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Multi-Mode Flight Control for an Unmanned Helicopter Based on Robust H_∞ D-Stabilization and PI Tracking Configuration

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Abstract: In this study, a novel multi-mode flight control strategy for unmanned helicopter, in presence of model uncertainty, atmospheric disturbances and handling qualities specification requirements (as in ADS-33E-PRF), based on multi-loop control structure is presented. Robust H-infinity D-stabilization optimal control technique is utilized in inner loop, which ensures the stability of flight control system in case of change of helicopter model uncertainty and effectively eliminates effect of gnst disturbance on helicopter states and collective/cyclic inputs. Sufficient condition for the solvability of robust H-infinity D-stabilization controller is derived in a form of Linear Matrix Inequality (LMI) for iuner-loop control design. By formulating command tracking problem as state feedback stabilization problem of augmented system, proportional-plus-integral (PI) tracking configuration is used in outer loop to improve the dynamic and static operation characteristics of the helicopter system. Based on the proposed control strategy, multi-mode flight control for the helicopter is designed and the flight performance is verified. Analysis and simulation results show that level 1 handling requirements as defined in ADS-33E-PRF are accomplished.

Key words: Regional pole assignment, gnst disturbance attenuation, model uncertainty, helicopter handling qnalities, LMI

INTRODUCTION

Unmauned Air Helicopters (UAH), because of their unique thrust generation and operation principle, have unique flight capabilities in comparison with fixed-wing aircraft, such as Vertical Takeoff/Landing (VTOL), hover, pirouette and slalom (ADS-33E-PRF, 2000). However, control design for unmanned helicopter has been considered as a challenge in aeronautical field over decades. The main difficulties, in designing controllers for UAH, can be generally characterized as underactuated, cross-coupled, large uncertainty, open-loop instabilities, highly noulinear dynamics and multiple flight modes (Gong and Yang, 2004; Oh *et al*., 2006; Padfield, 1996; Wang *et al*., 2007).

In spite of these challenges, a nnmber of researchers have worked on designing UAHs and a wide set of design techniques, from classical control to neural-based adaptive control, have been reported. Among early control technique, one-loop-at-a-time control design methods, based on classical Single Input/Single Output (SISO) techniques with a proportional-plus-integral (PI) configuration are the most widely used (Gong and Yang, 2004; Huang and Sun, 2003; Kim and Shim, 2003). The SISO approach has obvious advantages of simple structure, straightforward design process, low computing

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load and easy tuning. However, it does not provide a systematic way to consider uncertainties, disturbance, input saturation and cross-couplings among channels (Kim and Shim, 2003). Thereafter, multivariable techniques are investigated by researchers for design of stability augmentation and guidance systems for helicopter, such as eigenstructure assignment (Low and Garrard, 1992), LQG/LQG (Gribble, 1993), μ-synthesis (Lohar, 2000), H_2 (Takahashi, 1993) and H_∞ (Luo *et al*., 2003; Postlethwaite *et al*., 2005; Trentini and Pieper, 2001). Among MIMO approaches, H_∞ theory is the most widely used in recent decade as it provides robust stability for systems subject to uncertainty and disturbance. Luo *et al*. (2003) reported a method based on weighted H_∞ mixed sensitivity optimization to design helicopter control system in order to improve helicopter's stability, maneuverability and agility. Postlethwaite *et al*. (2005) utilized H_∞ loop shaping technique for control design of Bell 205 helicopter and various aspects of controller design are discussed. Trentini and Pieper (2001) carried out a mixed-norm optimization design methodology for helicopter control design. Though simulation and flight-test results show that the control laws designed using H_∞ approaches provide satisfactory results, their drawback is that order of the control system is equal to that of the augmented plant and can reach 15 to 30 states for a helicopter (Prempain and Postlethwaite, 2005). This may leads to implementation issues.

To provide maximum flexibility and to accommodate diverse situations that occur during flight, it is imperative to categorize flight of an unmanned helicopter in various flight states (e.g., Hover, Cruise) and operation modes (e.g., Attitude Command, ACAH), referred to as flight modes (Egerstedt *et al*., 1999; Boskovic and Mehra, 2000; ADS-33E-PRF, 2000). For different flight modes, different flight specifications are required to improve flight qualities and aviation safety. For instance, stability, gust disturbance attenuation and fast transient response are important if helicopter is in steady-state flight (Hover, Cruise); while in maneuvering flight, such as ACAH, tracking performance will be the most important. Thus, multi-mode flight control is necessary to take into account different flight performance requirements.

In this study, the problem of helicopter control design in presence of model uncertainty, gnst disturbance and multi-mode flight requirements as in ADS-33E-PRF is investigated. Instead of the frequency domain H_∞ based approaches mentioned previously, where unstructured uncertainty description is mainly concerned, time domain multivariable control methodology is utilized and a novel multi-mode control approach with multi-loop structure is presented. It is assumed that helicopter system uncertainty is due to parametric uncertainty and represented as interval variations in stability and control derivatives. This assumption is in accord with actual helicopter flight situations where high-frequency dynamics from main rotor and actuators are required not to be excited.

MIXED H_∞/PI MULTI-MODE CONTROL

In state space form helicopter mathematical model, with parameter uncertainties and gnst disturbance, can be represented as (Padfield, 1996; Etele, 2006):

$$\begin{aligned} \dot{x}(t) &= \hat{A}x(t) + B_1 w(t) + \hat{B}_2 u(t) \\ z(t) &= C_1 x(t) + D_{12} u(t) \end{aligned} \tag{1}$$

where, $x = [u, w, q, \theta, v, p, \varphi, r]^T$ is state vector with u, v and w as translation velocity components (m sec^{-1}) in body coordinates x, y, z-axis, respectively, θ and φ as pitch and roll angles (rad), p, q and r as angular velocity components (rad sec^{-1}) in body coordinate system x, y, z-axis, respectively. $u = [\delta_0, \delta_{1s}, \delta_{1c}, \delta_T]^T$ defines the helicopter operating control inputs with δ_0 as main rotor collective pitch, δ_{1s} and δ_{1s} longitudinal/lateral cyclic pitch and δ_T tail rotor collective pitch (rad). w(t) denotes gust disturbance signal. $\hat{A}$ and $\hat{B}$ are stability and control derivatives matrices, respectively given by:

$$\hat{A} = A + \Delta A,\ \hat{B}_2 = B_2 + \Delta B_2$$

where, ΔA and ΔB_2 denote uncertainties in system model. The structure of ΔA, ΔB_2 is given by:

$$[\Delta A \quad \Delta B_2] = H\Sigma[F_1 \quad F_2] \tag{2}$$

where, H, F_1, F_2 are known matrices with appropriate dimensions. Σ denotes range of parameter uncertainties, which satisfies $\Sigma \in \Omega$,

Where:

$$\Omega = \left\{\Sigma \middle| \Sigma^T\Sigma \le I\right\} \tag{3}$$

z(t) defines the evaluated output. C_1 and D_{12} are weighting matrices to system states x and control input u.

The structure of uncertainty is restricted to Eq. 2. It will be shown that when the stability and control derivatives vary in some finite interval, the state space parameter uncertainties in the helicopter model, Eq. 1, can be transformed into the form, represented by Eq. 2.

Inner Loop Control Design

Consider the state feedback control law given by Eq. 4,

$$u = Kx(t) + r_i \tag{4}$$

where, r_i is virtual control input.

The objective for the inner loop control is to design a state feedback law, Eq. 4 with $r_i = 0$, such that the closed-loop helicopter system satisfies the following performance specifications,

Objective 1: The closed-loop system is internally stable for any admissible uncertainty $\Sigma \in \Omega$

Objective 2: Poles of the closed-loop system lie within the disk D(−q, r) with center $-q+j_0$ and radius r, q>r>0, for any admissible uncertainty

Objective 3: Given gust disturbance suppression index γ, for any admissible uncertainty $\Sigma \in \Omega$, effect of gust disturbance to selected flight states and control input is in the given level, i.e.,

$$\int_0^\infty \left\{x^T(t)Qx(t) + u^T(t)Ru(t)\right\}dt < \gamma^2 \int_0^\infty w^T(t)w(t)dt \tag{5}$$

where, $w(t) \in L_2(0, \infty)$. Q and R are weighting matrices with appropriate dimensions and $Q = Q^T \ge 0$, $R = R^T > 0$,.

Selecting C_1 and D_{12} that satisfy $C_1^TC_1 = Q$, $D_{12}^TD_{12}$ and $C_1^T D_{12} = 0$, then from Eq. 5,

$$\int_0^\infty z^T(t)z(t)dt < \gamma^2 \int_0^\infty w^T(t)w(t)dt \tag{6}$$

i.e.,

$$\|z(t)\|_2 < \gamma \|w(t)\|_2 \tag{7}$$

From Parseval Equation and definition of H_∞ norm, Eq. 7 is equivalent to

$$\|T_{zw}(s)\|_\infty < \gamma \tag{8}$$

where, $T_{zw}(s)$ is transfer function matrix, from gust disturbance signal w(t) to evaluated output signal z(t), in closed-loop system represented by Eq. 1 and 4.

The following lemma has been taken from Chu *et al*. (2000).

Lemma 1: (Bounded-Real Lemma)

Given a constant $\gamma>0$, for system, G(s) = (A, B, C) the following two statements are equivalent,

- The system is stable $\|G(s)\|_\infty < \gamma$
- There exists a symmetric positive definite matrix P, such that

$$A^TP + PA + \gamma^{-2}PBB^TP + C^TC < 0$$

Lemma 2

Let $A \in R^{n\times n}$ be a given matrix. Eigenvalues of A belong to a disk D(−q, r), if and only if there exists a symmetric positive definite matrix P, such that:

$$(A + \alpha I)P + P(A + \alpha I)^T + r^{-1}(A + \alpha I)P(A + \alpha I)^T < 0$$

where, $\alpha = q-r$, $q>r>0$.

Proof

The proof is obtained from Theorem 1, in Garcia and Bernussou (1995) and standard Schur complement (Chu *et al*., 2000).

Theorem 1

Given a disk D (−q, r), q>r>0 and constant $\gamma>0$, the inner-loop performance specifications Obj. 1 ~ Obj. 3 are all satisfied via the feedback control law, Eq. 4, if there exists a symmetric positive definite matrix P, such that:

$$\begin{bmatrix} \Theta_{11} & B_1 & PC_K^T & (\hat{A}_K + \alpha I)P \\ B_1^T & -\gamma^2 I & 0 & 0 \\ C_K P & 0 & -I & 0 \\ P(\hat{A}_K + \alpha I)^T & 0 & 0 & -rP \end{bmatrix} < 0 \qquad (9)$$

where, $\Theta_{11} = (\hat{A}_K + \alpha I)P + P(\hat{A}_K + \alpha I)^T$, $\hat{A}_K = \hat{A} + \hat{B}_2K$, $C_K = C_1 + D_{12}K$, $\alpha = q - r$

Proof

From Schur complement, Eq. 9 is equivalent to:

$$(\hat{A}_K + \alpha I)P + P(\hat{A}_K + \alpha I)^T + \gamma^{-2}B_1B_1^T + PC_K^TC_KP + r^{-1}(\hat{A}_K + \alpha I)P(\hat{A}_K + \alpha I)^T < 0 \qquad (10)$$

That is,

$$\hat{A}_KP + P\hat{A}_K^T + \gamma^{-2}B_1B_1^T + PC_K^TC_KP + 2\alpha P + r^{-1}(\hat{A}_K + \alpha I)P(\hat{A}_K + \alpha I)^T < 0 \qquad (11)$$

Let $Q = P^{-1}$ and left and right multiplying Eq. 11 by Q, then:

$$Q\hat{A}_K + \hat{A}_K^TQ + \gamma^{-2}QB_1B_1^TQ + C_K^TC_K + 2\alpha Q + r^{-1}Q(\hat{A}_K + \alpha I)Q^{-1}(\hat{A}_K + \alpha I)^TQ < 0$$

Thus,

$$Q\hat{A}_K + \hat{A}_K^T Q + \gamma^{-2} Q B_1 B_1^T Q + C_K^T C_K < 0$$

By using Bounded-real lemma, it can be concluded that the closed-loop system is stable and $\|T_{zw}(s)\|_\infty < \gamma$. From Eq. 10,

$$(\hat{A}_K + \alpha I)P + P(\hat{A}_K + \alpha I)^T + r^{-1}(\hat{A}_K + \alpha I)P(\hat{A}_K + \alpha I)^T < 0$$

Hence, $\sigma(\hat{A}_K) \subset D(-q, r)$, from lemma 2.

The following lemma has been taken from Singh (2004).

Lemma 3

Let N_1 and N_2 be known matrices with appropriate dimensions, Y be symmetric matrix and Σ be matrix with appropriate dimensions satisfying $\Sigma^T\Sigma \leq I$, then the following two statements are equivalent:

- $Y + N_1 \Sigma N_2 + N_2^T \Sigma^T N_1^T < 0$
- There exists a constant $\varepsilon>0$, such that $Y + \varepsilon^{-1} N_1 N_1^T + \varepsilon N_2^T N_2 < 0$.

Theorem 2

For the uncertain system, Eq. 1 and 2, the matrix inequality, Eq. 9, is true, if and only if there exists a constant $\varepsilon>0$, matrix Z with appropriate dimensions and symmetric positive definite matrix P, such that:

$$\begin{bmatrix} \Psi_{11} & B_1 & XC_1^T + Z^T D_{12}^T & A_\alpha X + B_2 Z & XF_1^T + Z^T F_2^T & \varepsilon H \\ B_1^T & -\gamma^2 I & 0 & 0 & 0 & 0 \\ C_1 X + D_{12} Z & 0 & -I & 0 & 0 & 0 \\ XA_\alpha^T + Z^T B_2^T & 0 & 0 & -rX & XF_1^T + Z^T F_2^T & 0 \\ F_1 X + F_2 Z & 0 & 0 & F_1 X + F_2 Z & -\varepsilon I & 0 \\ \varepsilon H^T & 0 & 0 & 0 & 0 & -\varepsilon I \end{bmatrix} < 0 \qquad (12)$$

where, $\Psi_{11} = A_\alpha X + XA_\alpha^T + B_2 Z + Z^T B_2^T$, $A_\alpha = A + \alpha I$. Moreover, a suitable feedback control law, satisfying the inner-loop performance specifications, Obj. 1 ~ Obj. 3, is given by $u(t) = Kx(t)$ and $K = ZX^{-1}$.

Proof

Define

$$Y = \begin{bmatrix} Y_{11} & B_1 & P(C_1 + D_{12}K)^T & (A_\alpha + B_2 K)P \\ B_1^T & -\gamma^2 I & 0 & 0 \\ (C_1 + D_{12}K)P & 0 & -I & 0 \\ P(A_\alpha + B_2 K)^T & 0 & 0 & -rP \end{bmatrix}$$

where, $Y_{11} = (A_\alpha + B_2 K)P + P(A_\alpha + B_2 K)^T$.

Thus, Eq. 9 can be rearranged as:

$$Y + N_1\Sigma N_2 + N_2^T\Sigma^T N_1^T < 0 \quad (13)$$

where, $N_2 = [(F_1+F_2K)P \quad 0 \quad 0 \quad (F_1+F_2K)P]$, $N_1 = [H^T \quad 0 \quad 0 \quad 0]^T$.

By lemma 3, Eq. 13 is equivalent to:

$$Y + \zeta^{-1}N_1N_1^T + \zeta N_2^T N_2 < 0$$

Thus,

$$\begin{bmatrix} Y & N_2^T & N_1 \\ N_2 & -\zeta^{-1}I & 0 \\ N_1^T & 0 & -\zeta I \end{bmatrix} < 0$$

That is:

$$\begin{bmatrix} Y_{11} & B_1 & P(C_1+D_{12}K)^T & (A_\alpha + B_2K)P & P(F_1+F_2K)^T & H \\ B_1^T & -\gamma^2 I & 0 & 0 & 0 & 0 \\ (C_1+D_{12}K)P & 0 & -I & 0 & 0 & 0 \\ P(A_\alpha+B_2K)^T & 0 & 0 & -rP & P(F_1+F_2K)^T & 0 \\ (F_1+F_2K)P & 0 & 0 & (F_1+F_2K)P & -\zeta^{-1}I & 0 \\ H^T & 0 & 0 & 0 & 0 & -\zeta I \end{bmatrix} < 0 \quad (14)$$

Let $X = P$, $Z = KX$ and $\varepsilon = \xi^{-1}$. Substituting X and Z in Eq. 14 and left and right multiplying Eq. 14 by, $T = \mathrm{diag}(I, I, I, I, \varepsilon)$ Eq. 12 is obtained. The conclusion follows.

Model Uncertainty Description

Parameter uncertainties in the model of unmanned helicopter can be represented as variations in stability and control derivatives in certain intervals. In order to design controller for unmanned helicopter using Theorem 2, the uncertainty should be transformed to the structure represented by Eq. 2.

Define, $\hat{A} = \{a_{ij}\}$, $\hat{B}_2 = \{b_{ik}\}$ and $\underline{a}_{ij} \le a_{ij} \le \overline{a}_{ij}$, $\underline{b}_{ik} \le b_{ik} \le \overline{b}_{ik}$ where, $i, j = 1,...,n$, $k = 1,...,m$ and construct matrices $P = \{\underline{a}_{ij}\}$, $Q = \{\overline{a}_{ij}\}$, $M = \{\underline{b}_{ik}\}$ and $N = \{\overline{b}_{ik}\}$. Then $\hat{A}$ and $\hat{B}$ can be represented in the form of interval matrix:

$$\hat{A} \in [P,Q],\ \hat{B}_2 \in [M,N]$$

Define the following set of uncertain matrices pairs:

$$\Pi_1 = \{(\hat{A},\hat{B}_2) \mid \hat{A} \in [P,Q], \hat{B}_2 \in [M,N]\} \quad (15)$$

Let us define,

$$A = (P+Q)/2,\ S = (Q-P)/2 \triangleq \{s_{ij}\}$$
$$B_2 = (M+N)/2,\ T = (N-M)/2 \triangleq \{t_{ij}\}$$

Constructing matrices,

$$H_a = [\sqrt{s_{11}}e_1,\dots,\sqrt{s_{1n}}e_1,\dots,\sqrt{s_{n1}}e_1,\dots,\sqrt{s_{nn}}e_n]$$

$$F_a = [\sqrt{s_{11}}e_1,\dots,\sqrt{s_{1n}}e_1,\dots,\sqrt{s_{n1}}e_1,\dots,\sqrt{s_{nn}}e_n]^T$$

$$H_b = [\sqrt{t_{11}}e_1,\dots,\sqrt{t_{1m}}e_1,\dots,\sqrt{t_{n1}}e_n,\dots,\sqrt{t_{nm}}e_n]$$

$$F_b = [\sqrt{t_{11}}i_1,\dots,\sqrt{t_{1m}}i_m,\dots,\sqrt{t_{n1}}i_1,\dots,\sqrt{t_{nm}}i_m]$$

where, e_j is the jth column of n×n identity matrix and i_j is the jth column of n×m identity matrix. Defining two sets of matrices, Δ_a, Δ_b with diagonal uncertainty,

$$\Delta_a = \{\Sigma \in \mathbb{R}^{n^2\times n^2} | \Sigma = \text{diag}(\delta_{11},\dots,\delta_{1n},\dots,\delta_{n1},\dots,\delta_{nn}, \ |\delta_{ij}| \le 1, i, j = 1,\dots n\}$$
$$\Delta_b = \{\Sigma \in \mathbb{R}^{nm\times nm} | \Sigma = \text{diag}(\delta_{11},\dots,\delta_{1m},\dots,\delta_{n1},\dots,\delta_{nm}, |\delta_{ij}| \le 1, i = 1,\dots n; j = 1,\dots,m\}$$

Thus set of uncertain matrices pairs is:

$$\Pi_2 = \{(\hat{A},\hat{B}_2) \,|\, \hat{A} = A + H\Sigma F_1,\ \hat{B}_2 = B_2 + H\Sigma F_2\} \tag{16}$$

where, $H = [H_a \quad H_b]$, Σ = diag (Σ_a, Σ_b) and $\Sigma_a \in \Delta_a$, $\Sigma_b \in \Delta_b$. $F_1 = [F_a^T \quad 0]^T$, $F_2 = [0 \quad F_b^T]^T$. It's obvious that Σ satisfies Eq. 3.

It can be shown easily that, $\Pi_1 = \Pi_2$ i.e., the two descriptions, Eq. 15 and 16, are equivalent. Comparing Eq. 16 with Eq. 15, it can be concluded that:

$$[\Delta A \quad \Delta B_2] = H\Sigma[F_1 \quad F_2]$$

ACAH Out-Loop Control Design

Inner-loop controller ensures the stability of unmanned helicopter in case of parameter uncertainties, while the effect of gust disturbance is effectively reduced to minimum. Thus, unmanned helicopters have the capability of steady flight at given trim states. However, there are still significant inter-axis coupling with control law, Eq. 4. It's therefore necessary to take measures to improve the handling qualities further.

Under controller, given by Eq. 4, the state space description of the inner loop is:

$$\begin{aligned} \dot{x}(t) &= \bar{A}x(t) + B_1 w(t) + B_2 r_i(t) \\ y(t) &= C_2 x(t) \end{aligned} \tag{17}$$

where, $\bar{A} = \hat{A} + B_2 K_i$, K_i is inner-loop controller.

For ACAH flight mode, system output is $y = [w, \theta, \varphi, r]^T$. Without loss of generality, assume reference command r_i (t) as step signal with amplitude y_r.

Let tracking error be $e(t) = y(t) - y_r$. Consider the new system

$$\dot{q}(t) = y(t) - y_r \tag{18}$$

and let $\xi = [x^T \quad q^T]^T$. Then, from Eq. 17, the augmented system is:

$$\begin{aligned} \dot{\xi} &= \tilde{A}\xi + \tilde{B}_1 w + \tilde{B}_2 \tilde{u} - \tilde{y}_r \\ y &= \tilde{C}\xi \end{aligned} \tag{19}$$

Where:

$$\tilde{A} = \begin{bmatrix} \bar{A} & 0 \\ C_2 & 0 \end{bmatrix}, \tilde{B}_1 = \begin{bmatrix} B_1 \\ 0 \end{bmatrix}, \tilde{B}_2 = \begin{bmatrix} B_2 \\ 0 \end{bmatrix}, \tilde{C} = [C_2 \quad 0], \tilde{y}_r = [0 \quad y_r^T]^T \text{ and } \tilde{u} = r_i.$$

For augmented system, Eq. 19, consider the following state feedback control law:

$$\tilde{u} = [K_x K_q]\xi \triangleq K_o \xi \tag{20}$$

The closed loop system is thus given by:

$$\begin{aligned} \dot{\xi} &= \tilde{A}_d \xi + \tilde{B}_1 w - \tilde{y}_r \\ y &= \tilde{C}\xi \end{aligned} \tag{21}$$

Where:

$$\tilde{A}_d = \begin{pmatrix} \bar{A} + B_2 K_x & B_2 K_q \\ C_2 & 0 \end{pmatrix}$$

Theorem 3 Assume disturbance w(t) is time invariable. The output y(t) of system Eq. 19, can track reference signal y_r asymptotically, if there exists a state feedback control law, Eq. 20, such that $\tilde{A}_{cl}$ is Hurwitz.

Since, we have $\ddot{\xi} = \tilde{A}_d \dot{\xi}$, thus from Eq. 21,

$$\begin{pmatrix} \ddot{x} \\ \ddot{q} \end{pmatrix} = \tilde{A}_{cl} \begin{pmatrix} \dot{x} \\ \dot{q} \end{pmatrix} \tag{22}$$

Because $\tilde{A}_{cl}$ is Hurwitz, $\dot{\xi} \to 0$, then $\dot{q} \to 0$. The result follows.

By Theorem 3, the tracking problem to given constant signal is formulated as state feedback stabilization problem for the augmented system. For MIMO systems, asymptotically tracking to command signal mean that the system is steady state decoupled. In order to achieve fast response and speed up command tracking performance, pole assignment techniques can be utilized to select the closed loop poles of augmented system.

APPLICATION EXAMPLE

Here, present study shows the effectiveness of the control structure under obtained theorem through a flight control design example for Westland Lynx helicopter. The problem setup comes from Luo *et al*. (2003), where additional motivations and details can be found. The uncertainty is considered as 10% of parameter perturbation in stability and control derivative matrices. Using the technique proposed earlier, the interval uncertainty is transformed to the form of Eq. 2. Gust disturbance is formulated as, $w(t) = [u_g, w_g, v_g]^T$, gnst velocity components in body coordinates x, z, y-axis. The disturbance input matrix B_1 is determined, as stated by Etele (2006). The objective for iuner loop is to minimize effect of gnst disturbance to ACAH mode flight states output $[w, \theta, \varphi, r]^T$ and control input, $u = [\delta_0, \delta_{1s}, \delta_{1c}, \delta_T]^T$. Thus output z is chosen as $z = [y^T \; u^T]^T$ and the corresponding matrices C_1 and D_{12} are determined.

MatLAB LMI Toolbox is used for inner-loop robust control design. The desired inner-loop poles are in the disk region D(−10, 9), i.e., $\alpha = 1$ and r = 9. Using Theorem 2 and γ iteration, the optimal robust inner-loop controller is obtained with gust suppression index $\gamma = 0.1069$.

The outer loop controller is designed using pole assignment to augmented system, Eq. 19. The desired closed loop poles are:

$$\gamma_d = \{-0.0031, -0.0032, -3.92, -5.20, -10.20, -10.028, -10.028, -18.92, -4.56\pm4.91i, -4.56\pm4.91i\}$$

By using MatLAB, the outer-loop controller is designed. The poles obtained for the augmented system are,

$$\gamma = \{-0.0031, -0.0032, -4.3318, -5.1742, -10.0285, -10.1737, -11.9301, -16.6577, -4.56\pm4.91i, -4.56\pm4.91i\}$$

The related matrices and designed controllers are given in Appendix.

Based on the proposed multi-loop control structure and designed multi-mode flight controllers, flight performance of Westland Lynx helicopter is verified. Robust stability and robust gust suppression performance is evaluated when the helicopter is in steady flight with inner-loop controller. Response characteristics and command tracking performance are examined when the helicopter is in ACAH mode, with multi-loop controllers. The parameter uncertainties used in simulation are as -10% (sys_L), 0% (sys_0) and +10% (sys_H) of all aerodynamic derivatives. The gust model is the discrete gust model as defined in MIL-8785B (Gong and Yang, 2004). The parameters for the discrete gust model are V_m = 20, 10 and 10 m sec^{-1} and t_m = 0.5 sec for forward/vertical/lateral gust velocity components, respectively. The gust disturbance is beginning at 0 sec.

Figure 1 shows the inner-loop poles of the helicopter. As expected, all the inner-loop poles of the helicopter system are assigned in the prescribed locations for all admissible uncertainties.

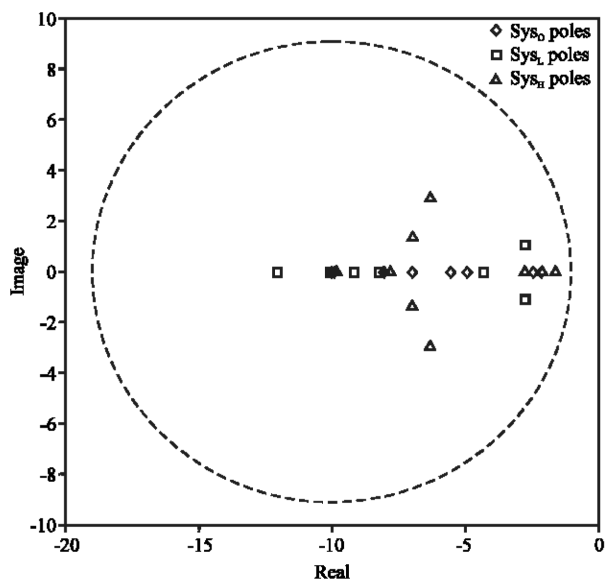

Fig. 1: Closed-loop poles for inner-loop system: dash circle denotes the disk region of D (-10, 9)

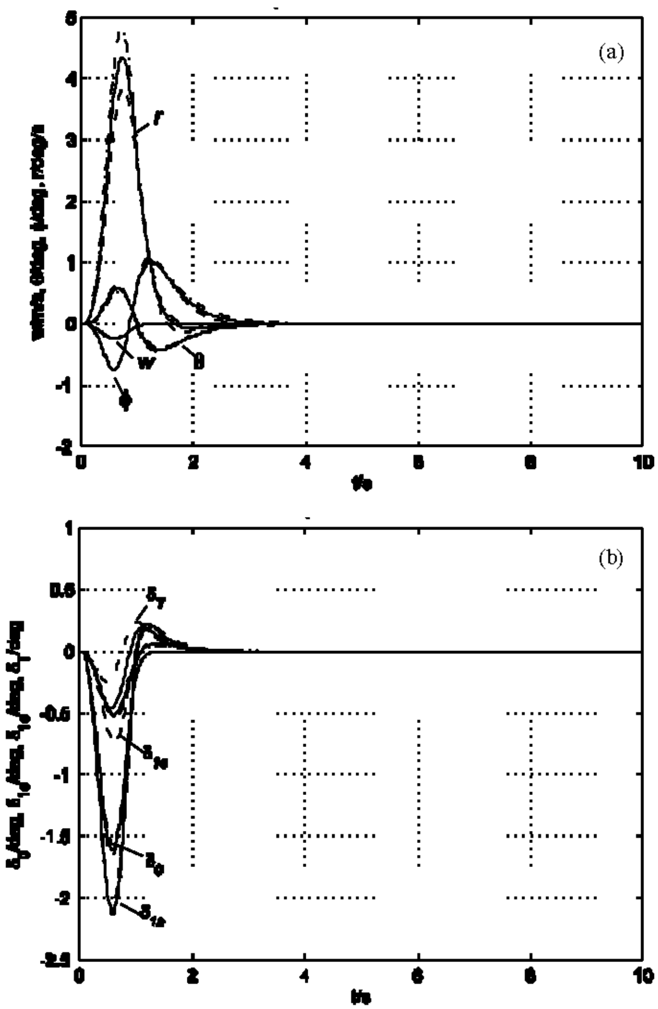

Fig. 2: Helicopter responses to gust in steady flight. Simulated states and control responses for perturbed system, sys_0 (solid), sys_L(dash-dotted) and sys_H (dash-dotted), (a) Responses of flight states and (b) Responses of control input

The helicopter responses to gust disturbance are as shown in Fig. 2. For the three different uncertainties, responses to gust disturbance show good consistent results. The maximal attitude change of the helicopter is less than 1° and peak of yaw rate is within 5 deg sec^{-1}, under given gust disturbance. The flight states concerned converge very fast with ouly one slightly oscillating. Moreover, the effects of gust to control inputs are small, less than -1° for δ_{1c} and δ_T and less than 2.5° for δ_0 and δ_{1s}. As expected, the helicopter with inner-loop controller, exhibits good robust stability and gust suppression performance.

Step input responses of the helicopter for ACAH flight mode are as shown in Fig. 3. The helicopter tracks the step commands fast and accurately. Though there still exists inter-axis coupling, the amplitude of off-axis response is quite small. It is seen that the couplings between pitch and yaw is less than 25%. Hence, level 1 requirements, as in ADS-33E-PRF are achieved.

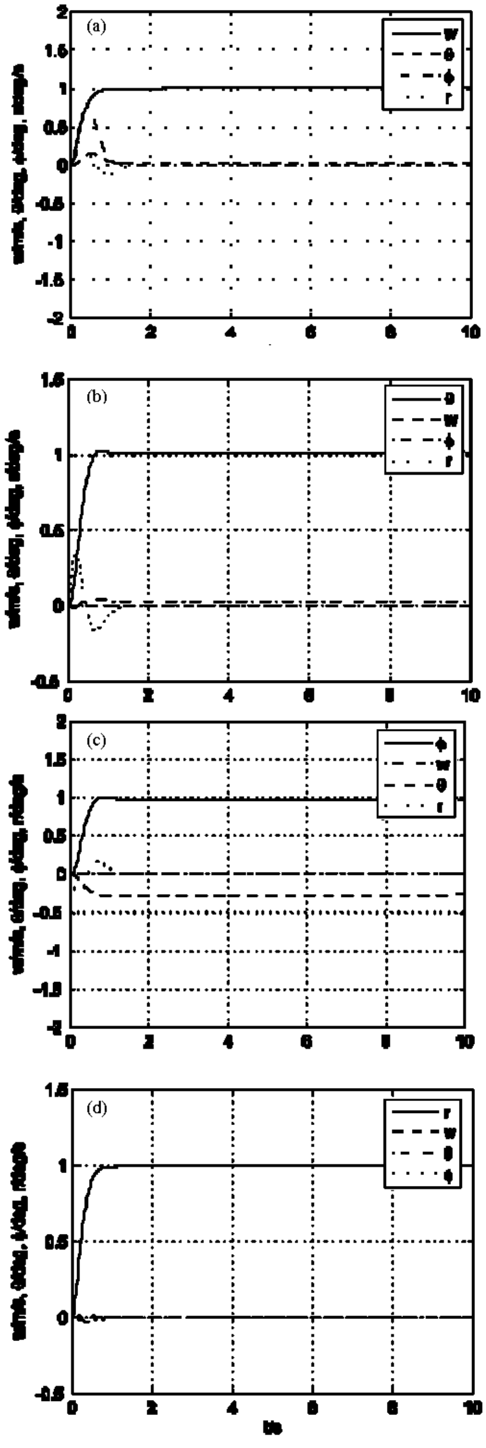

Fig. 3: Step input responses of helicopter in ACAH mode (a) Resonse to w command, (b) Response of pitch command, (c) Response of roll command and (d) Response of yaw rate comman

CONCLUSIONS

In this study, the most important aspects that affect the flight performance of helicopter, i.e., unstable flight dynamics, model uncertainty and gust disturbance, are considered during the design of helicopter flight controller. By integrating advanced control techniques with appropriate control structure, robust multi-mode helicopter flight control is designed which achieves, gust-disturbance rejection, good command following and insensitivity to parameter variations with flight conditions. Furthermore, using the multi-loop structure combining Robust H_∞ D-stabilization optimal control and PI tracking configuration, the order of the designed controller is much lower than that of other H_∞ based approaches such as mixed sensitivity optimization and loop shaping technique. Thus the implementation issue is also effectively addressed.

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APPENDIX

The relative matrices used in this study and designed controllers are given as follows:

- Helicopter model

$$A = \begin{bmatrix} -0.0191 & 0.0170 & 0.3839 & -9.7924 & -0.0008 & -0.3371 & 0 & 0 \\ 0.0136 & -0.2994 & 0.0237 & -0.5859 & -0.0017 & -0.0257 & 0.5374 & 0 \\ 0.0405 & -0.0026 & -1.8394 & 0 & 0.0024 & 0.5281 & 0 & -0.0015 \\ 0 & 0 & 0.9985 & 0 & 0 & 0 & 0 & 0.0549 \\ 0.0010 & -0.0017 & -0.3381 & 0.0322 & -0.0349 & -0.4032 & 9.7777 & 0.1168 \\ 0.0130 & 0 & -3.0470 & 0 & -0.2290 & -10.6199 & 0 & -0.0333 \\ 0 & 0 & -0.0033 & 0 & 0 & 1.0000 & 0 & 0.0598 \\ 0.0020 & 0.0060 & -0.5412 & 0 & 0.0039 & -1.8554 & 0 & -0.3487 \end{bmatrix}$$

$$B_2 = \begin{bmatrix} 5.2424 & -10.3456 & 1.0793 & 0 \\ -87.0103 & -0.7293 & 0.0755 & 0 \\ -1.5019 & 27.0900 & -4.7239 & -0.1857 \\ 0 & 0 & 0 & 0 \\ -0.3885 & -1.0820 & -10.3713 & 4.7239 \\ 7.5007 & -27.2884 & -156.4425 & -1.0690 \\ 0 & 0 & 0 & 0 \\ 17.7373 & -4.8969 & -27.9728 & -12.9304 \end{bmatrix}, \quad B_1 = \begin{bmatrix} -0.0191 & 0.0170 & -0.0008 \\ 0.0136 & -0.2994 & -0.0017 \\ 0.0405 & -0.0026 & 0.0024 \\ 0 & 0 & 0 \\ 0.0010 & -0.0017 & -0.0349 \\ 0.0130 & 0 & -0.2290 \\ 0 & 0 & 0 \\ 0.0020 & 0.0060 & 0.0039 \end{bmatrix}$$

- Controllers

$$V = \begin{bmatrix} -0.0454 & 0.3450 & 0.0641 & -0.0063 & -0.0132 & 0.0175 & 0.0013 & 0.0019 \\ 0.3497 & -0.1998 & -1.2591 & 0.0368 & -0.0831 & -0.0447 & 0.0127 & 0.0881 \\ 0.0051 & 0.0490 & 0.0917 & 0.0127 & -0.0568 & -0.0089 & 0.0271 & -0.0079 \\ 0.0899 & 0.0250 & -0.1839 & 0.0122 & -0.0243 & -0.1319 & -0.0019 & 0.6157 \end{bmatrix}$$

$$X = \begin{bmatrix} 8.3029 & -0.3742 & -3.6737 & 1.4965 & -0.2090 & 0.0991 & -0.0367 & 0.3762 \\ -0.3742 & 2.8540 & 0.4963 & -0.0576 & -0.0198 & 0.1998 & -0.0024 & -0.2651 \\ -3.6737 & 0.4963 & 6.9308 & -0.9934 & 0.3958 & -0.1352 & 0.0250 & -0.5079 \\ 1.4965 & -0.0576 & -0.9934 & 0.3335 & -0.0643 & -0.0022 & -0.0035 & 0.0665 \\ -0.2090 & -0.0198 & 0.3958 & -0.0643 & 4.8084 & 0.7679 & -0.8420 & -0.3465 \\ 0.0991 & 0.1998 & -0.1352 & -0.0022 & 0.7679 & 5.7219 & -0.5619 & 0.8264 \\ -0.0367 & -0.0024 & 0.0250 & -0.0035 & -0.8420 & -0.5619 & 0.2201 & -0.0802 \\ 0.3762 & -0.2651 & -0.5079 & 0.0665 & -0.3465 & 0.8264 & -0.0802 & 1.2969 \end{bmatrix}$$

The inner-loop controller,

$K_i = VX^{-1}$

$$= \begin{bmatrix} -0.0053 & 0.1237 & 0.0044 & 0.0340 & 0.0080 & -0.0030 & 0.0423 & 0.0350 \\ 0.2897 & -0.0175 & -0.3492 & -2.2201 & 0.0322 & -0.0009 & 0.2234 & -0.0196 \\ -0.0530 & 0.0132 & 0.0406 & 0.4107 & 0.0676 & 0.0323 & 0.4697 & 0.0333 \\ 0.0071 & 0.0743 & -0.0102 & -0.0954 & 0.1796 & -0.0685 & 0.7335 & 0.6257 \end{bmatrix}$$

The out-loop controller $K_o = [K_x, \quad K_q]$,

Where:

$$K_x = \begin{bmatrix} 0.0058 & 0.0784 & -0.0406 & -0.5895 & -0.0630 & 0.0265 & 0.0922 & -0.0563 \\ -0.2959 & 0.0372 & -0.3896 & -3.6163 & -0.5356 & 0.1181 & 0.3861 & -0.0413 \\ 0.0616 & 0.0099 & 0.0552 & 0.4325 & 0.0075 & 0.0154 & 0.3833 & -0.0158 \\ -0.0305 & 0.1655 & 0.0496 & 1.8060 & -0.0507 & -0.2868 & -4.1502 & 0.5474 \end{bmatrix}$$

$$K_q = \begin{bmatrix} 0.7839 & -2.0139 & 1.0568 & 0.0533 \\ 0.1295 & -18.1080 & 5.8510 & 1.4857 \\ 0.1551 & 2.6986 & 2.1181 & -0.1096 \\ 0.6278 & 4.6997 & -11.5657 & 3.7155 \end{bmatrix}$$

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