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## Predictive Determination of the Trajectory of an Electric Discharge

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**Abstract:** Some discharge models suppose that the discharge is developed mainly according to an electric field's line. In this study, a computer program is built in order to carry out accurate determination of the electric field's lines. Finite element method is implemented to solve the Laplace equation and then the electric field is derivate. Field's lines are built by successive jumps and their parameters are calculated by a polynomial approximation. Results are successfully compared to empirical formula established by earlier researches in rod to plane geometry. The field's lines and what could be a discharge line for an aerial insulator are also investigated and discussed.

**Key words:** High voltage, field line, numerical method, electric discharge, line of discharge

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### INTRODUCTION

In discharge or partial discharge researches, electric field's line has very important role in the analysis of initiation and propagation process. It's often assumed that the discharge begins at a local point on an electrode where the electrical field becomes enough strong and it follows the field's line going from that point (Bondiou and Gallimberti, 1994 ; Giralte and Buret, 2000; Gallimberti *et al.*, 2002). In these investigations, the rod to plan geometry is generally used because the discharge's line and the point where it's initiated are known. In a real configuration, the starting point of the discharge and its trajectory are random.

Electric field calculation's softwares have permitted a significant improvement in the design of high-voltage apparatus by bringing a detailed knowledge of the field in the actual geometry (Longeot *et al.*, 1991; Meunier *et al.*, 1991; Zaho and Comber, 2000). However, the software often used presents only some lines representing equal values, the field and its coordinates at a point. Therefore, any tool giving access to field line will be useful in discharge and partial discharge studies.

Finite elements method is used to solve the Laplace's equation with a scalar potential formulation (Chari and Salon, 2000). The electrical field is then derivate. It allows the determination of the first point of the field's line. Then, using approximation methods, the propagation of the field's line is established. Both convergent and divergent cases are presented. The calculation code is validated using the result of the rod to plane structure studied by Ibrahim (1988). The field's lines and what could be a discharge's line for an aerial insulator are also investigated.

The main aim of this study is to provide the electric field's lines in any kind of electrode gap. Thus, the electrical discharge can be initiated at any point of the electrode. It may follow any field line. This project poses the base of the development of a discharge model applicable to industrial structure.

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## MATERIALS AND METHODS

### Electric Field Calculation

Three dimensional tools must be used when solving electric discharge problems. Nevertheless, most of the electrical devices can be modelled using 2D or axisymmetric formulation. The electric field cartography is determined by solving the Laplace's equation (Chari and Salon, 2000; Yeo *et al.*, 1997):

$$\varepsilon_0 \cdot \varepsilon \cdot \Delta V = 0 \quad (1)$$

where,  $V$  is the scalar electric potential,  $\varepsilon$  and  $\varepsilon_0$  are the relative and absolute dielectric permittivities, respectively. In order to have a unique solution, boundary and interface conditions are necessary. They are of the type:

$$V = V_1, V_2 \dots \quad \text{On electrodes}$$

$$\frac{\partial V}{\partial n} = 0 \quad \text{On other boundaries}$$

where,  $\frac{\partial V}{\partial n}$  is the normal derivative of the potential. Interfaces conditions express the continuity of the potential on both sides of two mediums having different permittivities.

Equation 1 is discretized by a finite element method (Chari and Salon, 2000). Elements are linear triangles. The electric field  $E$  can then be derivate from the electric potential.

$$E = -\text{grad}V$$

This field is describe by its X-coordinate  $E_x$  and Y-coordinate  $E_y$ .

$$E_x = -\frac{\partial V}{\partial x} \quad \text{and} \quad E_y = -\frac{\partial V}{\partial y} \quad \text{for 2D plan problems}$$

$$E_r = -\frac{\partial V}{\partial r} \quad \text{and} \quad E_z = -\frac{\partial V}{\partial z} \quad \text{for axisymmetric problems}$$

### Starting Point and Orientation of the Field's Line

At every point of the field line, the electric field  $E$  is tangent to the line. Let us call  $dl$  an elementary displacement of the field's line. Therefore:

$$E \wedge dl = 0$$

The direction of the electric field makes an angle  $\alpha$  with a horizontal reference axis.  $dl$  is characterized by a X-coordinate  $dx$  and a Y-coordinate  $dy$ . They are expressed as:

$$dx = dl \cdot \cos\alpha \quad \text{and} \quad dy = dl \cdot \sin\alpha$$

An electrode is described by a set of mesh nodes. Each of them can be considered as a field's line starting point.

The direction of the field is determined by its X-coordinate and Y-coordinate. The orientation of the field's line differs, depending on whether the field is divergent (going from electrode) or a convergent (oriented toward electrode). As shown in Fig. 1, for divergent field,  $\alpha$  can be expressed as:

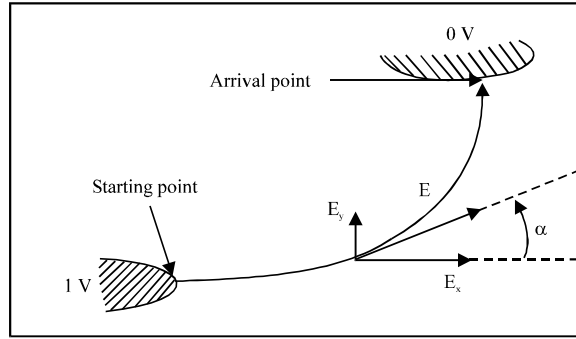


Fig. 1: Divergent field: The electric field and the electric field's line are oriented in the same direction

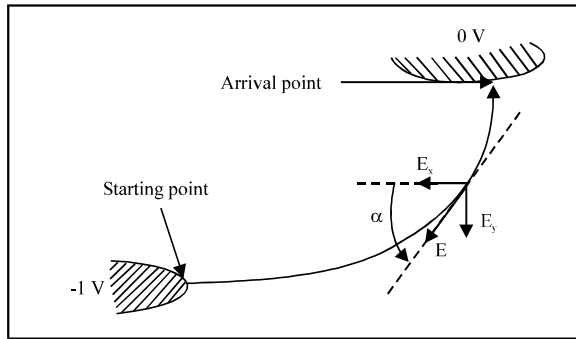


Fig. 2: Convergent field: The direction of the electric field is opposed to that of the electric field's line

$$\alpha = \tan^{-1} \left( \frac{E_y}{E_x} \right) \quad \text{if} \quad E_x \neq 0$$

$$\alpha = \text{sign}(E_y) \cdot \frac{\pi}{2} \quad \text{if} \quad E_x = 0$$

Figure 2 shows that in convergent field, field's line orientation is opposed to the direction of the field. In this study, the value of  $\alpha$  must be corrected as follows:

$$\alpha = \alpha + \pi$$

**Field Line Propagation**

The field's line evolves by successive jump according to the direction of the electric field. The length  $dl$  of a jump is a parameter depending on the geometry. A hundredth of the greatest dimension of the geometry is a best compromise between the CPU time and the computation precision. The evolution of the field line is calculated as shown by the following flowchart (Fig. 3). The last point of a field's line is reached when it attains an other electrode or goes outside the domain under consideration.

**Approximation at a Point**

The electric field's line, while progressing, goes through points other than the mesh's nodes. For these points, it is necessary to make an approximation of the geometrical and electric parameters. Let us consider two finite elements, triangles of the first order (Fig. 4).

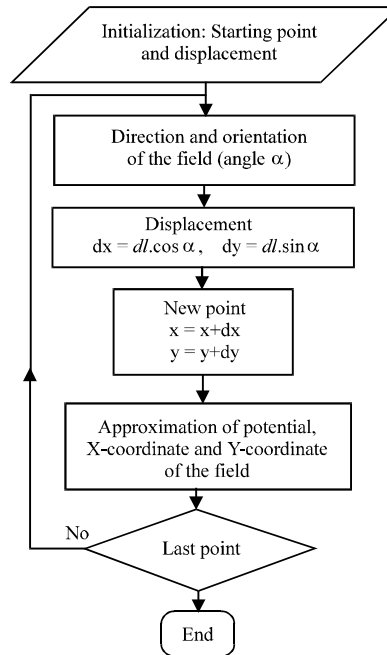


Fig. 3: Flowchart showing the stepwise propagation process of a field's line

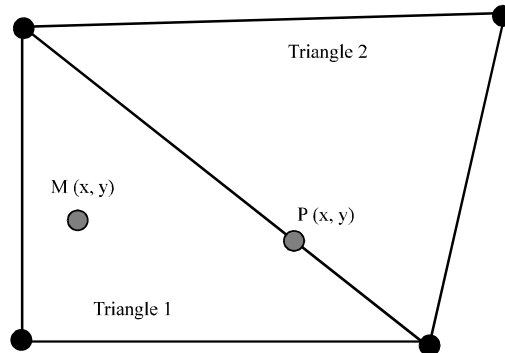


Fig. 4: Possible position of a point where the approximation must be made: Point M (x, y) belongs to one triangle, Point P (x, y) belongs to several triangles

If the point belongs to only one triangle, electric parameters are obtained by a polynomial approximation (Chari and Salon, 2000).

$$V(x, y) = a_1 + a_2x + a_3y$$

$$E_x(x, y) = b_1 + b_2x + b_3y$$

$$E_y(x, y) = c_1 + c_2x + c_3y$$

Coefficients  $a_i$ ,  $b_i$  and  $c_i$  are determined as follow:

$$[M]\{a_i\} = \{v_i\} \quad [M]\{b_i\} = \{E_{xi}\} \quad [M]\{c_i\} = \{E_{yi}\}$$

Where:

$$[M] = \begin{bmatrix} 1 & x_1 & y_1 \\ 1 & x_2 & y_2 \\ 1 & x_3 & y_3 \end{bmatrix}$$

When this point belongs to several triangles, as shown in Fig. 4, the approximation is made on each of them and an average is calculated. If H designates the potential, the X-coordinate or the Y-coordinate of the field, it can be calculated as:

$$H = \frac{\sum_{k=1}^N H_k}{N}$$

where, N being the number of triangles to which belongs the point and  $H_k$  being the approximation of H in the kth triangle.

### Line of Discharge

The field's lines previously given can be used in electric discharge's models. Indeed, it is commonly accepted that the electric discharge starts on one high voltage electrode and propagates following a field's line toward one low voltage electrode (Giralt and Buret, 2000).

The discharge's line is thus a field's line which leaves from a point where the electric field is sufficiently intense to start and maintain an electric discharge.

## RESULTS AND DISCUSSION

One difficulty, in discharge studies, is the validation of calculation codes by an analytic result or by experimental values. These difficulties have led most authors to use the rod-to-plane geometry (Ortega *et al.*, 1994; Goelian *et al.*, 1997). In this configuration, the discharge's line is simply the symmetry axis of the tip. Geometrical and electrical parameters of the field's line are reachable analytically. Photographs submitted by Hayakawa *et al.* (2005) showed that the discharge spreads along the field's line. Point-plane structure has played and continues to play an important role in the studies on the phenomena of electric discharge.

Ibrahim (1988) has proposed a formula (Eq. 2) for the rod plane gaps in the ranges 0.1-1 cm for the electrode radius and 2-12 cm for the gap spacing. If  $E(x)$  is the field per unit applied voltage at a distance x cm along the gap axis from the rod tip, then:

$$E(x) = \frac{A_1}{(r+x)^2} + A_2 \cdot \exp\left(-\frac{A_3}{x}\right) \quad (2)$$

Where:

$$\begin{cases} A_1 = (1.125 - 0.09 \ln d)(0.2625r^2 + 0.8555r - 0.017) \\ A_2 = \frac{0.25 + 0.24r}{d^{1.39}} \\ A_3 = 0.025 \exp(0.248d + 3.115r) \end{cases}$$

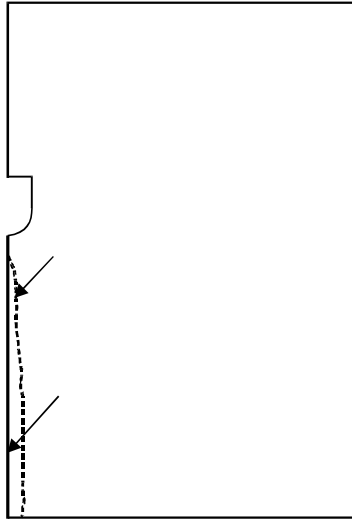


Fig. 5: Comparison between theoretical and calculated field's line, (—) Theoretical field's line and (.....) Simulated field's line

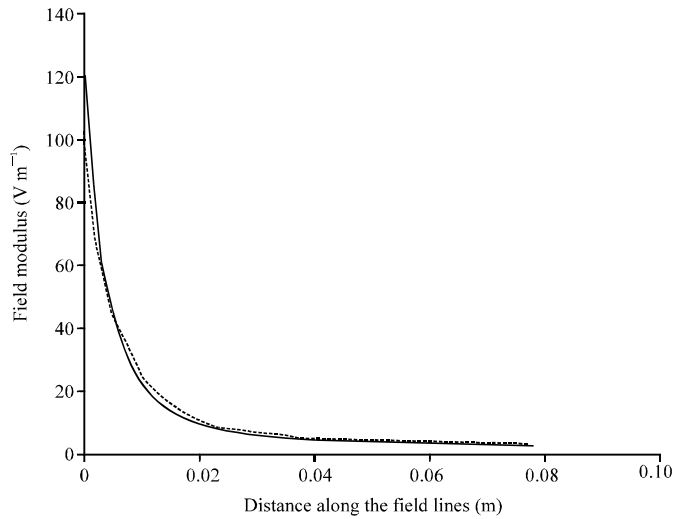


Fig. 6: Electrical field comparison between simulation and Eq. 2, (—) Eq. 2 and (.....) Simulation

First of all, let chose a geometry which observes these conditions in order to check simulation results (Electrode radius = 0.7 cm and Gap spacing = 8 cm). Figure 5 and 6 compare successfully simulations results with those of Ibrahim (1988).

It is necessary to extend the well known theories in point-plane geometry to intervals of any form. Indeed, most electrical devices can't be classed as rod-to-plane geometry (i.e., transformer, aerial insulator, ...). For an aerial insulator, a real industrial structure (Fig. 7), Fig. 8 shows all the field's lines leaving from the electrode. A discharge's model can thus be built for this geometry. The discharge would then be randomly initiated on a point of the electrode. It would follow the associated line of field.



Fig. 7: A photographs showing a link of an insulator string

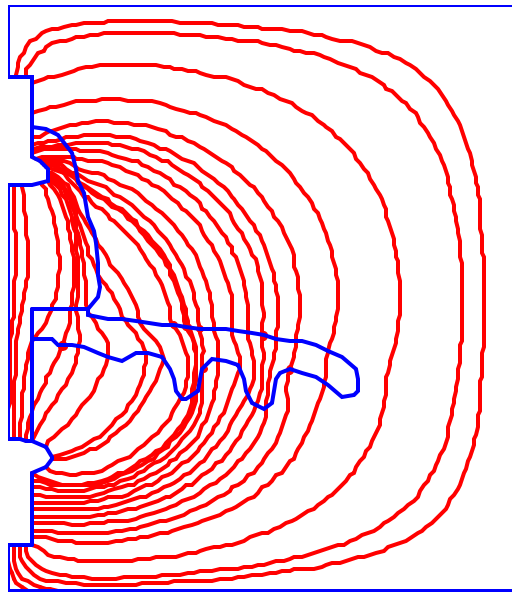


Fig. 8: Field's lines and definition of the geometry

Figure 8 shows two types of electric field lines. Some of them go through the dielectrics and others did not. These two types of electric field's lines, respectively, are useful in a perforation or flashover model. The electric field's lines are ranked in descending order according to field modulus of the starting point. The first line is therefore starting at the point where the field is most intense. It is the perforation discharge (Fig. 9). The first line which does not cross any dielectric will be the flashover line (Fig. 9). No field's line follows the shape of solid isolating material. The case of slipping discharge in which the discharge follows the dielectric's shape was not investigated.

During a discharge event, Giralt and Buret (2000) distinguishes the potential imposed by the sources and the potential due to space charges. Figure 10 and 11 show the potential and the field created by the sources for the perforation and the flashover lines.



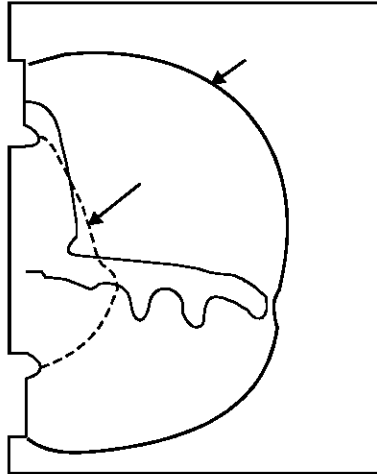


Fig. 9: The two field's lines under consideration, (.....) Perforation line and (—) Flash-over line

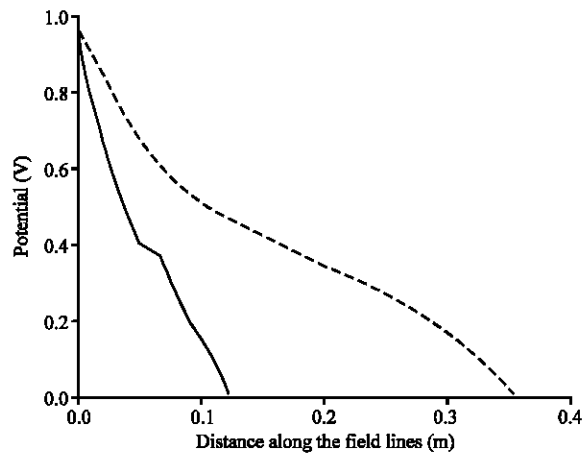


Fig. 10: Potential evolution along the lines, (—) Perforation line and (.....) Flash-over line

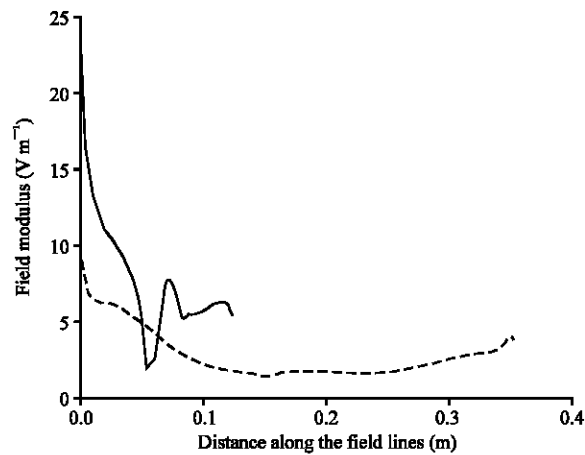


Fig. 11: Field evolution along the lines, (—) Perforation line and (.....) Flash-over line

## CONCLUSION

This study presents the interest of proposing a presumed trajectory of the discharge in high voltage devices and the possibility to expand the discharge's theory to realistic configuration. A new computer program, with the ability to determine electric field line in any arbitrary electrode's shape, has been described. The technique of the progression of the field line has been presented for divergent and convergent field. With this new method, the electric discharge can be initiated at any point on the high voltage electrode. That would make it possible to take into account the randomness of the starting point of a discharge.

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