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Dynamic Study of Double Layer Lattice Domes

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ABSTRACT
Space trusses have become widely popular, particularly in large areas such as sport center, exhibition halls and airport hangars. Base on their high degree of indeterminacy and great stiffness, the seismic designs of space trusses are neglected. Despite the earlier assumption, recent studies show that in the area which, the strong earthquakes are probable; these structures are vulnerable to seismic failures. As dynamic behavior of the structures is a function of the mass and stiffness matrixes, so in this study, the effects of structural geometry on the natural period of vibration which is depended to these two matrixes, are studied. The results show that the natural period of vibration and dynamic behavior of the double layer lattice domes depended on the geometry of these structures, especially on the span and finally, the natural period of vibration of the double layer lattice domes can be estimated by using this mentioned geometric parameter.

Key words: Space truss, lattice dome, double layer, dynamic study

INTRODUCTION
Space trusses are one of the lightest steel structures with three-dimensional and complex structural behaviours made of thousands of steel tubular bars joined together by different kind of connector such as MERO-type (Ghasemi *et al*., 2010). The very high degree of indeterminacy, their multiple redundancies and their appropriate three-dimensional geometrical form provide additional margins of safety to prevent them from the sudden collapse in the case of accidental local failure of one or more elements, when the overall loading is below the service load.

They have become widely popular as large span roof structures, particularly in areas such as sport centres, exhibition halls and airport hangars. The main advantages of these structures are that they are light in weight, have a high degree of indeterminacy and great stiffness, simple production and fast assembly, are totally prefabricated, do not need site welding, are easily formed into various attractive geometrical surfaces, have the ability to cover large areas with widely spaced column supports, have generally good response against earthquakes and are cost effective.

Study on the seismic behaviours is one of the interesting topic that is mentioned by most of the structural engineers (Adedeji and Ige, 2011; Sasan and Mohammadsadegh, 2011). With their low weight and great stiffness, space trusses are believed to attract low forces during seismic activities and can be considered to be amongst the least likely to suffer damage, when compared to other large span roofs (Marsh, 2000). Despite the earlier assumptions, recent studies show that in the area which, the strong earthquakes are probable; these structures are vulnerable to seismic failures, especially when roofs are covered with snow. Kawaguchi (1997) reported damages due to earthquake activities. Rezaiee-Pajand *el al.* (2011) presented a new method to calculate the viscous fictitious damping for dynamic relaxation on some structures such as space truss. Koohestani and Kaveh (2010) used a new method for vibration analysis of single and double layer shallow dome. Cai *el al.* (2008) studied on the seismic performance of space beam string structure. Coan and Plaut (1983) determined the dynamic response of lattice dome. Jamshidi *et al.* (2011) studied on dynamic behaviour of double layer cylindrical space truss. Sadeghi (2004) investigated the dynamic behavior of double layer barrel vaults; and showed that they are vulnerable to earthquakes and have a brittle behavior. Zhang and Lan (2000) have reviewed research findings on dynamic characteristics of space trusses. Further studies on seismic behavior of space structures are conduced by Ishikawa *et al.* (2000). The above study showed the importance of seismic evaluation of space trusses.

Equivalent static loading and dynamic analysis are the most common methods that used in the seismic analysis of civil engineering structures (Alsulayfani and Saaed, 2009). For the critical structure such as space truss, metro tunnel and some other structures, time history analysis which is a kind of dynamic analysis will be commonly used (Bagherzadeh and Ferdowsi, 2009). But it is noticeable that time history analysis is time consuming, so some of the researchers presented computer aid (El-Kafrawy and Bagchi, 2007) or try to use some new mathematical method such as wavelet method to mitigate its required time (Nadhim, 2006).

It also must regarded that dynamic analysis of a space truss is, in some manner, a cumbersome procedure due to extreme complexity of structural configuration and numerous degree of freedoms, as well as a node to node distribution of mass which would consequently results in a complicated dynamic response. So some engineers refuse to perform the dynamic analysis of space truss in their designs. avoiding of seismic evaluation of the special structure in horizontal or vertical direction (Nezamabadi *et al.*, 2008) based on the lacking of an appropriate seismic provision during their design or construction enhanced the probability of retrofitting in their service time which may need to spending too much money and complex analysis (Amiri *et al.*, 2008). The main aim of this study is to simplify the dynamic study of this type of structures. Twenty four Double Layer Lattice Domes (DLLDs) are modelled and their dynamic behaviors are studied and the effect of structural geometry on their dynamic response is investigated.

REVIEW ON DYNAMIC STUDY

First natural period of vibration as a dynamic characteristic: The free vibration dynamic equilibrium equation for a space frame with the Multi-degree of Freedom (MDF) which, their viscous damping is neglected, is presented by Eq. 1:

$$[M]\left[\ddot{v}(t)\right]+[K]\left[v(t)\right]=0 \qquad (1)$$

where, [M] and [K] are the structural mass and stiffness matrixes, respectively. $[\ddot{v}]$ and [v(t)] are the acceleration and displacement vectors of structures. Equation 2 is assumed to calculate the natural jth mode shape of the free vibration with frequency 'ω_j':

$$v_j(t)=\varphi_j \sin(\omega t-\alpha) \qquad (2)$$

and in a vector form:

$$[v]=[\varphi]\sin(\omega t-\alpha) \tag{3}$$

with replacing Eq. 3 into Eq. 1:

$$\{[M][\varphi](-\omega^2)+[K][\varphi]\}\sin(\omega t-\alpha)=0 \tag{4}$$

or:

$$\lfloor [K]-\varphi^2[M][\varphi]\rfloor=0 \tag{5}$$

This equation is called "characteristic or frequency equation" that used for a MDF system with 'N' as the number of freedom. Calculation of natural mode shapes of a Multi Degree Freedom structure is turned to finding the eigenvalues of Eq. 5. As soon as ω_j (j=1 to N) are calculated, the natural period of the vibration can be obtained by using of Eq. 6:

$$T=\frac{2\pi}{\omega} \tag{6}$$

As it is shown, in each mode, the natural period of vibration is depended on the mass and stiffness matrixes. On the other hand, the dynamic behavior of the structure is a function of these matrixes too (Eq. 1). So in this study, the natural period of vibration is chosen as a dynamic characteristic of space frames. A comprehensive study on the effect of structural configuration and geometry on main vibration mode is carried.

Effect of damping on the natural period of vibration: The free vibration dynamic equilibrium equation for Single-degree of Freedom (SDF) with viscous damping is shown by Eq. 7 and 8.

$$m\ddot{u}+c\dot{u}+ku=0 \tag{7}$$

$$\ddot{u}+\frac{c}{m}\dot{u}+\frac{k}{m}u=0 \tag{8}$$

where, m and k are the structural mass and stiffness, respectively. And 'c' is a damping factor that represented the energy dissipation in a cycle of amplitude or a period of forced harmonic vibration.

If we consider:

$$\omega n^2=\frac{k}{m}$$

$$2\xi\omega_n=\frac{c}{n}$$

Then the Eq. 8 will be:

$$\bar{u} + 2\xi\omega_n\dot{u} + \omega n^2 u = 0 \tag{9}$$

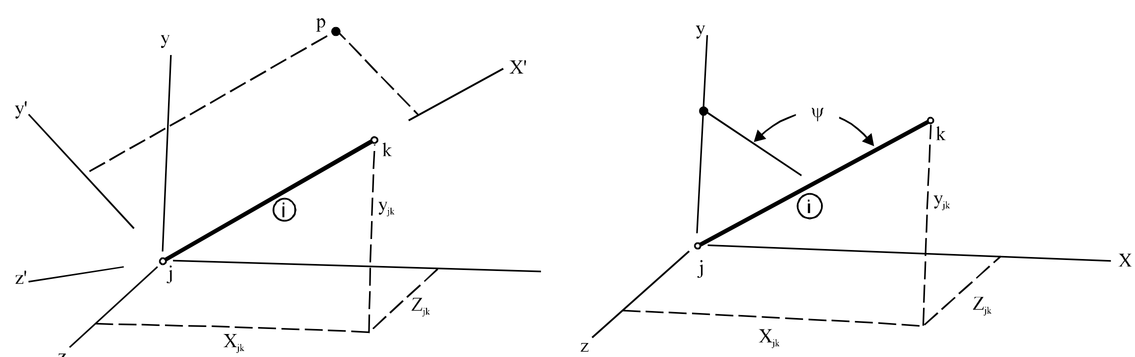
where, ξ is called, damping ratio and it's depend to mass and stiffness of the system. In most mention structure such as buildings, bridges, dams,... ξ is less than 0.1. By solving Eq. 9 for the system with ξ<1, displacement function will be arrived as Eq. 10:

$$u(t) = e^{-\xi\omega n t} * \left(u(0)\cos\omega_D t + \left(\frac{\xi\omega_n u(0) + u(0)}{\omega_D} \right) \right) \sin\left[(\omega_D)\right] \tag{10}$$

where, ω_D is equal to the $\omega_n\sqrt{1-\xi^2}$ that shows 'the natural frequency for the system with damping factor' (ω_D) is related to 'the natural frequency for the system without damping' (ω_n). Moreover, damping detracted natural frequency from ω_n to the ω_D and increased period from T_n to the T_D but for the system with ξ<20%, it's effect on the ω and T is neglected (Chopra, 1995). As the damping ratio for the common structure are located in this range, so ω_D and T_D are approximately equal to the ω_n and T_n.

Calculation the mass and stiffness matrix for the space trusses (Weaver and Johnston, 1987): An element of a truss that is hinged in the joint j and k is shown in the Fig. 1. In this study, the connections are considered in the ideal manner so the rotations in the each ends of the element are neglected. The two main flexural surfaces are defined by the surfaces that are built from y^l and z^l with x^l (local axis). In each ends, the translation in the direction x^l, y^l and z^l are shown with three numbered arrow. The stiffness matrix (6×6) for the prismatic element in the local direction is represented by the following matrix:

$$k^l = \begin{vmatrix} k^l_{jj} & k^l_{kj} \\ k^l_{kj} & k^l_{kk} \end{vmatrix} = \frac{EA}{L} \begin{bmatrix} 1 & & & & & \\ 0 & 0 & & & \text{sym} & \\ 0 & 0 & 0 & & & \\ -1 & 0 & 0 & 1 & & \\ 0 & 0 & 0 & 0 & 0 & \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$



(I) direction

(II) General direction

Fig. 1: An element of a truss

whereas, there isn't any stiffness due to connection at the joints in the perpendicular direction to the truss axis, most of the terms in the k^l will be zero. As the same way, for the local axis, the mass matrix will be shown as follows:

$$M^l = \begin{vmatrix} M_{jj}^l & M_{kj}^l \\ M_{kj}^l & M_{kk}^l \end{vmatrix} = \frac{\rho AL}{6} \begin{bmatrix} 2 & & & & & \\ 0 & 2 & & & \text{sym} & \\ 0 & 0 & 2 & & & \\ 1 & 0 & 0 & 2 & & \\ 0 & 1 & 0 & 0 & 2 & \\ 0 & 0 & 1 & 0 & 0 & 0 \end{bmatrix}$$

To form the rotation matrix, third point such as P (in addition of J and K) is used for defining flexural surfaces. This point is located in x^l-y^l surface but do not lie on the x^l axis. If possible, this point must be considered as another joint of the structure that its coordinate are determined. The sentences of the matrix will be driven by considering the properties of vector multiplication.

$$e_z{}^l = \frac{e_z{}^l \times e_{jp}}{\left|e_z{}^l \times e_{jp}\right|} \tag{11}$$

$$e_y{}^l = e_z{}^l \times e_x{}^l \tag{12}$$

where, e is a vector in its index direction. For example $e_x{}^l$ is equal to:

$$e_y{}^l = \left[c_x \; c_y \; c_z\right] \tag{13}$$

$$c_x = \frac{z_{jk}}{L}$$

$$c_y = \frac{y_{jk}}{L}$$

$$c_x = \frac{x_{jk}}{L}$$

$$L = \sqrt{x_{jk}{}^2 + y_{jk}{}^2 + z_{jk}{}^2} \tag{14}$$

The same description can be presented for the unite vector e_{jp} by using the coordinate of j and p. If the rotational matrix is used for this three unite vectors, then:

$$R = \begin{bmatrix} e_x{}^l \\ e_y{}^l \\ e_z{}^l \end{bmatrix} = \begin{bmatrix} c_x & c_y & c_z \\ \lambda_{21} & \lambda_{22} & \lambda_{23} \\ \lambda_{31} & \lambda_{32} & \lambda_{33} \end{bmatrix}$$

The $\hat{R}$ operator with 6*6 is used to transfer the stiffness matrix into the structural direction:

$$\hat{R} = \begin{bmatrix} R & 0 \\ 0 & R \end{bmatrix}$$

By using the above operator, the stiffness matrix will be presented in the structural direction as follow:

$$K = \hat{R}^T K' \hat{R} = \frac{EA}{L} \begin{bmatrix} c_x^2 & & & & & \\ c_x c_y & c_y^2 & & & \text{sym} & \\ c_x c_z & c_y c_z & -c_x c_z & & & \\ -c_x^2 & -c_x c_y & c_y c_z & c_x^2 & & \\ -c_x c_y & -c_y^2 & -c_y c_z & c_x c_y & c_y^2 & \\ -c_x c_z & -c_y c_z & -c_z^2 & c_x c_z & c_y c_z & c_z^2 \end{bmatrix}$$

Note that there are a lot of parameters that have influence on the stiffness matrix such as how the structure is connected to the earth or another structure as a support and how the elements are connected together, but when the stiffness matrix is written, the effects of these parameters are considered. So, any variation of these parameters made a change in the stiffness matrix and finally on the natural period of vibration. Consequently, in this study, the stiffness matrix is affected by the type of connection.

By using of the same method that described for the stiffness matrix, the mass matrix can be calculated by Eq. 15:

$$M = \hat{R}^T M' \hat{R} \tag{15}$$

METHOD OF STUDY

In this study, 24 Double Layer Lattice Domes roofs (DLLDs) are modelled. The geometrical features of these models are given in Table 1. In Table 1, H, S, D, α, R_t are the height, span, thickness, internal angle and outer radius of the DLLDs, respectively (Fig. 2a,b). All end nodes of inner layers are hinged to the rigid supports. They have three rotational degrees of freedom, but

Table 1: Geometrical properties of models

Name of models	S (m)	α (degree)	D (m)	R_t (m)	H (m)	Name of models	S (m)	α (degree)	D(m)	R_t (m)	H (m)
M1	10.0	45.0	0.500	7.07	2.07	M13	45.0	45.0	1.500	31.82	9.32
M2	10.0	45.0	1.000	7.07	2.07	M14	45.0	67.5	1.000	24.35	15.03
M3	10.0	67.5	0.750	5.41	3.34	M15	45.0	67.5	1.500	24.35	15.03
M4	20.0	45.0	0.750	14.14	4.14	M16	45.0	67.5	2.000	24.35	15.03
M5	20.0	67.5	0.500	10.82	6.68	M17	60.0	45.0	1.000	42.43	12.43
M6	20.0	67.5	0.750	10.82	6.68	M18	60.0	45.0	2.000	42.43	12.43
M7	20.0	67.5	1.000	10.82	6.68	M19	60.0	45.0	2.500	42.43	12.43
M8	30.0	45.0	0.500	21.21	6.21	M20	60.0	67.5	1.500	32.47	20.05
M9	30.0	45.0	1.000	21.21	6.21	M21	60.0	67.5	2.250	32.47	20.05
M10	30.0	45.0	2.000	21.21	6.21	M22	75.0	45.0	2.250	53.03	15.53
M11	30.0	67.5	0.750	16.24	10.02	M23	75.0	67.5	2.000	40.59	25.06
M12	30.0	67.5	1.500	16.24	10.02	M24	75.0	67.5	2.250	40.59	25.06

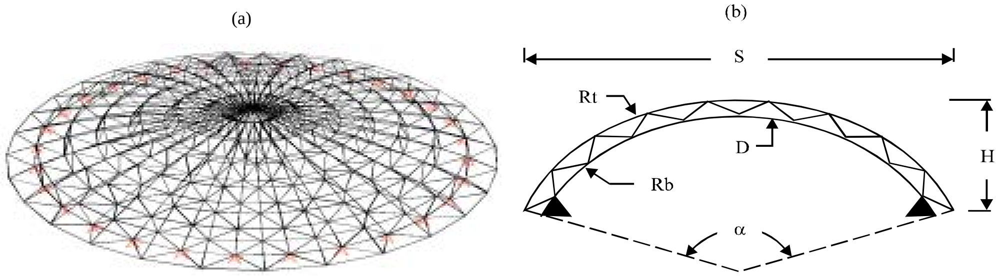


Fig. 2(a-b): General geometrical properties of models; (a) 3-d view and (b) Elevation

their transitional degrees are restrained. All end nodes that used for the connection of structural members have three transitional degrees of freedom.

Mechanical properties of material: It has been assumed that the same material is used for construction of all models. Mild steel material, with the Young's modulus of 200 GPa, Poisson's ratio of 0.3 and yield stress of 350 MPa selected for all members in of all models. The material behavior is proposed to be elastic perfectly plastic. However, in none of the models the nonlinear behavior is allowed and only linear part of material behavior is contributed in analysis.

Loading condition: One of the most significant loads on space structure is the effect of snow load. In space structures, the ratio of snow to dead load is considerably greater than the one in ordinary building. In regular buildings, the probability of coincidence of snow and earthquake loads does not play a significant role, because the snow loads are usually a negligible fraction of the total seismic weight. On the contrary, in space frames snow loads can easily reach 2 or 3 times the self-weight of a space structure. Therefore, even a very small probability of experiencing a strong seismic event when having heavy snow on the roof can lead to severe consequences such as the collapse of the roof. Hence, it is essential to consider the combination of snow and earthquakes in design (Moghaddam, 2000). In the analysis conducted in this paper, the seismic weight is assumed to be included with the whole gravity weight of structure which in a horizontal projection is assumed to be 490 Pa plus 30% of live load which is considered to be totally due to a snow load of 1370 kPa. Above conditions belongs to a weather zone with extreme cold winters. The effect of the milder situation in terms of fewer amounts of snow is considered to have negligible effect on the dynamic behavior of the space frame and is not included in the analysis presented in this paper.

Method of analysis: The mass and stiffness matrixes are calculated for any DLLDs by using of the equation that presented. To evaluate the effect of each geometrical characteristic on the mass and stiffness matrix the main parameter that defined the dynamic behavior of each structure- the natural period of vibration ((T0) are calculated by Eq. 5 and 6. And also and numerical analysis are fulfilled by finite element software-SAP2000.

RESULTS AND DISCUSSION

For any of DLLDs, the natural period of vibrations are given in Table 2. Comparing the results from analytical analysis for each DLLDs with numerical analysis that performed by finite element software SAP2000 shows the accuracy of the calculation-their difference is less than 0.001 sec.

Table 2: Natural period of vibration for each model.

Name of models	T_0 (sec)	Name of models	T_0 (sec)	Name of models	T_0 (sec)	Name of models	T_0 (sec)
M1	0.042	M7	0.119	M13	0.27	M19	0.381
M2	0.05	M8	0.216	M14	0.33	M20	0.472
M3	0.055	M9	0.182	M15	0.335	M21	0.497
M4	0.05	M10	0.143	M16	0.323	M22	0.557
M5	0.123	M11	0.21	M17	0.44	M23	0.628
M6	0.122	M12	0.205	M18	0.396	M24	0.655

Table 3: Dependence between the main time period and geometrical parameter Correlations

	S	α	D	H	R_t	S×H
Pearson's correlation	0.98	0.178	0.733	0.948	0.899	0.966

The correlation coefficient is calculated for each selected geometrical parameter to evaluate their effect on the dynamic behaviour and also the natural period of vibration of the structure. The correlation is a mathematic coefficient that determines the relation between two parameters. Two parameters are correlated together when their value change uniformly that means while one of the parameters is increased or decreased the other one is increased or decreased, too, that their relation can be defined by an equation. The correlation coefficient will be positive while these two parameters move in a same direction; otherwise if they moved in the contrary direction, its value must be negative.

In this paper the Eq. 16 (David 2009) is used to calculate the correlation coefficient.

$$r-\frac{\sum(x-\bar{x})(y-\bar{y})}{\left[\sum(x-\bar{x})^2(y-\bar{y})^2\right]^{\frac{1}{2}}} \tag{16}$$

According to Table 3, dependence between the natural period of vibration as a main characteristic of the structure's dynamic behaviour and the span of the structure is more than other characteristics, although the effect of 'Span * Rise' and 'Rise' is noticeable. In Table 3, it is also noticeable that the influence of α on the natural period of vibration is not significant but some of the researchers tried to categorize their model based on the internal angle (Coan and Plaut, 1983). It seems that the dependence of the dynamic behavior of structure to the internal degree (α) is not reasonable and it's better to study the variety of dynamic behavior based on the span.

Proposed an equation to calculate the natural period of vibration for DLLDs: The regression of Y to X is a direct line which is drawn between the sporadic points that the summation of the square deviation parallel to Y axis is minimum. If the equation of this line is y = a+bx, then the value of 'a' and 'b' can be calculated by the principle of minimum square, as follows:

$$\sum y=\sum a+b\sum x=an+b\sum x \tag{17}$$

$$\sum xy=a\sum x+b\sum x^2 \tag{18}$$

where, n is the number of the points. By solving Eq. 17 and 18, simultaneously, the value of 'a' and 'b' will be derived.

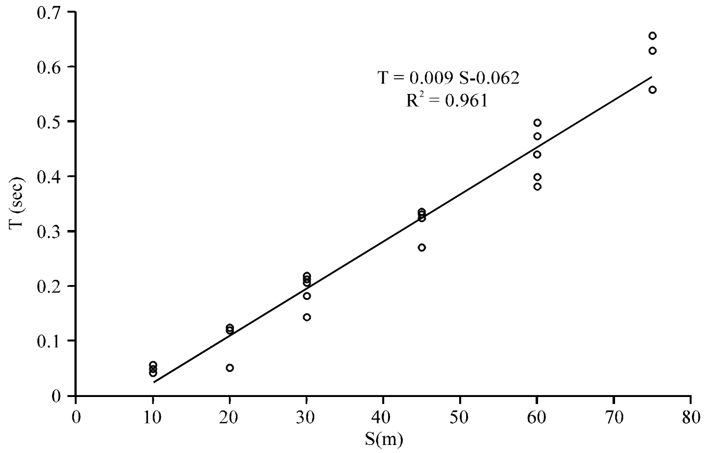


Fig. 3: Natural period of vibration's line

Table 4: The value of R square for each statistical model

Model	I	II	III	IV
Independent parameter	S×H×D	S×H	S	H
Dependent parameter	T_0	T_0	T_0	T_0
R square	0.854	0.934	0.961	0.899

The summation of the square regression is a criterion to evaluate the dispersion of the predicted Y value (derived from the regression line) from the average Y's value. If this value is near to one, then the trust to the regression line will be increased (Fig. 3).

For arriving to the approximate equation to calculate the natural period of vibration of the DLLDs, four statistic models are used. The accuracy of each model is evaluated based on 'R square'.

By considering Table 4, it can find that the maximum 'R square' is calculated for the model 'III', So it will be better to define T_0 as a function of S, opposite to the common seismic provision that assumed the H as a main factor to prediction T_0. Based on the calculated coefficient from Eq. 17 and 18, the following equation is suggested:

$$T_0 = 0.0009 \times S \quad 0.062 \tag{19}$$

CONCLUSION

Many code of practices used height (H) as one the main geometric parameters that affected the natural period of vibration of the structures. But, this is not true for DLLDs. This study shows that the span (S) of DLLDs is the most important geometric feature that should be used in the dynamic analysis of DLLDs. In addition, this study also found that the classification of DLLDs for dynamic study should not be on its internal angle, but it is recommended to category the DLLDs according to their span.

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