load factor is obtained by the integration of the output power spectral density

$$E(n_z^2) = \int_0^\infty \phi_n(\omega)\,d\omega \tag{17}$$

In an attempt to understand the nature of atmospheric turbulent better, to provide data through which mathematical modeling of turbulence may be made and an improve means for treating the response of aircraft in turbulent air, many experimental studies have been made to predict and measure the vertical gust velocity in various circumstances, using aircraft probing (such as NASA probe used in flight measurement of turbulence (Houbolt, 1973) and NASA B-757-200 research aircraft (Buck and Newman, 2005). In current study, the rms values of turbulent vertical gust (α_w) are detected experimentally from (Etkin, 1981).

Five new models with different gust excitation complexities are used in present work. The assumptions used in each models are presented in Table 2. Models 1 and 5 are the simplest models for short period and Lyapunov approach (Farrell, 1994). As shown, the value of $M_{\dot{\alpha}}$ (part of aerodynamic damping) and the pitch rate gust sigual q_g are zero. Models 3 and 4 are considered the effects of $M_{\dot{\alpha}}$ and ignore the pitch rate gust signal in two approaches while model 2, accounts all gust penetration effects of the aircraft in short period response. The numerical simulation model was built by using the MATLAB software.

RESULTS AND DISCUSSION

The maximum load factor determination is the primary purpose of the current work to predict the aircraft response resulting from flight within a different turbulence atmospheric enviroument in degrees of severity (moderate to severe turbulence, usually the latter is storm related, such as thunderstorm). The longitudinal equations of motion are modified to include the gust effect. New models are developed to estimate the mentioned purpose. The base line aircraft was taken into consideration in this analysis, Convair CV-880 jet transport, when operating at Mach 0.86 at altitude 7005 m (23000 ft) and 10661 m (35000 ft). The response of aircraft tested under two categories of turbulence excitation, including discrete gusts (usually 1-cos gust) as well as the phenomenon described as continuous turbulence. The maximum load factor $(\Delta n_z)_{max} = 1.79$ g at time = 0.285 sec is found from the time history response to one-minus-gust (Fig. 5). The effects on gust response of degree of freedom in present method can appear, with maximum load factor = (1.72 g) occurring at time = 0.252 sec. The addition of pitch angle rotation to response model results in maximum load factor decreasing by

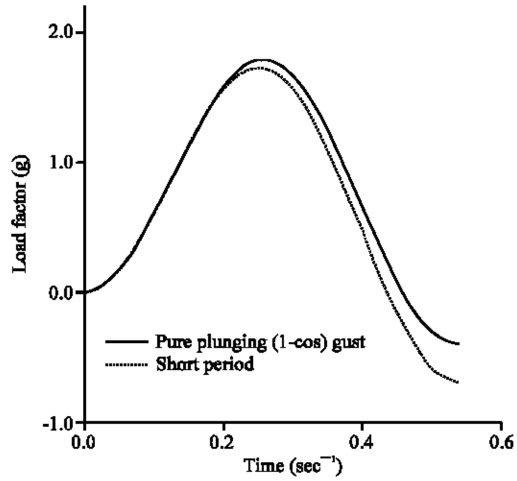

Fig. 5: Effects on gust response of degree of freedom for Convair CV-880 Jet transport at H = 7005 m (23000 ft): Mach = 0.86: W_g = 21 m sec^{-1}

about 0.07 g after the startup transient has occurred. The other type of turbulence under consideration in this study is a random turbulence, which was modeled by an appropriate power spectral density. The transfer function approach was applied here to determine the aircraft gust response based on short period approximation. The area under the power spectrum curve represented the mean square of load factor. The numerical estimation for σ_w (normalized input) and σ_w (output) were determined by using trapezoidal integral approximation for finite frequency range $0<\omega<30$ m. The normalized gust transfer function Eq. 11 yielded to $\sigma_w = 0.382$ m sec^{-1} (1 ft sec^{-1}) if ω_{max} were infinite; however, the frequency truncation results in σ_W estimation, which corresponds to 0.6% error. Figure 6a is a spectral representation of Dryden vertical gust model when normalized to unit area. The aircraft normal load factor transfer function due to vertical gust input shown in Fig. 6b with speak response value occuring near the short period frequency. Figure 6c represents the frequency distribution of aircraft normal acceleration (product of $|G_{wg}(\omega)|^2\,|G_{wg}(\omega)|^2$). The three sigma value for aircraft normal load factor estimation of 1.1 g with the probability of 99.7% does not exceed this value when encountering a turbulent vertical gust at variance $\sigma_w = 6.1$ m sec^{-1} (Fig. 7).

To validate the numerical results, a comparison between the present work and data in reference (Schmidt, 1998) was made to determine the load factor for Lockheed jet transport when operating at Mach 0.75 and altitude 6092 m (20000 ft). The results show a good agreement with 1.6% error. These verification results are shown in Fig. 8. Five new models (previously discussed) for the aircraft acceleration response are excited by vertical gust with different values of σ_w. The effects of σ_w (rms) values of stochastic vertical gust upon load factor (model 3 taken as an example for calculation) is illustrated in Fig. 9. The results show that the load factor and σ_w are directly related at different probabilities. Peak values of normal acceleration for all models are presented in Fig. 10. An alternate approach adopted in this study was the application of the Lyapunov equation, which directly yielded the mean square of the load factor (Farrell, 1994; Ogata, 1990), resulting in small error when the variance is estimated. This is noted when the values of load factor for models 1, 5 and 3, 4 is compared, respectively. The most energetic responses are models 1 and 5. As expected, these models exhibit higher frequency content due to the non-causal transfer function structure resulting from noted assumption. Models 1 and 5 predict the load factor with (2-3.5%) error compared with model 2 which considered all gust penetration effects (Fig. 10).

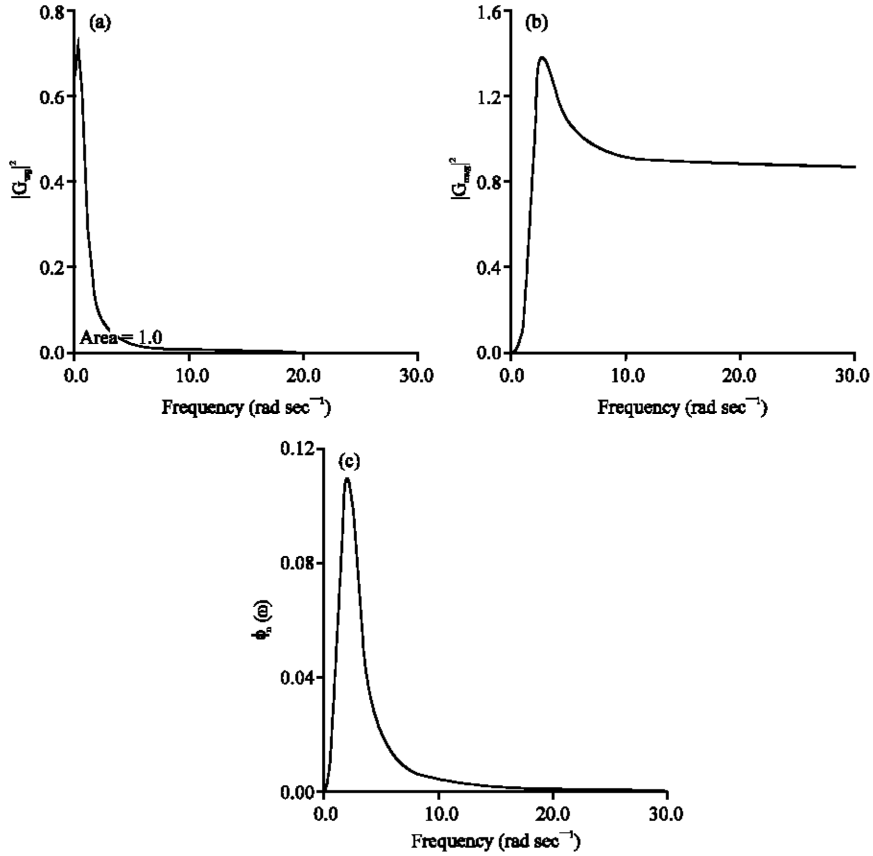

Fig. 6: Aircraft spectral response resulting from a vertical gust: H = 7005 m: Mach = 0.86, a) Dryden vertical gust input, b) Aircraft transfer function, c) Aircraft normal load factor ϕ_n (w) for σ_w = 0.328 m sec^{-1} (1 ft sec^{-1})

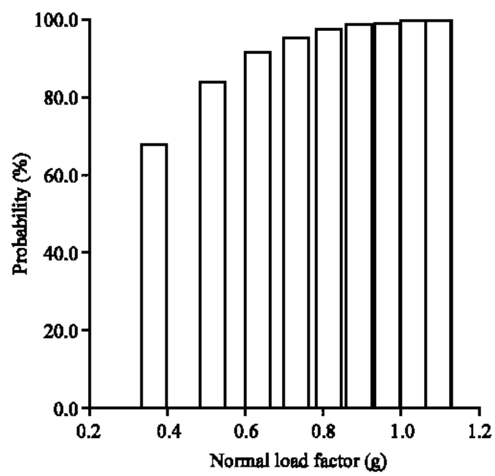

Fig. 7: Normal load factor estimation values for Convair 880M transport (Model 3): H = 7005 m: Mach = 0.86: at different probabilities not exceeding these values (load factor): σ_w= 6.1 m sec^{-1}

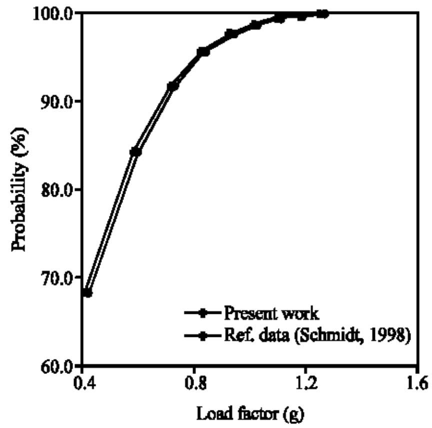

Fig. 8: Lockheed jet transport comparison results of peak normal load factor at different probabilities of not exceeding these values (model 3): H = 6092 m, Mach = 0.75:1.65% error in the estimation of load factor response

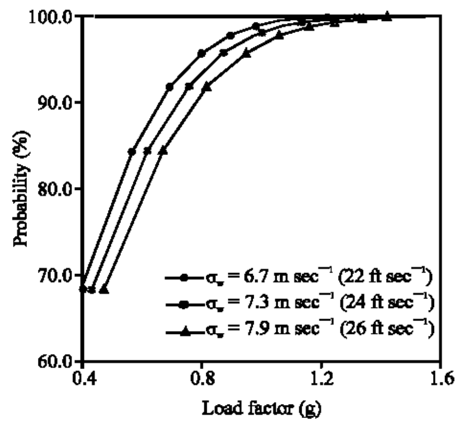

Fig. 9: Effects of σ_w values of stochastic vertical gust upon load factor (model 3)

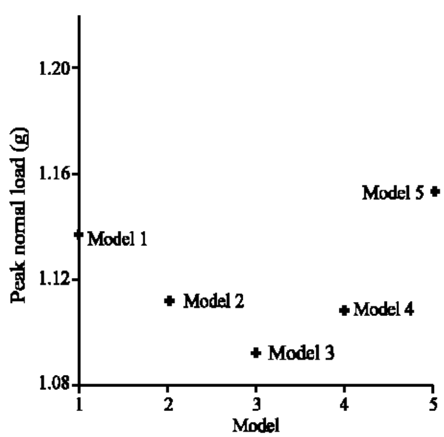

Fig. 10: Peak normal load factor for all suggestion models. Convair CV-880M, At H = 7005 m: Mach = 0.86: σ_w = 6.1 m sec^{-1}

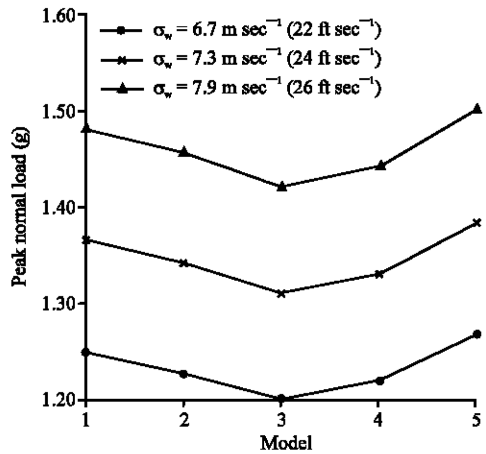

Fig. 11: Effects of σ_w on peak normal load factor for all models for CV-880M transport with 99.7% probability not exceeding the value of peak normal load

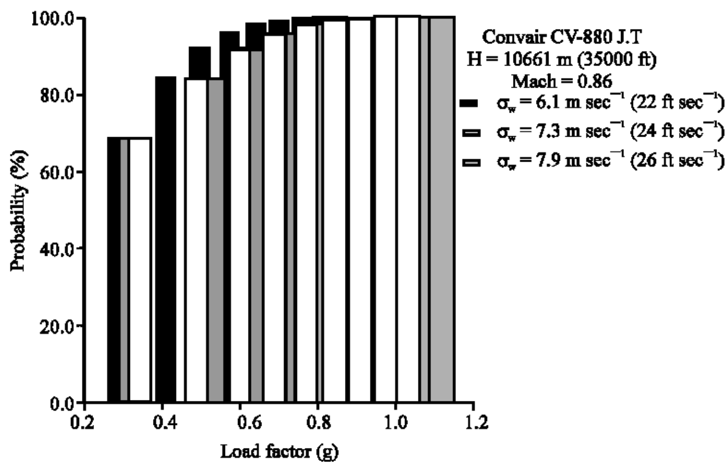

Fig. 12: The load factor predication (model 3) at different values of σ_w (rms) of vertical gust at high altitude H = 10661, m: Mach 0.86

Figure 11 introduces the effect of σ_w values on peak normal load factor. Other test conditions of aircraft at altitude 10661 m (35000 ft) was made at various σ_w (rms) according to probability not exceeding the predicted values of aircraft acceleration. These results are presented in Fig. 12.

Finally, this study provides increased motivation to improve airplane response in gust turbulence atmospheric by using modern optimal control methods.

CONCLUSIONS

Methodology to estimate aircraft transient response resulting from flight within turbulent atmosphere based on statistical and power spectral technique is presented. Modified longitudinal equations of motion which includes the effects of atmospheric gust are solved. The responses of aircraft are tested under two categories of turbulence excitation input (discrete and continuous random turbulence). Five linear dynamics models are developed to describe the normal acceleration throughout an aircraft when it encounters a vertical gust. The following conclusions have been obtained in the present research:

- The numerical results show dependencies on which gust excitation type and evaluation criteria are considered.
- Models 1 and 5 exhibit higher frequency contents and give a rapid estimation of normal load factor in case complete data are not readily available. These models predict the load factor with (2-3.5%) error compared with model 2 which considered all gust penetration effects.
- It can be concluded that the agreement between the finite frequency limit on integration of the spectral distribution and Lyapunov's results obtaining an estimate for the output deviation of load factor is well within the accuracy.

REFERENCES

Buck, B. and B.A. Newman, 2005. Aircraft acceleration prediction with flight data validation due to atmospheric disturbances. AIAA. 43rd Aerospace Sciences Meeting and Exhibit, 10-13 January 2005, Reno, Nevada, pp: 1-17.

Etkin, 1981. Turbulent wind and its effect on flight. J. Aircraft, 18: 327-345.

Farrell, 1994. Stochastic dynamics of mid latitude atmospheric jet. J. Atmosph. Sci., 52: 1642-1656.

Haddadpour and Shams, 2005. Sharp edge gust effects on aeroelastic behavior of a flexible wing with high aspect ratio. AIAA Aerospace Science Meeting and Exhibit. AIAA 2005-838,10-13 January 2005, Reno, Nevada, pp: 1-15.

Houbolt, 1973. Atmospheric turbulence. AIAA. J., 11: 421-437.

Kim, Kabe and Lee, 1999. Atmospheric flight gust loads analysis. AIAA Structure, Structure Dynamic and Materials Conference and Exhibit Vol. 1, A99-24601, 12-15 April 1999, pp: 552-562

McLean, 1990. Automatic Flight Control System, Prentice-Hall, Englewood Cliff, NJ. USA.

Ogata, 1990. Modern control engineering. Prentice-Hall, Englewood Cliff, NJ. USA.

Prince and Robinson, 2001. The effectiveness of linear aircraft simulations in predicting cabin loads induced by turbulence. AIAA, Atmosphere Flight Mechanics Conference and Meeting A01-37369,6-9 August 2001. Montreal, Ouebec, Canada, pp: 1-9.

Schmidt, 1998. Introduction to aircraft flight dynamics. Naval Postgraduate School, AIAA Education Series, Monterey. California, pp: 291-299, 314-315.