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Numerical Investigations of Aeroelastic Divergence Parameter of Unguided Launch Vehicles

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ABSTRACT
The present study focuses on the numerical investigations of stability margin for long, slender and relatively flexible unguided launch vehicles while avoiding the aeroelastic divergence conditions. The aerodynamic loads of the complete configuration are computed by using the modified three-dimensional low order panel method. The feasibility of using the transfer matrix method to estimate the natural frequencies and mode shapes for the launch vehicle is explored theoretically. Based on the vibration characteristics, the total dynamic response of the system has been evaluated when a sharp edged gust is encountered during the flight. The static aeroelastic characteristics, the divergence speed of a given launch vehicle have been studied by variation of the initial angle-of-attack and dynamic pressure. Computational analysis and discussion along with pertinent conclusions are presented.

Key words: Aeroelastic divergence, dynamic analysis, Transfer Matrix Method (TMM), panel method

INTRODUCTION
Aeroelasticity is concerned with problems in which there is substantial interaction among the aerodynamic, inertial and structural forces of an object. When a body moves through the atmosphere, or when a body is placed in a wind tunnel, aerodynamic forces act over its surface. If the body is deformed, there is a change in the magnitude and distribution of these surface forces. This redistribution causes additional deformations; the result is an interactive feedback loop between aerodynamic loads and aircraft deflections (Bisplinghoff *et al.*, 1996; Heeg, 2000). The speed of a launch vehicle above which no statically stable equilibrium condition exists and the deformation will increase up to a structural failure is called the divergence speed (Dowell *et al.*, 1996). The phenomenon of aeroelastic divergence is recognized to be of primary importance in the design of high performance, highly flexible launch vehicles. It is characterized by an unstable flight condition arising from the adverse interaction of aerodynamic forces and elastic deformation of the vehicle structure (Hodges and Pierce, 2002). Knowledge of the aeroelastic divergence characteristics of launch vehicles is essential to their design, since divergence is a function of the structural stiffness, mass distribution, distributed aerodynamic characteristics and engine thrust (Dowell *et al.*, 2003). Also, it is important to realize that conventional structural-design considerations are influenced primarily by the strength of the structure, whereas aeroelastic design focuses on the rigidity, aerodynamic shape, dynamic behavior and damping characteristics of the

structure. Failure to consider aeroelastic effects could very well lead to the destruction of the vehicle (Vernon and Harper, 1966; Clarence, 1967, 1968; Walters and Rister, 1971; Librescu *et al.*, 2002).

The evaluation of aeroelastic divergence characteristics is an important function in qualifying the aerodynamic and structural compatibility of a new launch vehicle configuration. Aeroelastic divergence theory specifically oriented to slender bodies is particularly necessary by virtue of the high performance of present day space launch vehicles (Capri *et al.*, 2006; Trikha and Pandiyan, 2008).

Among the tools routinely used in the aircraft/launch vehicles industry to analyze potential flow over complete launch vehicle configurations are the programs based on panel methods. These methods are, in principle, capable of analyzing almost entirely arbitrary configurations, within the limitations of potential flow computations. Also, panel methods have the distinct advantage over alternative field discretization techniques (finite difference, finite volume, etc.) in the fact that the unknowns are situated only on the surface of the configuration and not throughout the external space like the solution of the full Navier-Stokes equations which require discretization of the whole fluid domain (Wilcox, 2006). It is this quality of panel methods that makes them very attractive for routine use and also amenable for use on medium or even small computing facilities (Woodward, 1973, 1980).

The Transfer Matrix Method (TMM) has been developed for a long time and has been used widely in structure mechanics and rotor dynamics of linear time invariant system. Transfer matrices have been applied to a wide variety of engineering programs by a number of researchers. Holzer initially applied TMM to solve the problems of torsion vibrations of rods (Holzer, 1921). Myklestad (1945) applied TMM to determine the bending-torsion modes of beams, Thomson (1950) applied TMM to more general vibration problems and Pestel and Leckie (1963) listed transfer matrices for elastomechanical elements up to 12th-order. Many researchers, such as, Ohga and Shigematus (1987), Xue (1994), Loewy *et al.* (1985) and Loewy and Bhntani (1999) studied and improved the finite element transfer matrix for structure dynamics. Up to the present, Rui and Lu (1995) and Rui *et al.* (1993) developed TMM of Multibody System (MS-TMM) for vibrations analysis of linear multibody system by developing new transfer matrices and orthogonal property of multibody system.

The evaluation of the launch-vehicle design in view of its aeroelastic divergence characteristics requires a theoretical analysis which must be adequate for predicting this divergence behavior. Since, most launch vehicles are designed to perform orbital, probe and reentry missions, the dynamic conditions imposed on the vehicle are normally very stringent and require a high degree of sophistication in the aeroelastic divergence analysis.

In the present study, a theoretical method for analyzing the aeroelastic divergence behavior of unguided, slender-body, launch vehicles is presented. This accomplished by three steps. Firstly, a modified three dimensional low order panel method has been used to estimate the aerodynamic loads of the complete configuration; secondly, the launch vehicle vibration characteristics (linear eigenvalues and linear eigenvectors) are investigated using TMM; thirdly, a principle of superposition is utilized in finding the total dynamic response of the system.

MATERIALS AND METHODS

Theory: The major assumptions were considered in the derivations of the equations of motion are (Vernon and Harper, 1966):

- **Beam theory:** It is assumed that elastic deformations of the vehicle body are described by elementary bending theory of beams. The usual assumptions of small deflections are utilized. All effects due to axial loads are considered to be negligible
- **Structural representation:** The vehicle body structural characteristics are simulated by a lumped mass-spring system
- **Panel aerodynamic:** The solution requires linear aerodynamic coefficients and also the assumption of small angles of attack is made (less than 10 degrees) within the limitation of potential flow
- **Two-dimensional analysis:** The vehicle is considered as non-spinning and only lateral displacements and pitching rotations are considered
- **Steady-state analysis:** The analysis is applicable only for a specific time in the vehicle flight where structural characteristics and aerodynamics are constant
- **External forces:** The external forces are averaged over an integration time interval and assumed constant during a time interval

Computational aerodynamic analysis: Panel methods have been demonstrated over the past thirty years or more to be a most useful tool for computation of fluid flows, in different industrial applications as well as in research. Even though these methods are based on the formulation of the potential flow, many problems involving real viscous flows around solid bodies can be dealt with too close approximation with them. Hence, the wide acceptance of panel methods is found over years. Furthermore, it is important that panel methods can produce a solution of the flow with only discretization of the solid surface, compared with more exact approaches like the solution of full Navier-Stokes equations which require discretization of the whole fluid domain.

The problem of calculating the surface pressures, forces and moments acting on an arbitrary wing-body-tail combination is solved (after the configuration is subdivided into a large number of planar panels) by representing it in a system of sources, doublets and vortex singularities. The effects of the body volume, the incidence and the camber are simulated by line sources and doublets distributed along the body axis; the effects of the wing thickness are represented by planar source distributions; and wing camber, twist and incidence effects by planar vortex distributions. The interference effect of the wing on the body is provided by additional vortex distributions located on the body surface. The strengths of these singularities are determined by satisfying the condition of the tangential flow at panel control points for given Mach number and angle-of-attack (Woodward, 1973, 1980; Abbas *et al.*, 2006; Abbas and Chen, 2007).

For small perturbations, the governing equation of the potential flow can be simplified greatly. Thus, the solution of many problems becomes possible. The linearized three-dimensional potential equation for steady state flow is (Morino and Luo, 1974):

$$\beta^2\varphi_{xx} + \varphi_{yy} + \varphi_{zz} = 0 \tag{1}$$

where, φ is the velocity potential and β^2 is the compressibility parameter and depends upon Mach number Ma, except for the case of the transonic flow, Eq. 1 is valid for subsonic (β^2 = 1-Ma^2) and (β^2 = Ma^2-1) supersonic flows. The solution of the potential flow is used to predict the pressure distribution on the configuration. The pressure coefficient can be calculated using the exact isentropic formula at the control point i of a panel:

$$C_{p_i} = \frac{-2}{\gamma Ma^2}\left\{\left[1+\frac{\gamma-1}{2}Ma^2\left(1-p_i^2\right)\right]^{3.5}-1\right\} \tag{2}$$

where, γ is the ratio of specific heat, for ideal gas (air) = 1.4, $p_i^2 = u_i^2 + v_i^2 + w_i^2$ is the resultant velocity and u_i, v_i and w_i are components of the velocity at panel i in x, y and z coordinates. The velocity components in terms of the perturbation velocity potential are:

$$u = U_\infty + \partial\phi/\partial x \quad , \quad v = \partial\phi/\partial y \quad , \quad w = \partial\phi/\partial z \tag{3}$$

where, U_∞ is the stream velocity. To compute the surface pressures, normal force and pitching moment coefficients C_N and C_M acting on an arbitrary launch vehicle model, the same mathematical modeling of Abbas and Chen (2007) has been implemented.

Formulation of transfer matrix relation and vibration characteristics: A structure or a beam can be divided into segments. A typical segment of a beam consists of a massless span and point mass. The flexural property of the segment is described by the field transfer matrix of the span; the internal effect of the segment is described by the point transfer matrix of the mass. The state variables considered are Y_1, ϕ, M_s and V_s where Y_1 is the lateral deflection, ϕ is the slope, M_s is the structural moment and V_s is the structural shear. The corresponding state vector $\{Z_{state}\}_j$ at station-j is (Francis *et al.*, 1978).

$$\{Z_{state}\}_j = [Y_1 \quad \phi \quad M_s \quad V_s]_j^T \tag{4a}$$

The overall beam transfer equation and transfer matrix U, which relates the state vectors at ends of the beam, can be assembled and calculated. That is,

$$\{Z_{state}\}_j = \mathbf{U}\{Z_{state}\}_0 \tag{4b}$$

$$U = U_j U_{j-1} \cdots U_2 U_1 \tag{4c}$$

And the transfer matrix of the segment-j is (Francis *et al.*, 1978).

$$\mathbf{U}_j = \begin{bmatrix} 1 & L & \frac{L^2}{2EI} & -\frac{L^3}{6EI} \\ 0 & 1 & \frac{L}{EI} & -\frac{L^2}{2EI} \\ 0 & -\omega^2 I_r & 1-\omega^2 I_r \frac{L}{EI} & -L+\omega^2 I_r \frac{L^2}{EI} \\ -\omega^2 m & -\omega^2 mL & -\omega^2 m \frac{L^2}{2EI} & 1+\omega^2 m \frac{L^3}{6EI} \end{bmatrix}_j \tag{4d}$$

where, m is the lumped mass, L is the length between lumped masses, EI is the product of modulus of elasticity and bending moment of inertia (flexural rigidity), I_r is the rotary inertia about a diameter and ω is the natural frequency. The vibrational characteristics corresponding to the different boundary conditions can be obtained. For the problem under consideration, both edges of beam are free, i.e., Y_1 and ϕ are unknown and non-zero. Substituting these boundary conditions into Eq. 4b, the eigenvalue equations (frequency equations) can be derived. It is required that the determinant after eliminating the boundary conditions should be equal to zero. Hence, the natural frequencies of the system are determined, as shown below:

$$\Delta\,(\omega) = 0 \tag{5}$$

The procedure for natural frequency ω calculation is to assume a frequency as in Holzer method and proceed with the computation (Francis *et al*., 1978). Once ω is determined, the state vectors (normal modes) $(\hat{Z}_{state})_j$ can be calculated for each frequency (Rui *et al*., 2008).

System dynamic response analysis: The principle of superposition is utilized in finding the total dynamic response of the system. The structural system is linear and a set of external forces applied to the structure will independently excite a response in each normal modes of vibration. The total system dynamic response is then found by the summation of the contributions of each mode. Following the mathematical modeling which defined in Vernon and Harper (1966) and Walters and Rister (1971), the lateral displacement $\bar{x}_{jt}$ and $\dot{\bar{x}}_{jt}$ velocity for r degrees of freedom system at mass point j are:

$$\begin{aligned}\bar{x}_{jt} &= \sum_{k=1}^{k=r}\left[(A_k - D_k)\cos\omega_k t + B_k \sin\omega_k t + D_k\right] X_{jk}\\ \dot{\bar{x}}_{jt} &= \sum_{k=1}^{k=r}\omega_k\left[-(A_k - D_k)\sin\omega_k t + B_k \cos\omega_k t\right] X_{jk}\end{aligned} \tag{6a}$$

Where:

$$A_k = \sum_{j=1}^{j=r}\bar{x}_{j\,t_0} m_j X_{jk}\ ,\quad B_k = \sum_{j=1}^{j=r}\dot{\bar{x}}_{j\,t_0} m_j X_{jk}\big/\omega_k\ ,\quad D_k = \sum_{j=1}^{j=r}F_j X_{jk}\big/\omega_k^2 \tag{6b}$$

where, subscript k denotes each mode and F is the aerodynamic loads which are the normal force N and pitching moment M. These aerodynamic loads are calculated using the local coefficient as follows:

$$N = \lambda\times\alpha\times C_N\ ,\quad M = \lambda\times\alpha\times C_M \tag{6c}$$

where, $\lambda = q_\infty \times A_{ref}$ is the product of dynamic pressure q_∞ and the reference area of the launch vehicle A_{ref} and α is the angle of attack. The initial conditions will be designated for time t_0. Time t will be defined as $t = t_0+\Delta t$ where, Δt will be sufficiently small to permit essentially small change in the values required for the integration problem. X_{jk} is the normalized value of the modal eigenvector at mass point for mode j for mode k:

$$X_{jk} = (\hat{Z}_{state})_{jk}\Big/\left(\sum_{j=1}^{j=r} m_j (\hat{Z}_{state})_{jk}^2\right)^{1/2} \tag{6d}$$

Aeroelastic divergence analysis: The launch vehicle divergence has been studied by the variation of the initial conditions and the effective dynamic pressure. The model is subjected to different values of initial disturbance (initial condition) such as a wind gust, which creates a certain, all other conditions are constant. The effective dynamic pressure is a divergence parameter and was changed by using different values for λ. Aeroelastic divergence selected method does not directly yield a single value for divergence parameter λ_{div} (Walters and Rister, 1971). Two values used for determining the launch vehicle divergence parameter which are the slope for the nose section and the slope for the tail section of the model versus time functions. Generally speaking, these two slopes represent the extreme values of slope for the vehicle with slopes for all intermediate vehicle sections between these two extreme values. The effective value of the model slope is assumed to be an average of these extreme values, at the times selected for comparison. The launch vehicle stability is determined as a function of the change in average model slopes versus time.

RESULTS AND DISCUSSION

The unguided launch vehicle selected for a sample problem is a typical model with aeroelastic divergence tendencies. The length to diameter ratio is approximately 23. The model has been divided into 10 lumped elements with each having 2 degrees of freedom, translation and rotation. The structural properties representation of the model is shown in Fig. 1a-c.

The natural frequencies eigenvalues and the corresponding mode shapes eigenvectors were calculated by use of TMM as outlined earlier. The free-free beam representation of a launch vehicle in flight is a semi-definite system, which has the first two frequencies equal to zero due to rigid body mode. The eigenvectors of these zero frequencies must be calculated independently. The first zero mode eigenvector is a simple unit translation of all elements with all rotational displacements equal to zero while the second mode eigenvector is a rotation of the rigid body about its center of gravity. The natural frequencies and its mode shapes are shown in Fig. 2a-i.

The modified computer program (wing-body-tail) of the three-dimensional low-order panel method (Abbas and Chen, 2007) has been used to validate the computations of the aerodynamic parameters (lift and pitching moment coefficients) along the length of the selected model with the data in reference (Walters and Rister, 1971) which they used an empirical method for determining the distributed aerodynamic loads on axisymmetric launch vehicle. The model surface has been divided into a suitable number of panels as shown in Fig. 3. The results show a good agreement with approximate 1.15% error. These verification results are shown in Fig. 4.

Different initial wind gust conditions were investigated for their influence on the stability of the launch vehicle and comparative calculations were made with α = 1°, 2° and 5° all other conditions are constant. The dynamic response is calculated as a function of time t = 0→0.018 sec

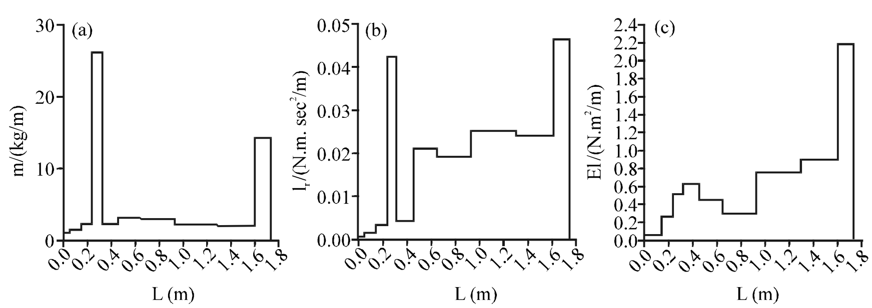

Fig. 1: Selected model-structural properties along the launch vehicle length (a) Lumped masses (b) Rotary inertia (c) Flexural rigidity

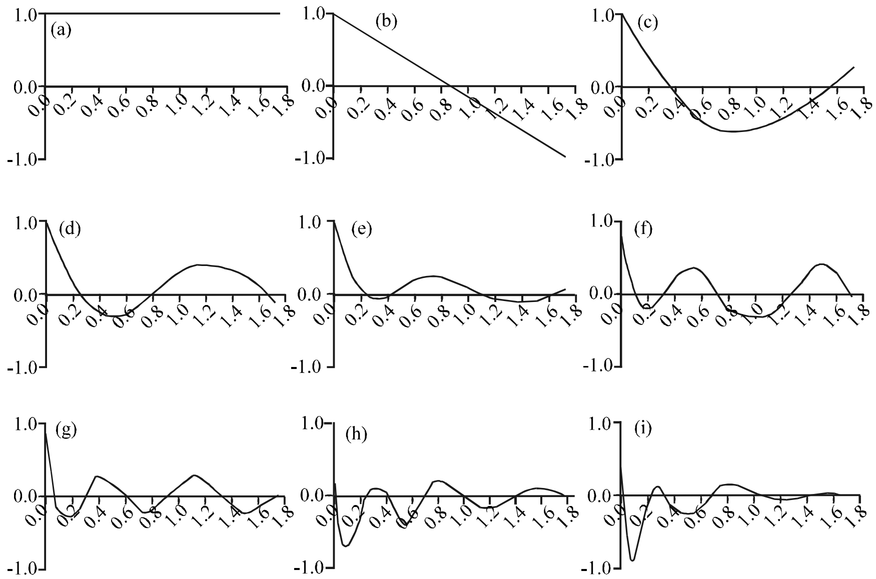

Fig. 2: Selected model-nine natural frequencies and its mode shapes along the launch vehicle length, (a) $\omega_1 = 0$ rad sec^{-1} (b) $\omega_2 = 0$ rad sec^{-1} (c) $\omega_3 = 462.9$ rad sec^{-1} (d) $\omega_4 = 1283$rad sec^{-1} (e) $\omega_5 = 2042$ rad sec^{-1} (f) $\omega_6 = 3370$ rad sec^{-1} (g) $\omega_7 = 4278$ rad sec^{-1} (h) $\omega_8 = 5830$ rad sec^{-1} (i) $\omega_9 = 6549$ rad sec^{-1}

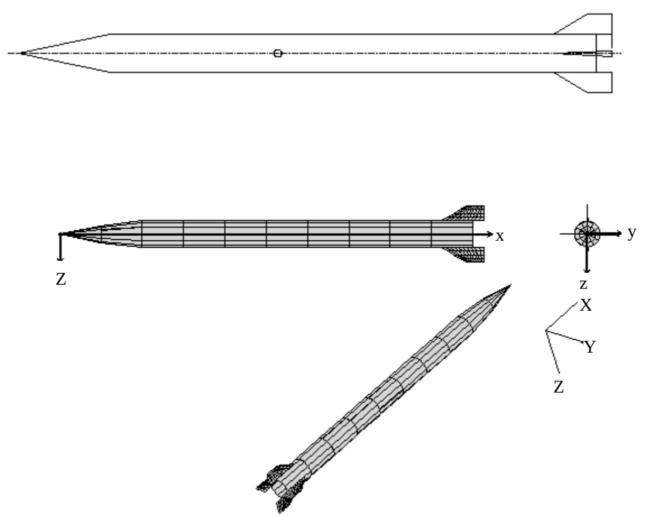

Fig. 3: Selected model-panel discretization

with time interval $\Delta t = 0.001$ sec. Figure 5a-c show the launch vehicle deflection versus time curves. The behaviors of the response curves were essentially identical with the response magnitude being direct function of the initial conditions. At t $\approx$ 0.015 sec, the values of the nose and tail section slopes are less than the starting conditions, hence average launch vehicle slope has been decreased and the vehicle is stable. It can be concluded that those initial conditions of gust strength as related to α will not affect on the stability of the launch vehicle for a specific λ.

Figure 6a highlights the effect played by the variation of λ for constant value of $\alpha = 5°$. As it can be seen, at $\lambda = 8000$ and 8500 N, the launch vehicle averaged slope crossed the starting value

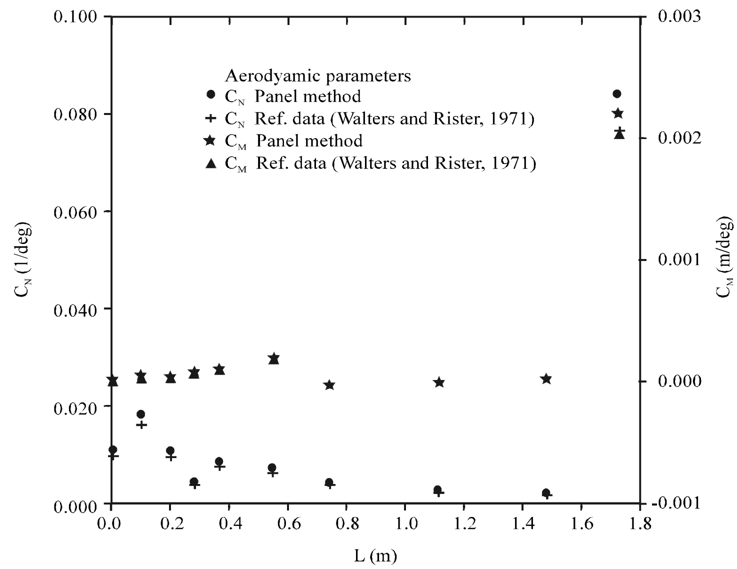

Fig. 4: Selected model-aerodynamic parameters

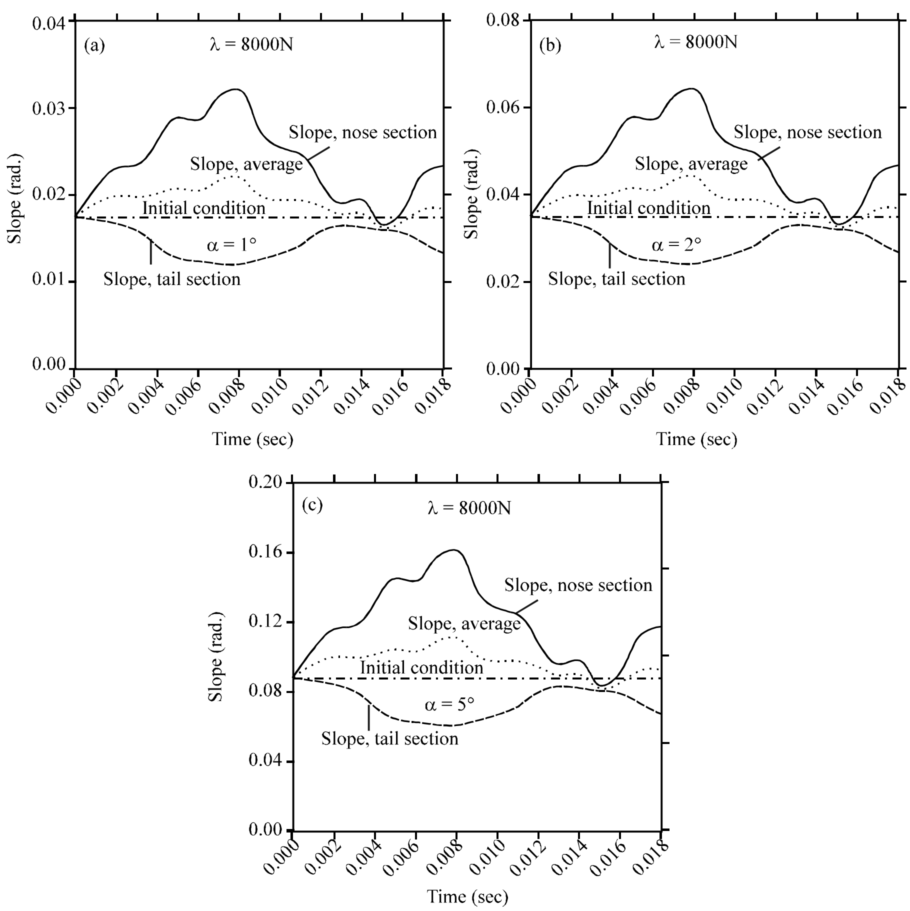

Fig. 5: (a-c) Effects of variation of α on the slope of the selected model for λ

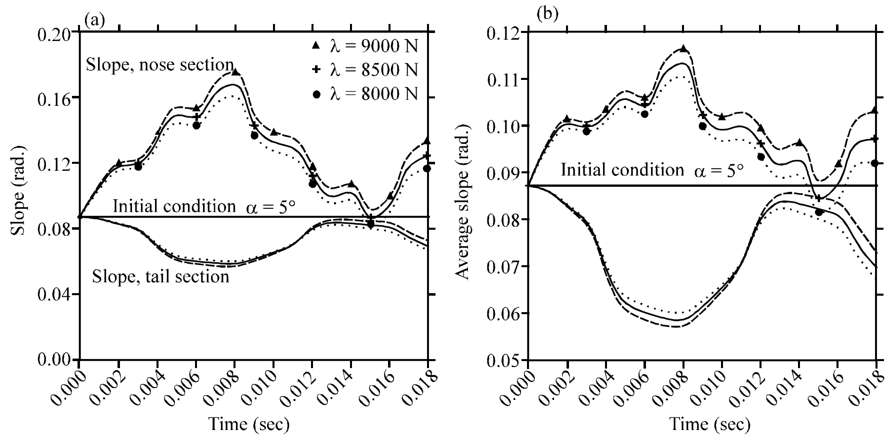

Fig. 6: Effects of variation of λ on the slope of the selected model for α = 5°. (a) Slope of nose and tail sections and (b) Average slope

Table 1: Divergence dynamic pressure (α = 5°, t ≈ 0.015 sec)

λN	ϕ_{nose} rad	ϕ_{tail} rad	$\phi_{average}$ rad	ϕ_{intial} rad	$\Delta\phi$ rad	State
8000	0.0830	0.0801	0.0816	0.0873	-0.0057	Stable
8500	0.0870	0.0823	0.0846	0.0873	-0.0027	Stable
9000	0.0918	0.0847	0.0883	0.0873	0.001	Unstable

$$\lambda_{div} \approx 8500+\frac{0.0027}{0.0037}(9000-8500)\approx 8865 \text{ N} \quad , \quad \lambda_{\%}=\frac{8865-8000}{8000}\approx 11\%$$

at t ≈ 0.015 sec (Fig. 6b). This implies that the launch vehicle is stable. Increasing λ to a value 9000 N, the averages launch vehicle slope has increased after a certain period of time which reveals that the vehicle is unstable or aeroelastically divergent. Table 1 shows the summary of the results of Fig. 6b. The stability margin based on dynamic pressure then is approximately $\lambda_{\%}$ ≈ 11%. For rigid bodies, the dynamic pressure ratio $(\lambda/\lambda_{div})\leq 0.5$ is suggested (Vernon and Harper, 1966; Clarence, 1967, 1968; Walters and Rister, 1971), static margin greater than 1 diameter is recommended. Depending on the confidence in the accuracy of the structural representation of the launch vehicle and the computations of the aerodynamic loads, it is reasonable to assume that this ratio could be closer to 1.0 for relatively elastic bodies.

CONCLUSION

Depending on the flight condition requirements during sharp edged gust wind of the unguided launch vehicle selected model and with the assistance of the three-dimensional low-order panel method program for aerodynamic computations; transfer matrix method for natural frequencies and mode shapes for vibration computations, aeroelastic divergence parameter is numerically investigated based on the principle of superposition in finding the total dynamic response of the system. A panel method has the advantage of reducing the dimension of the problem. Therefore, it has important application to the early stages of a design when the geometry of the body can change frequently in the process of optimization and the rapid response time is essential. It has been shown that variation of the initial conditions of gust strength as related to the angle-of-attack will not affect on the stability of the launch vehicle. Depending on the confidence in the accuracy of the structural representation of the launch vehicle and the computations of the aerodynamic loads, it is reasonable to assume that the ratio of (λ/λ_{div}) could be closer to 1.0 for relatively elastic bodies. Based upon results of this investigation, the theoretical method presented appears to be adequate for predicting aeroelastic divergence and for evaluating the design of any kind of unguided, single or multiple stages, relatively flexible launch vehicles in view of their aeroelastic divergence characteristics. It is desirable to employ the formulations of discrete time of transfer matrix method (DT-TMM) as a direction of future research.

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