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Application of Parameter Expansion Method and Variational Iteration Method to Strongly Nonlinear Oscillator

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ABSTRACT

In this study, we illustrate the nonlinear oscillator with a discontinuous term which is arising from the motion of rigid rod on the circular surface without slipping. At first we explain the problem. Two powerful methods, Parameter Expansion Method (PEM) and Variational Iteration Method (VIM), which has good accuracy and efficiency applied to solution. The results compared with the forth order Runge-Kutta. The advantages of using these methods are high accuracy and simple procedure in comparison to exact solution.

Key words: Nonlinear oscillation, variational iteration method, parameter expansion method, analytical solution

INTRODUCTION

Nonlinear phenomena play important roles in engineering. Moreover, obtaining exact solutions for these problems has many difficulties. To overcome the shortcomings, many new analytical methods have purposed nowadays. Some of these methods are: Parameter Expansion Method (PEM) (Ghasempour *et al*., 2009), Energy Balance Method (EBM) (Bayat and Pakar, 2011; Ganji *et al*., 2008; Mehdipour *et al*., 2010), Variational Iteration Method (VIM) (Choobbasti *et al*., 2008; Ganji *et al*., 2007; Khatami *et al*., 2008; Shakeri *et al*., 2009; He, 1999, 2007), Homotopy Perturbation Method (HPM) (He, 2008a; Fazeli *et al*., 2008; Mirgolbabaei and Ganji, 2009; Sharma and Methi, 2011; Ghotbi *et al*., 2008; Chowdhury, 2011), Amplitude Frequency Formulation (AFF) (He, 2008b), The Max-Min Approach (MMA) (Zeng, 2009; He, 2008c) and Homotopy Analysis Method (HAM) (Zahedi *et al*., 2008).

In this study, we clarify the rigid rod rocks on the circular surface problem by using Parameter Expansion Method (PEM), Variational Iteration Method (VIM). And numerical Runge-Kutta method of order 4 will be compare with the analytical results.

PROBLEM DESCRIPTIONS

The motion's equation of the rigid rod which rocks on the circular surface without slipping is: (1) Where l is rigid rod's length, r is radius of circular surface and u is function of angle in each time:

$$\ddot{u}+\frac{12r^2}{l^2}u^2\ddot{u}+\frac{12r^2}{l^2}u\dot{u}^2+\frac{12gr}{l^2}\cos(u)=0 \tag{1}$$

SOLUTION PROCEDURE

Parameter expansion method (PEM): We can rewrite Eq. 1 as following form:

$$\ddot{u}+a\left(u^2\ddot{u}+u\dot{u}^2\right)+bu\cos\left(u\right)=0,u\left(0\right)=A,\dot{u}\left(0\right)=0 \tag{2}$$

where, Substitution of approximation:

$$\cos\left(u\right)=1-\frac{u^2}{2}+\frac{u^4}{24} \tag{3}$$

into Eq. 2, yields:

$$\ddot{u}+a\left(u^2\ddot{u}+u\dot{u}^2\right)+b\left(u-\frac{u^3}{2}+\frac{u^5}{24}\right)=0 \tag{4}$$

To apply parameter expansion method to Eq. 3, we rewrite it as follows:

$$\ddot{u}+0\cdot u+a\left(u^2\ddot{u}+u\dot{u}^2\right)+b\left(u-\frac{u^3}{2}+\frac{u^5}{24}\right)=0 \tag{5}$$

According to the parameter expansion method, the solution and coefficients of Eq. 4 can be expanded as following terms:

$$u=\sum_{i=0}^{n}p^iu_i\left(t\right)=u_0\left(t\right)+pu_1\left(t\right)+p^2u_2\left(t\right)+\cdots \tag{6}$$

$$0=\omega^2+\sum_{i=1}^{n}p\omega_1=\omega^2+p\omega_1+p^2\omega_2+\cdots \tag{7}$$

$$a=\sum_{i=1}^{n}p^ia_i=pa_1+p^2a_2+\cdots \tag{8}$$

$$b=\sum_{i=1}^{n}p^ib_i=pb_1+p^2b_2+\cdots \tag{9}$$

where, p is a book keeping parameter. Inserting Eq. 5-8 into Eq. 4, we have:

$$\left(\ddot{u}_0+p\ddot{u}_1+p^2\ddot{u}_2+\cdots\right)+\left(\omega^2+p\omega_1+p^2\omega_2+\cdots\right)\times\left(u_0+pu_1+p^2u_2+\cdots\right)$$
$$+\left(pa_1+p^2a_2+\cdots\right)\times\left[\ \left(\left(u_0+pu_1+p^2u_2+\cdots\right)^2\times\left(\ddot{u}_0+p\ddot{u}_1+p^2\ddot{u}_2+\cdots\right)\right)\right.$$

$$+\left(\left(u_0+pu_1+p^2u_2+\cdots\right)\times\left(\dot{u}_0+p\dot{u}_1+p^2\dot{u}_2+\cdots\right)\right)\Bigg]+\left(pb_1+p^2b_2+\cdots\right)$$
$$\times\left(\left(u_0+pu_1+p^2u_2+\cdots\right)-\frac{\left(u_0+pu_1+p^2u_2+\cdots\right)^3}{2}+\frac{\left(u_0+pu_1+p^2u_2+\cdots\right)^5}{24}\right)=0 \quad (10)$$

Equating terms with the identical powers of p yields:

$$p^0: \ddot{u}_0+\omega^2u_0=0 \quad (11)$$

$$p^1: \ddot{u}_1+\omega^2u_1+\omega_1u_0+b_1\left(u_0-\frac{1}{2}u_0^3+\frac{1}{24}u_0^5\right)+a_1\left(u_0^2\ddot{u}_0+u_0\dot{u}_0^2\right) \quad (12)$$

$$p^2: \ddot{u}_2+\omega^2u_2+\omega_1u_1+\omega_2u_0+b_1\left(u_1-\frac{3}{2}u_0^2u_1+\frac{5}{24}u_0^4u_1\right) \quad (13)$$

$$+b_2\left(u_0-\frac{1}{2}u_0^3+\frac{1}{24}u_0^5\right)+a_1\left(u_0^2\ddot{u}_1+2u_0u_1\ddot{u}_0+2u_0\dot{u}_0\dot{u}_1+u_1\dot{u}_0^2\right)+a_2\left(u_0^2\ddot{u}_0+u_0\dot{u}_0^2\right) \quad (14)$$

Considering the initial conditions $u_0(0) = A, \dot{u}_0(0) = 0$, the solution of Eq. 10 is $u_0 = A\cos(\omega t)$. Substituting the result into Eq. 7, yields:

$$\ddot{u}_1+\omega^2u_1=-\Big(\omega_1 A\cos(\omega t)+b_1\left(A\cos(\omega t)-\frac{1}{2}A^3\cos(\omega t)^3+\frac{1}{24}A^5\cos(\omega t)^5\right)$$
$$+a_1\left(-A^3\cos(\omega t)^3\omega^2+A^3\cos(\omega t)\sin(\omega t)^2\omega^2\right)\Big) \quad (15)$$

To find secular term, we use Fourier expansion as follows:

$$\ddot{u}_1+\omega^2u_1=\Psi(\omega t)\Psi(\omega t)=\sum_{n=0}^{\infty}\delta_{2n+1}\cos\left[(2n+1)\omega t\right]\approx\delta_1\cos(\omega t)$$
$$\delta_1=\frac{4}{\pi}\int_0^{\frac{\pi}{2}}\Big(\left(\omega_1 A\cos(\varphi)+b_1\left(A\cos(\varphi)-\frac{1}{2}A^3\cos(\varphi)^3+\frac{1}{24}A^5\cos(\varphi)^5\right)\right.$$
$$\left.+a_1\left(-A^3\cos(\varphi)^3\omega^2+A^3\cos(\varphi)\sin(\varphi)^2\omega^2\right)\cos(\varphi).d\varphi\right.$$
$$=A\omega_1+\frac{5}{192}A^5b_1-\frac{3}{8}A^3b_1-\frac{1}{2}A^3a_1\omega^2+Ab_1 \quad (16)$$

Avoiding secular term, needs:

$$\delta_1=0 \quad (17)$$

Let p = 1 into Eq. 5-7, gives:

$$\omega_1 = -\omega^2,\ a_1 = a,\ b_1 = b$$

Frequency can be yield:

$$\omega_{PEM} = \frac{\sqrt{6}}{24}\frac{\sqrt{\left(aA^2+2\right)b\left(-72A^2+5A^4+192\right)}}{aA^2+2} \tag{18}$$

where, g is gravitational acceleration:

$$a = \frac{12r^2}{l^2}$$

$$b = \frac{12gr}{l^2}$$

Variational iteration method (VIM): To clarify the basic ideas of VIM, we consider the following differential equation:

$$Lu+Nu = g(u) \tag{19}$$

where, L is a linear operator and N is a nonlinear operator.

According to the Variational Iteration Method we can construct the following iteration formulation:

$$u_{n+1}(t)=u_n(t)+\int_0^t \lambda\left(Lu(\tau)+N\tilde{u}(\tau)-g(\tau)\right)d\tau \tag{20}$$

where, λ is general Lagrange multiplier which can be identified optimally via the variational theory. The subscript indicates the nth approximation and $\tilde{u}_n(\tau)$ is considered as restricted variation, i.e., $\delta\tilde{u}_n = 0$.

Assuming that the angular frequency is ω, we can rewrite Eq. 2 as following form:

$$u''+\omega^2u+a(u^2u''+uu'^2)+bu\cos(u)-\omega^2u = 0 \tag{21}$$

We can write following equation:

$$u_{n+1}(t)=u_n(t)+\int_0^t \lambda\left(Lu(\tau)+N\tilde{u}(\tau)-g(\tau)\right)d\tau \tag{22}$$

where, $Lu = u''+\omega^2 u$, $Nu = a(u^2u'')+bu\cos(u)-\omega^2 u$ and $\tilde{u}$ is considered as restricted variation, i.e., $\delta\tilde{u} = 0$. Making the above correction functional stationary, we obtain the following stationary conditions:

$$\begin{cases} \lambda''(\tau)+\omega^2\lambda(\tau)=0, \\ \lambda(\tau)_{\tau=t}=0, \\ 1-\lambda'(\tau)_{\tau=t}=0. \end{cases} \tag{23}$$

The Lagrange multipliers, so, can be identified as:

$$\lambda=\frac{1}{\omega}\sin\omega(\tau-t) \tag{24}$$

Substituting Eq. 25 into Eq. 23 obtained as follows:

$$u_{n+1}(t)=u_n(t)+\int_0^t \frac{1}{\omega}\sin\omega(\tau-t)\left(Lu(\tau)+N\tilde{u}(\tau)\right)d\tau \tag{25}$$

Substituting $u_0(t) = A\cos(\omega t)$ as trial function into Eq. 2, yields the residual as follows:

$$R_0(t) = -A\cos(\omega t)\omega^2-aA^3\cos(\omega t)^3\,\omega^3+aA^3\cos(\omega t)\sin(\omega t)^2\omega^2+bA\cos(\omega t)\cos(A\cos(\omega t)) \tag{26}$$

Using power Fourier series, we can obtain:

$$bA\cos(\omega t)\cos\left(A\cos(\omega t)\right)=\left(bA\cos(A)+\frac{b}{2}A^2\sin(A)\omega^2t^2+O\left(t^4\right)\right)\cos(\omega t) \tag{27}$$

Now we apply Fourier expansion series on to achieve secular term:

$$\begin{aligned} R_0(t)&=\sum_{n=0}^{\infty}\delta_{n+1}\cos\left[(2n+1)\omega t\right]\approx\delta_1\cos(\omega t), \\ \delta_1&=\frac{4}{\pi}\int_0^{\frac{\pi}{2}}\Bigg(-A\cos(\omega t)\omega^2-aA^3\cos(\omega t)^3\omega^2+aA^3\cos(\omega t)\sin(\omega t)^2\omega^2 \\ &+\left(bA\cos(A)+\frac{b}{2}A^2\sin(A)\omega^2t^2+O\left(t^4\right)\right)\Bigg)\cos(\omega t)\,d(\omega t) \end{aligned} \tag{28}$$

Substituting R_0 (t) into Eq. 26, we have:

$$u_1(t)=A\cos(\omega t)+\int_0^t \frac{1}{\omega}\sin(t-\tau)R_0(\tau)d\tau \tag{29}$$

Avoiding secular term needs avoid resonance, so relationship between amplitude A and frequency ω can be yield:

$$\omega_{VIM} = \frac{\sqrt{2}\sqrt{\left(2 + aA^2\right)\cos(A)\,b}}{\left(2 + aA^2\right)} \quad (30)$$

Like previous part, g is gravitational acceleration:

$$a = \frac{12r^2}{l^2}$$

$$b = \frac{12gr}{l^2}$$

RESULTS AND DISCUSSION

Now, we want to investigate the approximate solutions in some numerical cases. Table 1-2 give information about the values of approximate solutions and numerical forth order Runge-Kutta results and also, analytical error between these quantities for two numerical cases in each 0.5 sec.

As can be seen in Fig. 1 and 2 the approximate solutions have a good adjustment with numerical method Runge-Kutta. High accuracy and validity reveal that both methods are powerful and effective to use Solution gives us possibility to obtain frequency in different cases.

Table 1: Comparison between PEM, VIM and Runge-Kutta for g = 9.8 m sec^{-1}, r = 1 m, l = 2 m, a = 3, b = 29.4 and $A = \frac{\pi}{18}$

t	U_{VIm}	U_{PEM}	$U_{Runge\text{-}Kutta}$	$\left\lvert\frac{U_{Runge-Kutta} - U_{PEM}}{U_{Runge-Kutta}}\times 100\right\rvert$	$\left\lvert\frac{U_{Runge-Kutta} - U_{VIM}}{U_{Rnge-Kutta}}\times 100\right\rvert$
0	0.174532	0.1745329	0.1745329	0	0
0.5	-0.15227	-0.1527	-0.153552	0.55484932	0.83477151
1	0.091161	0.0926641	0.0943344	1.77061225	3.36294385
1.5	-0.00679	-0.0094448	-0.00981	3.72313226	3.71171204
2	-0.0793	-0.076137	-0.077464	1.71386202	2.3708183
2.5	0.145169	0.142671	0.1436533	0.68375908	1.0555315
3	-0.17412	-0.173510	-0.17353	0.0111347	0.2728375

Table 2: Comparison between PEM, VIM and Runge-Kutta for g = 9.8 m sec^{-2}, r = 1 m, l = 2 m, a = 3, b = 29.4 and $A = \frac{\pi}{6}$

t	U_{VIm}	U_{PEM}	$U_{Runge\text{-}Kutta}$	$\left\lvert\frac{U_{Runge-Kutta} - U_{PEM}}{U_{Runge-Kutta}}\times 100\right\rvert$	$\left\lvert\frac{U_{Runge-Kutta} - U_{VIM}}{U_{Runge-Kutta}}\times 100\right\rvert$
0	0.5235987	0.52359877	0.52359877	0	0
0.5	-0.275009	-0.2927083	-0.3304665	11.42572261	16.7815688
1	-0.234713	-0.1963322	-0.2192023	10.43331238	7.0761996
1.5	0.5215655	0.51222027	0.51298617	0.14930257	1.672442
2	-0.313168	-0.3763625	-0.4148982	9.28799545	14.51913105
2.5	-0.192595	-0.091423	-0.0840465	8.7766053	19.1527698
3	0.5154817	0.47857931	0.4806779	0.43659252	7.2405733

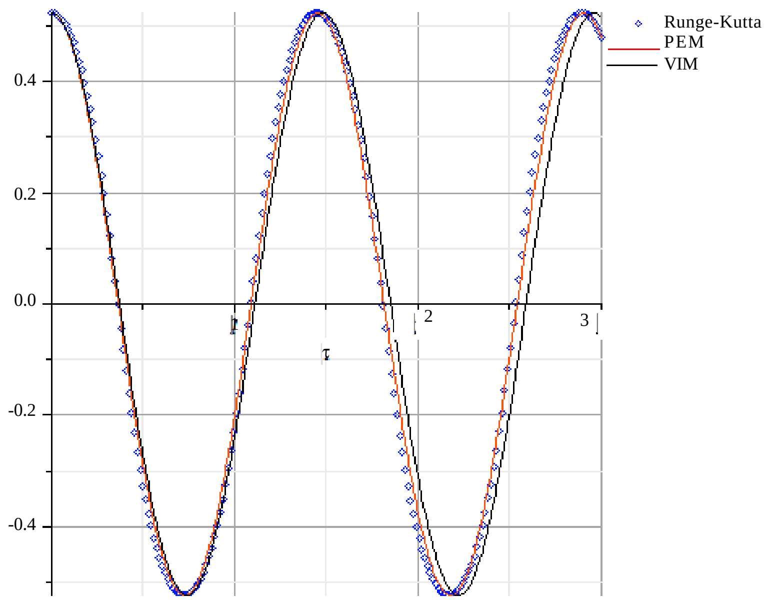

Fig. 1: Comparison between PEM, VIM and Runge-Kutta for g = 9.8 m sec^{-1}, r = 1 m, l = 2 m, a = 3, b = 29.4 and $A=\frac{\pi}{18}$

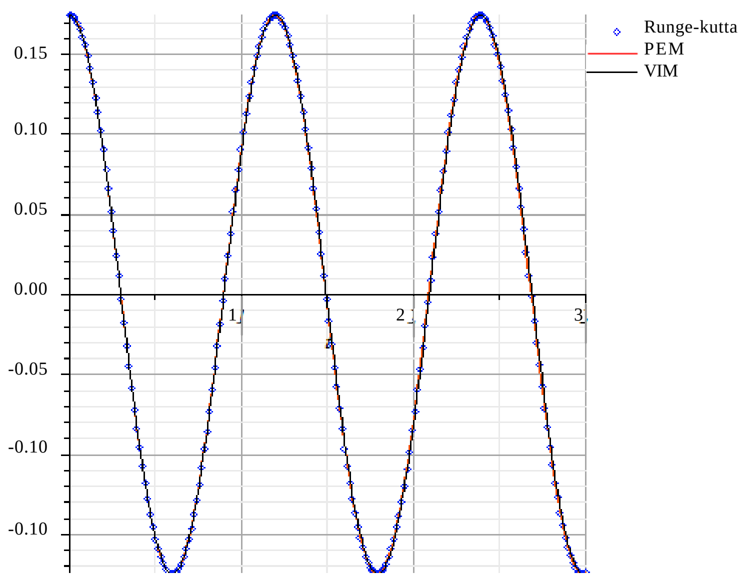

Fig. 2: Comparison between PEM, VIM and Runge-Kutta for g = 9.8 m sec^{-1}, r = 1 m, l = 2 m, a = 3, b = 29.4 and $A=\frac{\pi}{6}$

CONCLUSION

In this study, we applied the parameter-expansion method and VIM which are two powerful and efficient methods, to obtain the relationship between amplitude and frequency for the nonlinear equation which comes from the motion of rigid rod on the circular surface without slipping. Also, the solution compared with the numerical method forth order Runge-Kutta. The high accuracy and validity of approximate solutions assure us about the solution and reveal these methods can be used for nonlinear oscillators even with high order of nonlinearity.

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