



Asian Journal of Algebra

ISSN 1994-540X

ERRATUM

Asian Journal of Algebra published a manuscript entitled "About Catalan-Mihăilescu Theorem" 2 (1): 11-16, 2009. Now on the request of author Page No. 14 and 15 has been changed due to some technical problems Page No. 14 and 15 republished as under:

Lemma 4

The solution of lemmas 2, 3, 4 is

$$xy(x-y) = 0$$

Proof of Lemma 4

$$x_i - x_{i+1} = y_i - y_{i+1} = \sqrt{x_{i+1}y_{i+1}}$$

$$\forall i \geq 1$$

$$\begin{aligned} \sum_{j=2}^{j=i} ((-1)^j \sqrt{x_j y_j}) &= x - x_2 - x_3 + x_4 + \dots + (-1)^i (x_{i-1} - x_i) \\ &= x - 2x_2 + 2x_3 - \dots + 2(-1)^i x_{i-1} + (-1)^{i+1} x_i \\ &= y - y_2 - y_3 + y_4 + \dots + (-1)^i (y_{i-1} - y_i) \\ &= y - 2y_2 + 2y_3 - \dots + 2(-1)^i y_{i-1} + (-1)^{i+1} y_i \\ \sum_{j=2}^{j=i} ((-1)^j \sqrt{x_j y_j}) &= 2 \sum_{j=1}^{j=i} ((-1)^{j+1} x_j) - x - (-1)^{i+1} x_i = 2 \sum_{j=1}^{j=i} ((-1)^{j+1} y_j) - y - (-1)^{i+1} y_i \\ &= 2 \sum_{j=2}^{j=i-1} ((-1)^{j+1} x_j) + x + (-1)^{i+1} x_i = 2 \sum_{j=2}^{j=i-1} ((-1)^{j+1} y_j) + y + (-1)^{i+1} y_i \\ \Rightarrow \sum_{j=2}^{j=i} ((-1)^j \sqrt{x_j y_j}) - 2 \sum_{j=1}^{j=i} ((-1)^{j+1} x_j) + x_i + (-1)^{i+1} x = x_i - x + (-1)^{i+1} (x - x_i) &= (x_i - x)(1 + (-1)^i) \\ = y_i - y + (-1)^{i+1} (y - y_i) &= (y_i - y)(1 + (-1)^i) = \sum_{j=2}^{j=i} ((-1)^j \sqrt{x_j y_j}) - 2 \sum_{j=1}^{j=i} ((-1)^{j+1} y_j) + y_i + (-1)^{i+1} y \end{aligned}$$

and

$$\begin{aligned} \sum_{j=2}^{j=i} ((-1)^j \sqrt{x_j y_j}) &= 2 \sum_{j=2}^{j=i-1} ((-1)^{j+1} x_j) + x + (-1)^{i+1} x_i = 2 \sum_{j=2}^{j=i-1} ((-1)^{j+1} y_j) + y + (-1)^{i+1} y_i \\ \Rightarrow \sum_{j=2}^{j=i} ((-1)^j \sqrt{x_j y_j}) - x_i - (-1)^{i+1} x - 2 \sum_{j=2}^{j=i-1} ((-1)^{j+1} x_j) &= x - x_i - (-1)^{i+1} (x - x_i) = (x - x_i)(1 + (-1)^i) \\ = y - y_i - (-1)^{i+1} (y - y_i) &= (y - y_i)(1 + (-1)^i) \\ = \sum_{j=2}^{j=i} ((-1)^j \sqrt{x_j y_j}) - y_i - (-1)^{i+1} y - 2 \sum_{j=2}^{j=i-1} ((-1)^{j+1} y_j) &= \sum_{j=2}^{j=i} ((-1)^j \sqrt{x_j y_j}) - 2 \sum_{j=1}^{j=i} ((-1)^{j+1} x_j) - x - (-1)^{i+1} x_i = 2 \sum_{j=1}^{j=i} ((-1)^{j+1} y_j) - y - (-1)^{i+1} y_i \\ = 2 \sum_{j=2}^{j=i-1} ((-1)^{j+1} x_j) + x + (-1)^{i+1} x_i &= 2 \sum_{j=2}^{j=i-1} ((-1)^{j+1} y_j) + y + (-1)^{i+1} y_i \end{aligned}$$

$$\begin{aligned}
 &= \sum_{j=1}^{j=i} ((-1)^{j+1} x_j) + \sum_{j=2}^{j=i-1} ((-1)^{j+1} x_j) = \sum_{j=1}^{j=i} ((-1)^{j+1} y_j) + \sum_{j=2}^{j=i-1} ((-1)^{j+1} y_j) \\
 &\sum_{j=2}^{j=i} ((-1)^j \sqrt{x_j y_j}) + x_i + (-1)^{i+1} x - 2 \sum_{j=1}^{j=i} ((-1)^{j+1} x_j) = x_i - x + (-1)^{i+1} (x - x_i) = -(x - x_i)(1 + (-1)^i) \\
 &= - \sum_{j=2}^{j=i} ((-1)^j \sqrt{x_j y_j}) + x_i + (-1)^{i+1} x + 2 \sum_{j=2}^{j=i-1} ((-1)^{j+1} x_j) \\
 &= y_i - y + (-1)^{i+1} (y - y_i) = -(y - y_i)(1 + (-1)^i) \\
 &= - \sum_{j=2}^{j=i} ((-1)^j \sqrt{x_j y_j}) + y_i + (-1)^{i+1} y + 2 \sum_{j=2}^{j=i-1} ((-1)^{j+1} y_j) \\
 &= \sum_{j=2}^{j=i} ((-1)^j \sqrt{x_j y_j}) + x_i + (-1)^{i+1} x - 2 \sum_{j=1}^{j=i} ((-1)^{j+1} x_j) \\
 &= \frac{1}{2} (x_i + y_i) + \frac{(-1)^{i+1}}{2} (x + y) - \sum_{j=2}^{j=i} ((-1)^j \sqrt{x_j y_j}) + \sum_{j=2}^{j=i-1} ((-1)^{j+1} y_j) + \sum_{j=2}^{j=i-1} ((-1)^{j+1} x_j) \\
 &= \frac{1}{2} (x_i + y_i) + \frac{(-1)^{i+1}}{2} (x + y) - \sum_{j=1}^{j=i} ((-1)^{j+1} x_j) - \sum_{j=2}^{j=i-1} ((-1)^{j+1} x_j) + \sum_{j=2}^{j=i-1} ((-1)^{j+1} y_j) + \sum_{j=2}^{j=i-1} ((-1)^{j+1} x_j) \\
 &= \frac{1}{2} (x_i + y_i) + \frac{(-1)^{i+1}}{2} (x + y) + \sum_{j=2}^{j=i-1} ((-1)^{j+1} y_j) - \sum_{j=1}^{j=i} ((-1)^{j+1} x_j) \\
 &= \frac{1}{2} (x_i + y_i) + \frac{(-1)^{i+1}}{2} (x + y) + \sum_{j=2}^{j=i-1} ((-1)^{j+1} y_j) - \sum_{j=2}^{j=i-1} ((-1)^{j+1} x_j) - x - (-1)^{i+1} x_i \\
 &= \frac{1}{2} (x_i + y_i - 2x) + \frac{(-1)^{i+1}}{2} (x + y - 2x_i) + \sum_{j=2}^{j=i-1} ((-1)^{j+1} (y_j - x_j)) \\
 &= \frac{1}{2} (x_i + y_i - 2x) + \frac{(-1)^{i+1}}{2} (x + y - 2x_i) + \sum_{j=2}^{j=i-1} ((-1)^{j+1} (y - x)) \\
 &= \frac{1}{2} (x_i + y_i - 2x) + \frac{(-1)^{i+1}}{2} (x + y - 2x_i) + \frac{1}{2} (1 + (-1)^{i+1})(y - x) \\
 &= \frac{1}{2} (x_i + y_i - 2x) + \frac{1}{2} (y_i - x_i) + \frac{(-1)^{i+1}}{2} (x + y - 2x_i) + \frac{(-1)^{i+1}}{2} (y - x) \\
 &= (y_i - x) + (-1)^{i+1} (y - x_i) = x_i - x + (-1)^{i+1} (x - x_i) \\
 &\Rightarrow (y_i - x) + (-1)^{i+1} (y - x_i) - (x_i - x + (-1)^{i+1} (x - x_i)) = 0 \\
 &= y_i - x_i + (-1)^{i+1} (y - x) = (y - x)(1 + (-1)^{i+1}) = 0 \\
 &\forall i \geq 2 \\
 &\Rightarrow y - x = 0
 \end{aligned}$$

We conclude that

$$\begin{aligned}
 x_i &= x_{i+1} = x_{i+1} + \sqrt{x_{i+1} y_{i+1}} \Rightarrow y_{i+1} = 0 \Rightarrow y = 0 \\
 y_i &= y_{i+1} = y_{i+1} + \sqrt{x_{i+1} y_{i+1}} \Rightarrow x_{i+1} = 0 \Rightarrow x = 0 \\
 x_i &> x_{i+1}, y_i > y_{i+1} \Rightarrow x = y
 \end{aligned}$$

We have proved the lemma 4.

And there is no solution, it means that $p = 2$, effectively, the expression of the sequences for $p = 1$, is

$$x_i = \frac{x^{2^{i-1}}}{x^{2^{i-1}} - y^{2^{i-1}}} (x - y) = \frac{(XY)^{2^{i-2}2}}{(XY)^{2^{i-2}2} - X^{2^{i-2}2}} X^i (XY + X) = \frac{(XY)^{2^{i-3}4}}{(XY)^{2^{i-3}4} - X^{2^{i-3}4}} X^i (XY + X)$$

As there is an infinity of solutions for $p = 1$, the expressions of the sequences imply the existence of solutions for $p = 2$ and does not guarantee at all the existence of the sequences for $p > 2$ and $I = 2$. Then, $p = 2$ and $q = 3$ is the only solution.

The equation becomes

$$\begin{aligned} Y^2 &= X^3 + 1 = (X+1)(X^2 - X + 1) \\ Y^q &= Y^3 = Y^p(X^p - 1) = Y^2(X^2 - 1) \Rightarrow Y = (X-1)(X+1) \\ Y^p &= Y^2 = (X-1)^2(X+1)^2 = X^4 + 1 = X^3 + 1 = (X+1)(X^2 - X + 1) \\ \Rightarrow (X-1)^2(X+1) &= X^2 - X + 1 \Rightarrow (X^2 - 2X + 1)(X+1) = X^2 - X + 1 \\ \Rightarrow X^3 - X^2 - X + 1 &= X^2 - X + 1 \Rightarrow X^2 - X - 1 = X - 1 \Rightarrow X - 1 = 1 \Rightarrow X = 2 \\ \Rightarrow Y^2 &= X^3 + 1 = 2^3 + 1 = 9 \Rightarrow Y = 3 \end{aligned}$$

The only solution of Catalan equation is effectively
 $(X, Y, p, q) = (2, 3, 2, 3)$

Now above part of the paper should be read as under:

Catalan equation is $z^{c+2} = x^{a+2} + 1$ with x and z coprime and greater than zero. z is odd and x is even.
 We have

$$2a_1S_1 + 2a_2S_2 = (a_1 + a_2)(S_1 + S_2) + (a_1 - a_2)(S_1 - S_2)$$

But

$$z^{c+2} = xx^{a+1} + 1 \cdot 1^{b+1}$$

$$2xx^{a+1} + 2 \cdot 1 \cdot 1^{b+1} = (x+1)(x^{a+1} + 1^{b+1}) + (x-1)(x^{a+1} - 1^{b+1})$$

Thus, for $a \geq 0, c \geq 0$

$$\begin{aligned} (x+1)(x^{a+1} + 1^{b+1}) &= \frac{1}{2}(x+1)((x+1)(x^a + 1) + (x-1)(x^a - 1)) \\ \frac{1}{2}(x+1)^2(x^a + 1) &= \frac{1}{2^2}(x+1)^2((x+1)(x^{a-1} + 1) + (x-1)(x^{a-1} - 1)) \end{aligned}$$

Until a

$$\frac{1}{2^{a-1}}(x+1)^a(x^2 + 1) = \frac{1}{2^a}(x+1)^a((x+1)(x+1) + (x-1)(x-1))$$

We add and simplify

$$\begin{aligned} 2z^{c+2} &= 2(x^{a+2} + 1) \\ &= \frac{1}{2^a}((x+1)^{a+1}(x+1) + (x-1)\sum_{m=0}^{m=a}(\frac{(x+1)^m}{2^m}(x^{a+1-m} - 1)) \end{aligned}$$

If we multiply by 2^a

$$2^{a+1}(x^{a+2} + 1) = ((x+1)^{a+1}(x+1) + (x-1)\sum_{m=0}^{m=a} (2^{a-m}(x+1)^m(x^{a+1-m} - 1))$$

But x and z are coprimes, z is odd and x is even. And, also

$$2x^{a+1} - 2 = (x-1)(x^{a+1} + 1^{b+1}) + (x+1)(x^{a+1} - 1^{b+1})$$

And

$$\frac{1}{2}(x+1)(x^{a+1} - 1) = \frac{1}{2^2}(x+1)((x-1)(x^a + 1) + (x+1)(x^a - 1))$$

Until a

$$\frac{1}{2^{a-1}}(x+1)^a(x^2 - 1) = \frac{1}{2^a}(x+1)^a((x-1)(x+1) + (x+1)(x-1))$$

We add and simplify

$$2^{a+1}(x^{a+2} - 1) = ((x+1)^{a+1}(x-1) + (x-1)\sum_{m=0}^{m=a} (2^{a-m}(x+1)^m(x^{a+1-m} + 1))$$

We remember

$$2^{a+1}(x^{a+2} + 1) = ((x+1)^{a+1}(x+1) + (x-1)\sum_{m=0}^{m=a} (2^{a-m}(x+1)^m(x^{a+1-m} - 1))$$

We deduce

$$2^{a+2}x^{a+2} = 2x(x+1)^{a+1} + 2(x-1)\sum_{m=0}^{m=a} (2^{a-m}(x+1)^m x^{a+1-m})$$

And

$$2^{a+2} = 2(x+1)^{a+1} - 2(x-1)\sum_{m=0}^{m=a} (2^{a-m}(x+1)^m)$$

Thus

$$2^{a+1}x^{a+1} = (x+1)^{a+1} + (x-1)\sum_{m=0}^{m=a} (2^{a-m}(x+1)^m x^{a-m})$$

And

$$2^{a+1} = (x+1)^{a+1} - (x-1) \sum_{m=0}^{m=a} (2^{a-m}(x+1)^m)$$

If we subtract these two expressions

$$\begin{aligned} 2^{a+1}(x^{a+1} - 1) &= (x-1) \sum_{m=0}^{m=a} (2^{a-m}(x+1)^m(x^{a-m} + 1)) \\ &= (x-1)(2^a(x+1)(x^{a-1} + 1) + \dots + 2(x+1)) \end{aligned}$$

Or with $a > 1$

$$\begin{aligned} 2^a(x^{a+1} - 1) &= (x-1) \sum_{m=0}^{m=a} (2^{a-m-1}(x+1)^m(x^{a-m} + 1)) \\ &= (x-1)(2^{a-1}(x+1)(x^{a-1} + 1) + \dots + (x+1)) \end{aligned}$$

In the left, we have an even number, in the right an odd one ! It is impossible ! The equation is possible only for $a = 1$ or $a = 0$ and $c = 0$ or $c = 1$. Those cases have been studied in the past. The only solution is $(X, Y, a, c) = (2, \pm 3, 2)$.

$$\begin{aligned} \sum_{k=1}^{k=2m} (-1)^{k+1} x_k e^{\frac{-k}{\sqrt{2m}}} &= (e^{\frac{1}{\sqrt{2m}}} - 1) \sum_{k=1}^{k=m} x_{2k-1} e^{\frac{-2k}{\sqrt{2m}}} + \sum_{k=1}^{k=m} \sqrt{x_{2k} y_{2k}} e^{\frac{-2k}{\sqrt{2m}}} \\ \sum_{k=1}^{k=2m} (-1)^{k+1} y_k e^{\frac{-k}{\sqrt{2m}}} &= (e^{\frac{1}{\sqrt{2m}}} - 1) \sum_{k=1}^{k=m} y_{2k-1} e^{\frac{-2k}{\sqrt{2m}}} + \sum_{k=1}^{k=m} \sqrt{x_{2k} y_{2k}} e^{\frac{-2k}{\sqrt{2m}}} \\ (e^{\frac{1}{\sqrt{2m}}} - 1) \sum_{k=1}^{k=m} y_{2k-1} e^{\frac{-2k}{\sqrt{2m}}} &= S \\ (e^{\frac{1}{\sqrt{2m}}} - 1) \sum_{k=1}^{k=m} y_{2k-1} e^{\frac{-2k-1}{\sqrt{2m}}} &< S < (e^{\frac{1}{\sqrt{2m}}} - 1) \sum_{k=1}^{k=m} y_{2k-1} e^{\frac{-2k+1}{\sqrt{2m}}} \\ (e^{\frac{1}{\sqrt{2m}}} - 1) e^{\frac{-3}{\sqrt{2m}}} \sum_{k=1}^{k=m} y_{2k-1} e^{\frac{-2k+2}{\sqrt{2m}}} &< S < (e^{\frac{1}{\sqrt{2m}}} - 1) e^{\frac{-1}{\sqrt{2m}}} \sum_{k=1}^{k=m} y_{2k-1} e^{\frac{-2k+2}{\sqrt{2m}}} \\ \lim_{m \rightarrow \infty} (S) &= \lim_{m \rightarrow \infty} ((e^{\frac{1}{\sqrt{2m}}} - 1) \sum_{k=2}^{k=m} y_{2k-1} e^{\frac{-2k+2}{\sqrt{2m}}}) \\ A &= (e^{\frac{1}{\sqrt{2m}}} - 1) \sum_{k=p}^{k=m} y_{2k-1} e^{\frac{-2k+p}{\sqrt{2m}}} \\ (e^{\frac{1}{\sqrt{2m}}} - 1) \sum_{k=p}^{k=m} y_{2k-1} e^{\frac{-2k+p-1}{\sqrt{2m}}} &> A > (e^{\frac{1}{\sqrt{2m}}} - 1) \sum_{k=p}^{k=m} y_{2k-1} e^{\frac{-2k+p+1}{\sqrt{2m}}} \\ (e^{\frac{1}{\sqrt{2m}}} - 1) e^{\frac{-2}{\sqrt{2m}}} \sum_{k=p}^{k=m} y_{2k-1} e^{\frac{-2k+p+1}{\sqrt{2m}}} &> A > (e^{\frac{1}{\sqrt{2m}}} - 1) \sum_{k=p}^{k=m} y_{2k-1} e^{\frac{-2k+p+1}{\sqrt{2m}}} \\ \lim_{m \rightarrow \infty} (S) &= \lim_{m \rightarrow \infty} (A) = \lim_{m \rightarrow \infty} ((e^{\frac{1}{\sqrt{2m}}} - 1) \sum_{k=p+1}^{k=m} y_{2k-1} e^{\frac{-2k+p+1}{\sqrt{2m}}}) \\ p = m-1 \Rightarrow \lim_{m \rightarrow \infty} (S) &= \lim_{m \rightarrow \infty} (A) = \lim_{m \rightarrow \infty} ((e^{\frac{1}{\sqrt{2m}}} - 1)(y_{2m-1} e^{\frac{-m}{\sqrt{2m}}})) = 0 \end{aligned}$$

$$\begin{aligned}
 \lim_{m \rightarrow \infty} (S) &= \lim_{m \rightarrow \infty} ((e^{\frac{1}{\sqrt{2m}}} - 1) \sum_{k=2}^{k=m} y_{2k-1} e^{\frac{-2k+2}{\sqrt{2m}}}) = 0 \\
 \lim_{m \rightarrow \infty} ((e^{\frac{1}{\sqrt{2m}}} - 1) \sum_{k=2}^{k=m} x_{2k-1} e^{\frac{-2k+2}{\sqrt{2m}}}) &= 0 \\
 0 < \lim_{m \rightarrow \infty} \left(\sum_{k=1}^{k=2m} (-1)^{k+1} x_k e^{\frac{-k}{\sqrt{2m}}} \right) &= \lim_{m \rightarrow \infty} \left(\sum_{k=1}^{k=2m} (-1)^{k+1} y_k e^{\frac{-k}{\sqrt{2m}}} \right) < \lim_{m \rightarrow \infty} \left(\sum_{k=1}^{k=m} \sqrt{x_{2k} y_{2k}} e^{\frac{-2k}{\sqrt{2m}}} \right) < \lim_{m \rightarrow \infty} \left(\sum_{k=1}^{k=m} \sqrt{x_{2k} y_{2k}} \right) = y \\
 \lim_{m \rightarrow \infty} \left(\sum_{k=1}^{k=2m} (-1)^{k+1} (x_k - y_k) e^{\frac{-k}{\sqrt{2m}}} \right) &= \lim_{m \rightarrow \infty} \left(\sum_{k=1}^{k=2m} (-1)^{k+1} (x - y) e^{\frac{-k}{\sqrt{2m}}} \right) = (x - y) \lim_{m \rightarrow \infty} \left(\sum_{k=1}^{k=2m} (-1)^{k+1} e^{\frac{-k}{\sqrt{2m}}} \right) = \frac{x - y}{2} = 0
 \end{aligned}$$