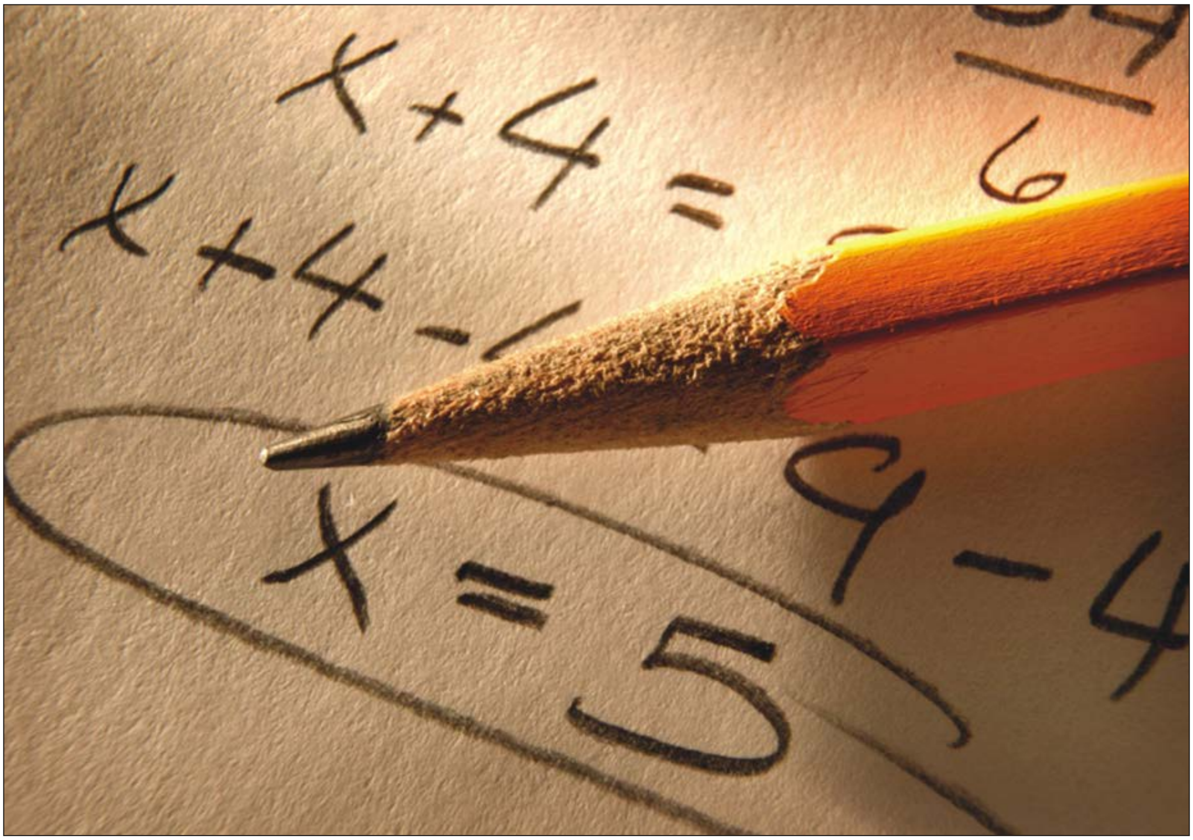

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Some Operations of Intuitionistic Fuzzy Primary and Semi-primary Ideal

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ABSTRACT
In this study, some operations of intuitionistic fuzzy primary ideals as well as intuitionistic semi-primary ideal were defined. Some results based on intuitionistic fuzzy primary and semi-primary ideals are also established.

Key words: Intuitionistic fuzzy set, intuitionistic fuzzy primary ideal, intuitionistic fuzzy semi-primary ideal

INTRODUCTION
Ever since an introduction of fuzzy sets by Zadeh (1965), the fuzzy concept has invaded almost all branches of mathematics. The concept of intuitionistic fuzzy set and its operations were introduced by Atanassov (1986, 1994), as a generalization of the notion of fuzzy set. Kumbhojkar and Bapat (1991) discussed on correspondence theorem for fuzzy ideals. Palanivelrajan and Nandakumar (2012) introduced the definition of intuitionistic fuzzy primary and semi-primary ideal. In this study, some operations on intuitionistic fuzzy primary and semi-primary ideal are introduced and some properties of the same are proved.

PRELIMINARIES
Definition 1: Let S be any nonempty set. A mapping μ: S → [0, 1] is called a fuzzy subset of S.

Definition 2: A fuzzy ideal μ of a ring R is called fuzzy primary ideal, if for all a, b$\in$R either:

$$\mu(ab) = \mu(a) \text{ or else } \mu(ab) \leq \mu(b^m) \text{ for some } m \in z_+$$

Definition 3: A fuzzy ideal μ of a ring R is called fuzzy semi-primary ideal, if for all a, b$\in$R, either:

$$\mu(ab) \leq \mu(a^n), \text{ for some } n \in z_+ \text{ or else } \mu(ab) \leq \mu(b^m) \text{ for some } m \in z_+$$

Definition 4: An Intuitionistic Fuzzy Set (IFS) A in X is defined as an object of the form:

$$A = \{<x, \mu_A(x), \gamma_A(x)>/x \in X\}$$

Where:

$$\mu_A: X \rightarrow [0, 1]$$

and:

$$\gamma_A: X \rightarrow [0, 1]$$

Define the degree of membership and the degree of non-membership of the element $x \in X,R$ respectively and for every $x \in X$ satisfying:

$$0 \leq \mu_A(x)+\gamma_A(x) \leq 1$$

Definition 5: A fuzzy ideal A of a ring R is called Intuitionistic fuzzy primary ideal if for all a, b$\in$R either:

$$\mu_A(ab) = \mu_A(a) \text{ and } \gamma_A(ab) = \gamma_A(a) \text{ or } \mu_A(ab) \leq \mu_A(b^m) \text{ and } \gamma_A(ab) \geq \gamma_A(b^m)$$

for some $m \in Z_+$.

Example: Consider:

$$\mu_A(x) = \begin{cases} 1 \text{ if } x = 0 \\ 0.8 \text{ if } x \in <4> \sim <0> \\ 0.6 \text{ if } x \in Z \sim <4> \end{cases}$$

$$\gamma_A(x) = \begin{cases} 0 \text{ if } x = 0 \\ 0.1 \text{ if } x \in <4> \sim <0> \\ 0.3 \text{ if } x \in Z \sim <4> \end{cases}$$

Definition 6: A fuzzy ideal A of a ring R is called intuitionistic fuzzy semi-primary ideal if for all a, b$\in$R either:

$$\mu_A(ab) \leq \mu_A(a^n) \text{ and } \gamma_A(ab) \geq \gamma_A(a^n), \text{ for some } n \in Z_+ \text{ or else } \mu_A(ab)\ \mu_A(b^m) \text{ and } \gamma_A(ab) \geq \gamma_A(b^m), \text{ for some } m \in Z_+$$

SOME OPERATIONS ON INTUITIONISTIC FUZZY PRIMARY AND SEMI-PRIMARY IDEAL

Theorem 1: If A and B are any two intuitionistic fuzzy semi-primary ideal of R then A+B is an intuitionistic fuzzy semi-primary ideal of R.

Proof: Consider:

$$x, y \in R \text{ then } x, y \in A+B$$

Since, A is an intuitionistic fuzzy semi-primary ideal of R, $\mu_A(xy) \leq \mu_A(x^n)$ and $\gamma_A(xy) \geq \gamma_A(x^n)$. Since, B is an intuitionistic fuzzy semi-primary ideal of:

R, $\mu_B(xy) \le \mu_B(x^n)$ and $\gamma_B(xy) \ge \gamma_B(x^n)$

Consider:

$$\mu_{A+B}(xy) = \begin{cases} \mu_A(xy)+\mu_B(xy)-\mu_A(xy).\mu_B(xy) \\ \le \mu_A(x^n)+\mu_B(x^n)-\mu_A(x^n).\mu_B(x^n) \\ = \mu_{A+B}(x^n) \end{cases}$$

Therefore:

$$\mu_{A+B}(xy) \le \mu_{A+B}(x^n)$$

Consider:

$$\gamma_{A+B}(xy) = \begin{cases} \gamma_A(xy)\gamma_B(xy) \\ \ge \gamma_A(x^n)\gamma_B(x^n) \\ \gamma_{A+B}(x^n) \end{cases}$$

Therefore:

$$\gamma_{A+B}(xy) \ge \gamma_{A+B}(x^n)$$

Hence, A+B is an intuitionistic fuzzy semi-primary ideal of R.

Theorem 2: If A and B are any two intuitionistic fuzzy semi-primary ideal of R, then A.B is an intuitionistic fuzzy semi-primary ideal of R.

Proof: Consider x, yϵR then x, yϵA.B. Since, A is an intuitionistic fuzzy semi-primary ideal of R, $\mu_A(xy) \le \mu_A(x^n)$ and $\gamma_A(xy) \ge \gamma_A(x^n)$. Since, B is an intuitionistic fuzzy semi-primary ideal of R, $\mu_B(xy) \le \mu_B(x^n)$ and $\gamma_B(xy) \ge \gamma_B(x^n)$.

Consider:

$$\begin{aligned} \mu_{A.B}(xy) &= \mu_A(xy).\mu B(xy) \le \mu_A(x^n).\mu_B(x^n) \\ &= \mu_{A.B}(x^n) \end{aligned}$$

Therefore:

$$\gamma_{A.B}(xy) \ge \mu_{A.B}(x^n)$$

Consider:

$$\begin{aligned} \gamma_{A.B}(xy) &= \gamma_A(xy)+\gamma_B(xy)-\gamma_A(xy).\gamma_B(xy) \ge \gamma_A(x^n)+\gamma_B(x^n)-\gamma_A(x^n).\gamma_B(x^n) \\ &= \gamma_{A.B}(x^n) \end{aligned}$$

Therefore:

$$\mu_{A.B}(xy) \geq \gamma_{A.B}(x^n)$$

Hence, A. B is an intuitionistic fuzzy semi-primary ideal of R.

Theorem 3: If A and B is an intuitionistic fuzzy semip-rimary ideal of R then A*B is an intuitionistic fuzzy semi-primary ideal of R.

Proof: Consider: x, y$\in$R then x, y$\in$A*B. Since, A is an intuitionistic fuzzy semi-primary ideal of R, $\mu_A(xy) \leq \mu_A(x^n)$ and $\gamma_A(xy) \geq \gamma_A(x^n)$. Since, B is an intuitionistic fuzzy semi-primary ideal of R, $\mu_B(xy) \leq \mu_B(x^n)$ and $\gamma_B(xy) \geq \gamma_B(x^n)$. Consider:

$$\begin{aligned}\mu_{A*B}(xy) &= \mu_A(xy)+\mu_B(xy)/(2(\mu_A(xy)\mu_B(xy)+1)) \leq \mu_A(x^n)+\mu_B(x^n)/(2(\mu_A(x^n)\mu_B(x^n)+1)\\ &= \mu_{A*B}(x^n)\end{aligned}$$

Therefore:

$$\mu_{A*B}(xy) \leq \mu_{A*B}(x^n)$$

and:

$$\gamma_{A*B}(xy) = \gamma_A(xy)+\gamma_B(xy)/(2(\gamma_A(xy)\gamma_B(xy)+1)) \geq \gamma_A(x^n)+\gamma_B(x^n)/(2(\gamma_A(x^n)\gamma_B(x^n)+1)) = \gamma_{A*B}(x^n)$$

Therefore:

$$\gamma_{A*B}(xy) \geq \gamma_{A*B}(x^n)$$

Hence, A*B is an intuitionistic fuzzy semi-primary ideal of R.

Theorem 4: If A and B is an intuitionistic fuzzy semi-primary ideal of R then A$\bowtie$B is an intuitionistic fuzzy semi-primary ideal of R.

Proof: Consider: x, y$\in$R then x, y$\in$A$\bowtie$B. Since, A is n intuitionistic fuzzy semi-primary ideal of R, $\mu_A(xy) \leq \mu_A(x^n)$ and $\gamma_A(xy) \geq \gamma_A(x^n)$. Since, B is an intuitionistic fuzzy semi-primary ideal of R, $\mu_B(xy) \leq \mu_B(x^n)$ and $\gamma_B(xy) \geq \gamma_B(x^n)$. Consider:

$$\mu_{A\bowtie B}(xy) = 2\mu A(xy).\mu B(xy)/(\mu_A(xy)+\mu_B(xy)) \leq 2\mu_A(x^n).\mu_B(x^n)/(\mu_A(x^n)+\mu_B(x^n)) = \mu_{A\bowtie B}(x^n)$$

Therefore:

$$\mu_{A\bowtie B}(xy) \leq \mu_{A\bowtie B}(x^n)$$

Consider:

$$\gamma_{A\bowtie B}(xy) = 2\gamma_A(xy).\gamma_B(xy)/(\gamma_A(xy)+\gamma_B(xy)) \geq 2\gamma_A(x^n).\gamma_B(x^n)/(\gamma_A(x^n)+\gamma_B(x^n)) = \gamma_{A\bowtie B}(x^n)$$

Therefore:

$$\gamma_{A\bowtie B}(xy) \geq \gamma_{A\bowtie B}(x^n)$$

Hence, $A\bowtie B$ is an intuitionistic fuzzy semi-primary ideal of R.

Theorem 5: If A and B is an intuitionistic fuzzy semi-primary ideal of R then A$B is an intuitionistic fuzzy semi-primary ideal of R.

Proof: Consider: x, y∈R then x, y∈A$B. Since, A is an intuitionistic fuzzy semi-primary ideal of R, $\mu_A(xy) \leq \mu_A(x^n)$ and $\gamma_A(xy) \geq \gamma_A(x^n)$. Since, B is an intuitionistic fuzzy semi-primary ideal of R, $\mu_B(xy) \leq \mu_B(x^n)$ and $\gamma_B(xy) \geq \gamma_B(x^n)$. Consider:

$$\mu_{A\$B}(xy) = \sqrt{\mu_A(xy).\mu_B(xy)} \leq \sqrt{\mu_A(x^n).\mu_B(x^n)} = \mu_{A\$B}(x^n)$$

Therefore:

$$\mu_{A\$B}(xy) \leq \mu_{A\$B}(x^n)$$

Consider:

$$\gamma_{A\$B}(xy) = \sqrt{\gamma_A(xy).\gamma_B(xy)} \geq \sqrt{\gamma_A(x^n).\gamma_B(x^n)} = \gamma_{A\$B}(x^n)$$

Therefore:

$$\gamma_{A\$B}(xy) \geq \gamma_{A\$B}(x^n)$$

Hence, A$B is an intuitionistic fuzzy semi-primary ideal of R.

Theorem 6: If A and B is an intuitionistic fuzzy semi-primary ideal of R then A@B is an intuitionistic fuzzy semi-primary ideal of R.

Proof: Consider: x, y∈R then x, y∈A@B. Since, A is an intuitionistic fuzzy semi-primary ideal of R, $\mu_A(xy) \leq \mu_A(x^n)$ and $\gamma_A(xy) \geq \gamma_A(x^n)$. Since B is an intuitionistic fuzzy semi-primary ideal of R, $\mu_B(xy) \leq \mu_B(x^n)$ and $\gamma_B(xy) \geq \gamma_B(x^n)$. Consider:

$$\mu_{A@B}(xy) = (\mu_A(xy)+\mu_B(xy))/2 \leq (\mu_A(x^n)+\mu_B(x^n))/2 = \mu_{A@B}(x^n)$$

Therefore:

$$\mu_{A@B}(xy) \leq \mu_{A@B}(x^n)$$

Consider:

$$\gamma_{A@B}(xy) = (\gamma_A(xy)+\gamma_B(xy))/2 \geq (\gamma_A(x^n)+\gamma_B(x^n))/2 = \gamma_{A@B}(x^n)$$

Therefore:

$$\gamma_{A@B}(xy) \geq \gamma_{A@B}(x^n)$$

Hence, A@B is an intuitionistic fuzzy semi-primary ideal of R.

Theorem 7: If A is an intuitionistic fuzzy primary ideal of R then $\bar{A}$ is also an intuitionistic fuzzy primary ideal of R.

Proof: Consider: x, y $\in$ R then x, y$\in\bar{A}$. Since, A is an intuitionistic fuzzy primary ideal of R, $\mu_A(xy) = \mu_A(x)$ and $\gamma_A(xy) = \gamma_A(x)$. Consider:

$$\mu_{\bar{A}}(xy) = \gamma_{\bar{A}}(xy) = \gamma_{\bar{A}}(x) = \mu_{\bar{A}}(x)$$

Therefore:

$$\mu_{\bar{A}}(xy) = \mu_{\bar{A}}(x)$$

Consider:

$$\gamma_{\bar{A}}(xy) = \mu_A(xy) = \mu_A(x) = \gamma_A\,(x)$$

Therefore:

$$\gamma_{\bar{A}}(xy) = \gamma_{\bar{A}}(x)$$

Hence, $\bar{A}$ is an intuitionistic fuzzy primary ideal of R.

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