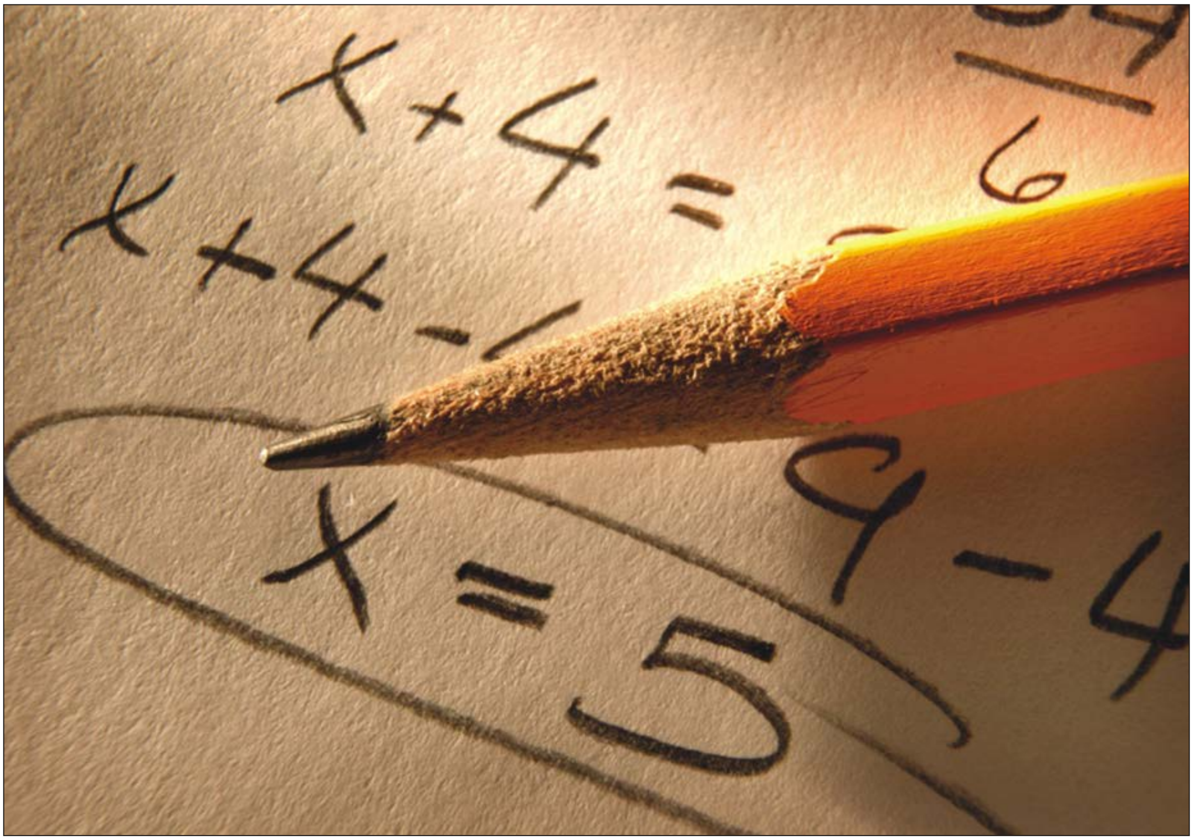

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Some Structure Properties of Anti L-Q-Fuzzy and Normal Fuzzy Subgroups

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ABSTRACT

The study introduces the concept of Q-fuzzy groups. In this paper, based on this, the concept of anti L-Q-fuzzy subgroups and anti L-Q-normal fuzzy subgroups are given and its some elementary properties are discussed. We shall also extend some results on this subject.

Key words: L-Fuzzy subset, L-Q-Fuzzy set, anti L-Q-Fuzzy subgroups, anti L-Q-Normal fuzzy subgroups, conjugate anti L-Q-Fuzzy subgroups, anti homomorphism groups

INTRODUCTION

Zadeh (1965) introduced the concepts of fuzzy sets. Fuzzy set theory has been developed in many directions by many researches. Such as Rosenfeld (1971) constituted the elementary concepts of fuzzy groups, fuzzy subgroupoid and fuzzy ideals. Many researchers Muthuraj *et al.* (2010), Aktas and Cagman (2006), Aktas (2004), Sulaiman and Ahmad (2011), Sulaiman and Ahmad (2010) and Tarnauceanu and Bentea (2008) studied the properties of groups and subgroups by definition of fuzzy subgroups. Motivated by this, many mathematicians started to review the various concepts and theorem of abstract algebra in the broader frame work of fuzzy settings (Solairaju and Nagarajan, 2009), Biswas (1990) introduced the concept of anti-fuzzy subgroups. Palaniappan and Mathuraj (2004) defined the homomorphism, anti homomorphism of fuzzy and anti fuzzy groups. Pandiammal *et al.* (2010) defined a new algebraic structure of anti L-fuzzy normal M-subgroups. On the other Nagarajan and Solairaju (2011). hand studied introduced the notion of Q-fuzzy subgroups and upper Q-fuzzy order In this study, based on the reference Solairaju and Nagarajan (2009) the concept of anti L-Q-fuzzy subgroups and anti L-Q-normal fuzzy subgroups are given and its some elementary properties are discussed, some results in references Solairaju and Nagarajan (2009) and Pandiammal *et al.* (2010) are extended.

PRELIMINARIES

Definition Pandiammal *et al.* (2010): Let X be a non-empty set and L = (L, ≤) be a lattice with least element 0 and greater element 1. A L-fuzzy subset δ of X is just a function:

$$\delta: X \rightarrow L$$

Definition: Let Q and G be a set and group, respectively. A mapping $\delta: G \times Q \rightarrow L$ is called L-Q-fuzzy set in G.

Definition: An L-Q-fuzzy set δ is called anti L-Q-fuzzy subgroups of G if:

- δ (xy, q)$\geq$min {δ (x, q), δ (y, q)}
- δ (x^{-1}, q) = δ (x, q)
- δ (e, q) = 1; $\forall$ x, y$\in$G, q$\in$Q

Example: Let G = {e, a, b, ab} be the Klein four group and δ = {(e, 0.75), (a, 0.25) (b, 0.25) (ab, 0.75)} be a L-fuzzy subset, then δ is anti L-Q-fuzzy subgroups.

Definition: Let G be a group. An anti L-Q-fuzzy subgroup δ of G is said to be anti L-Q-normal fuzzy subgroup of G if δ (xy, q) = δ (yx, q) or δ (xyx^{-1}, q)$\geq\delta$ (y, q) for all x, y$\in$G, q$\in$Q.

PROPERTIES OF ANTI L-Q-FUZZY AND NORMAL FUZZY SUBGROUPS

Proposition: Let G be a group and let δ be anti L-Q-fuzzy subgroups of G. Then δ (xy, q) = min {δ (x, q), δ (y, q)} if δ (x, q)$\neq\delta$ (y, q) for all x, y$\in$G, q$\in$Q.

Proof: Assume that δ (x, q)>δ (y, q):

- δ (y, q) = δ (x^{-1} xy, q) $\geq$ min {δ (x^{-1}, q), δ (xy, q)} = min {δ (x, q), δ (xy, q)} = δ (xy, q) also δ is anti L-Q-fuzzy subgroup of G, then δ (xy, q)$\geq$min
- {δ (x, q), δ (y, q)} = δ (y, q) thus we get δ (y, q)$\geq\delta$ (xy, q)$\geq\delta$ (y, q) this means that δ (xy, q) = min {δ (x, q), δ (y, q)}. The same way of δ (x, q)>δ (y, q)

Proposition: Let G be a group, if δ, μ are two anti L-Q-fuzzy subgroups of G. Then their intersection $\delta\cap\mu$ is anti L-Q-fuzzy subgroup of G.

Proof: Straight forward.

Corollary: Let G be a group, if δ, μ are two anti L-Q-normal fuzzy subgroups of G. Then their intersection $\delta\cap\mu$ is anti L-Q-normal fuzzy subgroup of G.

Proof: Let x, y$\in$G, q$\in$Q and let:

$$\delta = \{<(x, q), \delta (x, q)>; x\in G, q\in Q\}$$

$$\mu = \{<(x, q), \mu (x, q)>; x\in G, q\in Q\}$$

Be anti L-Q-fuzzy subgroups of G, suppose that $\lambda = \delta\cap\mu$ and $\lambda = \{<(x, q), \lambda (x, q)>; x\in G, q\in Q\}$. Then, we know that λ is an anti L-Q-fuzzy subgroup of G. Since δ, μ are two anti L-Q-fuzzy subgroups of G and λ (xy, q) = min {δ (xy, q), μ (xy, q)} as, δ, μ are anti L-Q-normal fuzzy subgroups of G, = min {δ (yx, q), μ (yx, q)} = λ (yx, q), therefore, λ (xy, q) = λ (yx, q).

Hence, $\delta\cap\mu$ is an anti L-Q-normal fuzzy subgroup.

Theorem: Let G be a group. The intersection of a family of anti L-Q-normal fuzzy subgroups is anti L-Q-normal fuzzy subgroup of G.

Proof: Let $\{\delta_i\}_{i\in\Delta}$ be a family of anti L-Q-normal fuzzy subgroup of G and $\delta = \cap\delta_i$, then for x, y$\in$G. We know that, the intersection of a family of anti L-Q-fuzzy subgroups of G is anti L-Q-fuzzy subgroup of G. Now δ (xy, q) = inf {δ_i (xy, q); i$\in\Delta$} = inf {δ_i (yx, q); I$\in\Delta$} = δ (yx, q). Therefore, δ (xy, q) = δ (yx, q). Hence, the intersection of a family of anti L-Q-normal fuzzy subgroups is anti L-Q-normal fuzzy subgroup of G.

Definition: Let G be a group. An anti L-Q-fuzzy subgroup λ of G is said to be anti L-Q-fuzzy characteristic subgroup of G if λ (x, q) = λ (φ (x), q), for all x$\in$G and $\varphi\in$ Aut G.

Theorem: If λ is an anti L-Q-fuzzy characteristic subgroup of G, then λ is anti L-Q-normal fuzzy subgroup of G.

Proof: Let λ be anti L-Q-fuzzy characteristic subgroup of G and let x, y$\in$G. Consider φ: G$\rightarrow$G be a map defined by φ (x) = yxy^{-1}, we know that, $\varphi\in$Aut G.

λ (xy, q) = λ (φ (xy), q) = λ ($y(xy)y^{-1}$, q) = λ (yx, q). Therefore, λ (xy, q) = λ (yx, q), thus λ is an anti L-Q-normal fuzzy subgroup of G.

Lemma: If δ is anti L-Q-fuzzy subgroup of G, then δ (e, q)$\geq\delta$ (x, q) for every x$\in$G, q$\in$Q.

Proposition: Let μ be anti L-Q-fuzzy subgroup of G. Then for any integer n, x$\in$G, q$\in$Q, we have μ (x^n, q)$\geq\mu$ (x, q).

Proof: Straight forward.

Lemma: An anti L-fuzzy subset μ of a group G is anti L-Q-fuzzy subgroup of G iff μ (xy^{-1}, q)$\geq$min {μ (x, q), μ (y, q)} for every x, y$\in$G, q$\in$Q.

Proposition: If λ is anti L-Q-fuzzy subgroup of G, then for any x, y$\in$G, q$\in$Q if λ (xy, q) = λ (e, q) then λ (x, q) = λ (y, q).

Proof: Straight forward.

Corollary: Let μ be anti L-Q-fuzzy subgroup of G, such that μ (x, q) = μ (e, q) then μ (xy, q) = μ (y, q).

Proof: Assume that μ is an anti L-Q-fuzzy subgroup:

Since, μ (xy, q)$\geq$min {μ (x, q), μ (y, q)} = min {μ (e, q), μ (y, q)}
Then, μ (xy, q) $\geq$ μ (y, q)
Also, μ (y, q) = μ (x^{-1} x y, q)$\geq$min {μ (x^{-1}, q), μ (xy, q)} = min {μ (x, q), μ (xy, q)}
= min {μ (e, q), μ (x y, q)} = μ (xy, q)
Hence, μ (y, q) $\geq$ μ (xy, q) and we get μ (xy, q) = μ (y, q)

Corollary: Let μ be anti L-Q-fuzzy subgroup of G and μ(x^2, q) = μ(e, q) for all x$\in$G, then μ is anti L-Q-normal fuzzy subgroup.

Proof: Let x, y$\in$G, q$\in$Q and μ be anti L-Q-fuzzy subgroup of G such that μ (x^2, q) = μ (e, q) then, we have μ(e, q) = μ($(xy)^2$, q) = μ((x yxy), q) = μ ((x yx^{-1})x^2y), q), hence, by the above proposition we get $\mu(x\,yx^{-1}, q) = \mu((xy)^2, q) \geq \min\{\mu(x^2, q), \mu(y, q)\} \geq \min\{\mu(e, q), \mu(y, y)\} = \mu(y, q)$. Therefore, $\mu(x\,yx^{-1}, q) \geq \mu(y, q)$, thus then μ is anti L-Q-normal fuzzy subgroup.

Proposition: If λ is anti L-Q-fuzzy subgroup of G. Then:

$\lambda(xy^{-1}, q) \geq \min\{\lambda(x, q), \lambda(y, q)\}$ for every $x, y \in G$, $q \in Q$

Proof: Since λ is anti L-Q-fuzzy subgroup, then $\lambda(xy^{-1}, q) \geq \min\{\lambda(x, q), \lambda(y^{-1}, q)\}$ and we know λ is ant L-Q-fuzzy subgroup. Then:

$\min\{\lambda(x, q), \lambda(y^{-1}, q)\} = \min\{\mu(x, q), \mu(y, q)\}$. Therefore, $\lambda(xy^{-}, {}^{1}q) \geq \min\{\lambda(x, q), \lambda(y, q)\}$ for every $x, y \in G$, $q \in Q$

Theorem: If δ is anti L-Q-fuzzy subgroup of the group G, then the following conditions are equivalent:

- δ is anti L-Q-normal fuzzy subgroup
- $\delta(xyx^{-1}, q) = \delta(y, q)$; $x, y \in G$, $q \in Q$
- $\delta(xy, q) = \delta(yx, q)$; $x, y \in G$, $q \in Q$

Proof: Straight forward.

Proposition: Let δ be an anti L-Q-normal fuzzy subgroup of G, then for any $y \in G$ we have $\delta(yxy^{-1}, q) = \delta(y^{-1}xy, q)$ for every $x \in G$, $q \in Q$.

Proof: For any $y \in G$, $q \in Q$ we have:

$$\delta(yxy^{-1}, q) = \delta(y^{-1}yx, q) = \delta(x, q) = \delta(xyy^{-1}, q) = \delta(y^{-1}xy, q)$$

Thus, $\delta(yxy^{-1}, q) = \delta(y^{-1}xy, q)$ for every $x, y \in G$, $q \in Q$.

Definition: Let δ, μ be two anti L-Q-fuzzy subgroups of G. Then, δ, μ are said to be conjugate anti L-Q-fuzzy subgroups of G if for some $g \in G$ such that:

$$\delta(x, q) = \mu(g^{-1}xg, q) \text{ for every } x \in G$$

Theorem: An anti L-Q-normal fuzzy subgroup δ of G is anti L-Q-normal fuzzy subgroup of G iff δ is constant on the conjugate classes of G.

Proof: Suppose that δ is an anti L-Q-normal fuzzy subgroup of G and let $x, y \in G$. Now, $\delta(y^{-1}xy, q) = \delta(xyy^{-1}, q) = \delta(x, q)$.

Therefore, $\delta(y^{-1}xy, q) = \delta(x, q)$ and hence, δ is constant on the conjugate classes of G.

Conversely, suppose that δ is constant on the conjugate classes of G. Then:

$$\delta(xy, q) = \delta(xyxx^{-1}, q) = \delta(x(yx)x^{-1}, q) = \delta(yx, q)$$

Therefore, $\delta(xy, q) = \delta(yx, q)$.

Hence, δ is anti L-Q-normal fuzzy subgroup of G.

Proposition: Let δ, μ be two anti L-Q-fuzzy subgroups of abelian group G, then μ, δ are conjugate anti L-Q-fuzzy group of G iff $\delta = \mu$.

Proof: Suppose that δ, μ are conjugate anti L-Q-fuzzy subgroup of G, therefore, for all $g \in G$. $\mu(x, q) = \delta(g^{-1}xg, q) = \delta(g^{-1}gx, q) = \delta(x, q)$ for all $x \in G$, $q \in Q$. Therefore, $\mu(x, q) = \delta(x, q)$, then $\mu = \delta$ for all $x \in G$, $q \in Q$. Now suppose $\mu = \delta$, then for the identity element $e \in G$, we have $\mu(x, q) = \delta(e^{-1}xe, q)$ for all $x \in G$, $q \in Q$. Then μ, δ are conjugate anti L-Q-fuzzy subgroups of G.

Corollary: Let $\mathfrak{S}$ be the set of all anti L-Q-fuzzy subgroups of G, let ~ be the relation in $\mathfrak{S}$ defined by $\lambda \sim \mu \Leftrightarrow \lambda(g, q) = \mu(x^{-1}gx, q)$ for all $x, g \in G$, $q \in Q$. Then ~ is an equivalence relation in $\mathfrak{S}$.

Definition: Let, μ be an anti L-Q-fuzzy subgroup of G, then μ is commutative anti L-Q-fuzzy subgroup of G if $\mu(xyz, q) = \mu(yxz, q)$ for all $x, y, z \in G$, $q \in Q$.

Lemma: Let, μ be an anti L-Q-fuzzy subgroup of G, then μ is commutative anti L-Q-fuzzy subgroup of G iff $\mu(xyx^{-1}y^{-1}, q) = \mu(xy(yx)^{-1}, q) = \mu(e, q)$ for all $x, y \in G$, $q \in Q$.

Proof: Assume that μ is commutative anti L-Q-fuzzy subgroup, then $\mu(xyz, q) = \mu(yxz, q)$ we need to show $\mu(xyx^{-1}y^{-1}, q) = \mu(e, q)$:

$$\mu(xyx^{-1}y^{-1}, q) = \mu(yxx^{-1}y^{-1}, q) = \mu(e, q). \text{ Therefore, } \mu(xyx^{-1}y^{-1}, q) = \mu(e, q)$$

Conversely, suppose that $\mu(xyx^{-1}y^{-1}, q) = \mu(e, q)$ we need to show μ is commutative anti L-Q-fuzzy subgroup of G. Let $x, y, z \in G$, $q \in Q$, then $\mu((xyz)(yxz)^{-1}, q) = \mu(xyzz^{-1}x^{-1}y^{-1}, q) = \mu(xyx^{-1}y^{-1}, q) = \mu(e, q)$ by the above proposition we get $\mu(xyz, q) = \mu((yxz)^{-1}, q)$. Since μ is anti L-Q-fuzzy subgroup, then $\mu((yxz)^{-1}, q) = \mu(yxz, q)$. Therefore, $\mu(xyz, q) = \mu(yxz, q)$ for all $x, y, z \in G$, $q \in Q$, thus, μ is commutative anti L-Q-fuzzy subgroup of G.

Theorem: Let δ, λ be two anti L-Q-fuzzy subgroups with $\delta(e, q) = \lambda(e, q)$, then, If δ is commutative anti L-Q-fuzzy subgroup and $\delta \leq \lambda$ then λ is commutative anti L-Q-fuzzy subgroup.

Proof: Suppose that δ, λ be two anti L-Q-fuzzy subgroups also $\delta(e, q) = \lambda(e, q)$.

$\lambda(e, q) = \delta(e, q) = \delta(xyx^{-1}y^{-1}, q) \leq \lambda(xyx^{-1}y^{-1}, q)$, then $\lambda(e, q) \leq \lambda(xyx^{-1}y^{-1}, q)$ also $\lambda(e, q) \geq \lambda(xyx^{-1}y^{-1}, q)$, therefore, we obtain $\lambda(e, q) = \lambda(xyx^{-1}y^{-1}, q)$. This means that $\lambda(xy(yx)^{-1}, q) = \lambda(e, q)$ by the Lemma give above we get λ is commutative anti L-Q-fuzzy subgroup.

ANTI L-Q-NORMAL FUZZY SUBGROUPS OF GROUP UNDER HOMOMORPHISM AND ANTI-HOMOMORPHISM

Proposition: Let G_1, G_2 be any 2 groups. The homomorphic image of an anti L-Q-fuzzy subgroup of G_1 is an anti L-Q-fuzzy subgroup of G_2.

Proof: Straight forward.

Theorem: Let G_1, G_2 be any to groups. The homomorphic image of an anti L-Q-normal fuzzy subgroup of G_1 is an anti L-Q-normal fuzzy subgroup of G_2.

Proof: Suppose that Φ: $G_1 \to G_2$ be a group homomorphism, let $\lambda = \Phi(\mu)$ where , μ is an anti L-Q-normal fuzzy subgroup of G_1, we need to prove that λ is an anti L-Q-normal fuzzy subgroup of G_2. Now, for Φ (x), Φ (y)$\in G_2$ we have λ is an anti L-Q-fuzzy subgroup of G_2. Also λ (Φ (x). Φ (y), q) = λ (Φ (xy), q)$\leq\mu$ (xy, q) = μ (yx, q)$\geq\lambda$ (Φ (yx), q) = λ (Φ (y). Φ (x), q). Thus, we get λ (Φ (x). Φ (y), q) = λ (Φ (y). Φ (x), q), therefore, λ is an anti L-Q-normal fuzzy subgroup of G_2.

Proposition: Let G_1 and G_2 be any 2 groups. The homomorphic pre-image of an anti L-Q-fuzzy subgroup of G_2 is an anti L-Q-fuzzy subgroup of G_1.

Proof: Straight forward.

Theorem: Let G_1 and G_2 be any 2 groups. The homomorphic pre-image of anti L-Q-normal fuzzy subgroup of G_2 is an anti L-Q-normal fuzzy subgroup of G_1.

Proof: Suppose that Φ: $G_1 \to G_2$ be a group homomorphism, let $\lambda = \Phi$ (μ) where λ is an anti L-Q-normal fuzzy subgroup of G_2, we have to prove that μ is an anti L-Q-normal fuzzy subgroup of G_1. Let x, y$\in G_1$, then we know that μ is anti L-Q-fuzzy subgroup of G_1. Since λ is anti L-Q-fuzzy subgroup of G_2, now μ (xy, q) = λ (Φ (xy), q) = λ (Φ (x). Φ (y), q) = λ (Φ (y). Φ (x), q) = λ (Φ (yx), q) = μ (yx, q). Thus, we get μ is an anti L-Q-normal fuzzy subgroup of G_1.

Proposition: Let G_1 and G_2 be any 2 groups. The anti-homomorphic image of an anti L-Q-fuzzy subgroup of G_1 is an anti L-Q-fuzzy subgroup of G_2.

Proof: Straight forward.

Theorem: Let G_1 and G_2 be any 2 groups. The anti -homomorphic image of an anti L-Q-normal fuzzy subgroup of G_1 is an anti L-Q-normal fuzzy subgroup of G_2.

Proof: Let Φ: $G_1 \to G_2$ be a group anti-homomorphism, let $\lambda = \Phi$ (μ) where μ is an anti L-Q-normal fuzzy subgroup of G_1, we have to prove that λ is an anti L-Q-normal fuzzy subgroup of G_2. Now, for Φ (x), Φ (y)$\in G_2$ we have λ is an anti L-Q-fuzzy subgroup of G_2. Since, μ is an anti L-Q-fuzzy subgroup of G_1. Now λ (Φ (y). Φ (x), q) = λ (Φ (yx), q) $\leq$ μ (yx, q) =μ (xy, q) $\geq$ λ (Φ (xy), q) = λ (Φ (x). Φ (y), q). Thus, we get λ (Φ (x). Φ (y), q) = λ (Φ (y). Φ (x), q), hence, λ is an anti L-Q-normal fuzzy subgroup of G_2.

Proposition: Let G_1 and G_2 be any 2 groups. The anti-homomorphic pre-image of an anti L-Q-fuzzy subgroup of G_2 is an anti L-Q-fuzzy subgroup of G_1.

Proof: Straight forward.

Theorem: Let G_1, G_2 be any to groups. The anti-homomorphic pre-image of anti L-Q-normal fuzzy subgroup of G_2 is an anti L-Q-normal fuzzy subgroup of G_1.

Proof: Suppose that Φ: $G_1 \to G_2$ be anti-homomorphism group, let $\lambda = \Phi$ (μ) where λ is an anti L-Q-normal fuzzy subgroup of G_2, we have to prove that μ is an anti L-Q-normal fuzzy subgroup of G_1. Let x, y$\in G_1$, then we know that μ is anti L-Q-fuzzy subgroup of G_1. Since, λ is anti L-Q-fuzzy subgroup of G_2, now μ (xy, q) = λ (Φ (xy), q) = λ (Φ (x). Φ (y), q) = λ (Φ (y). Φ (x), q) = λ (Φ (yx), q) = μ (yx, q). Thus, we get μ(xy, q) = μ(yx, q) and μ is an anti L-Q-normal fuzzy subgroup of G_1.

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