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An SIR Estimation for Pandemic by influenza A During the Haj

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Abstract: In this study, a system of SIR models depended on effective contact rate of the disease and potential size of the community is developed for simulating patterns of pandemic by influenza A during the Haj. Indeed, we consider three different SIR models such that each of them is calibrated to simulate a separate part of the ceremony and then we link these models together to obtain system simulating whole the Haj ceremony. Considering different numerical values of effective contact rate and potential size of the community, we estimate the portion of susceptible hosts in the community and also the portion of total infection during the ceremony and portion of infective individuals at any time of the ceremony. Our goal is to suggest doable optimal situations keeping the pandemic under the control and the number of infected people as low as possible.

Key words: SIR model, effective contact rate, transmission rate, influenza A, Haj ceremony

INTRODUCTION

For who are interested in study of infectious diseases, the influenza is of great importance. Since, influenza virus continues always evolving, so there exists a menace of pandemic by mutant influenza virus and it is still an important problem for the public health systems. In 1918, the great pandemic of influenza on modern history occurred. About 40000000 humans died of this pandemic all over the world. After 1918, a pandemic of influenza occurred twice in 1957 and 1968 and today again, the World Health Organization (WHO) has warned of a substantial risk of pandemic of influenza A throughout the globe. Apart from the mortality associated with complications due to influenza, the costs of workdays lost by the infected people, vaccines, antiviral drugs and so on must be taken in account among the consequences of probably pandemic by influenza. Meanwhile, Islamic countries with near to 2 billions populations, mostly dwelled in Asia, are threaten by pandemic of influenza during the Haj ceremony when almost 2 millions of Moslems gather in Mecca every year. Islamic republic of Iran, as country with more than 70 millions populations joints this ceremony every year with tens of thousands of Iranian people. In 2009, because of the threats existing for pandemic by influenza A, the government put some limitation as they did not let the people with more that 63 years old and children to joint the ceremony. Also they stated a restriction of number of people jointing the ceremony. Similar policies took by a few other Islamic countries in this regards. These facts convinced the author that any effort on developing models for illustrating, estimation and simulation patterns of infection within the Haj ceremony may be helpful.

In this regards, mathematical models are considered as easily accessible tools for simulating patterns of infection by influenza. Especially, when these models are supported by exact statistical data, the consequent results are applicably reliable. Many mathematical models have been proposed in the study to describe the inter-pandemic ecology of influenza A in humans. A typical approach is modeling of the interactions between individuals who are (or have been) infected by one or different viral strains (Castillo-Chavez *et al*., 1989; Brauer *et al*., 2008; Brauer and Castillo-Chavez, 2001). An advanced modeling is done by Gupta *et al*. (1998), where the researchers showed that the simultaneous circulation of several antigenic variants of the same pathogen can give rise to complex dynamics, including sustained oscillations and chaos in disease. This may occur when a genetic mutation for influenza A causes simultaneous existence of two or more various strain. Then the results were illustrated more specifically by Lin *et al*. (1999), where an epidemiological model consisting of a linear chain of three co-circulating influenza A strains that provide hosts exposed to a given strain with partial immune cross-protection against other strains is analyzed. They could find a sub-model that exhibits sustained oscillations. Since, genetic mutation for influenza A is occurred usually ones within per several years, so the models which simulate mid time patterns, consider pandemic by a single strain. Such simulations have been done by Hethcote (1976), Casagrandi *et al*. (2006) and Thieme and Yang (2002), where a SIR or SIRC model develops a simple ordinary differential equation for studying patterns of infection in a certain mid-time interval. For an especial case by Andreasen *et al*. (1997), a SIRC model is considered to simulate the epidemiological consequences of the drift mechanism for influenza A viruses. Their model could predict a rich variety of complex temporal patterns that are realistic for influenza A. One year later, a mathematical model is proposed to interpret the spread of avian influenza from the bird world to the human world (Iwami *et al*., 2007). They gave their results on the spread of avian influenza and mutant avian influenza and measures to control the spread of avian influenza. Since, the complete immunity is usually lost within a more that several weeks, so the short time patterns are usually simulated by simple SIR models (Brauer and Castillo-Chavez, 2001; Hethcote, 2000; Anderson and May, 1991).

In this study, we consider a set of SIR models which simulate patterns of infection by influenza A during the stages of the Haj ceremony. These models depend on several parameters, including the effective contact rate and the potential size of the community. The parameters are tried to be well calibrated for the Iranian Haj ceremony; however; these systems can be implemented simply for other communities or diseases by selecting the values of the parameters appropriately.

MATERIALS AND METHODS

Introducing the SIR System and Parameters

In a short period of time, infectious diseases follow epidemiological patterns of infection which is called in brief SIR models. That is : individuals in susceptible subgroup are infected by the disease and expend a period of time as member of infective subgroup and finally a portion of them are recovered from the disease and the rest of them died. In a short time, recovered individuals are assumed to have complete immunity against the disease. Figure 1 shows the corresponding model.

Since, the Haj ceremony is usually accomplished within a period of 30 days, so SIR models are appropriate for modeling of pandemic by influenza A. Therefore, we divide the Haj community at time t into three compartments: susceptible individuals S(t) of who have no specific immunity against influenza A; the fraction I(t) of those individuals that are infected and infective; the fraction R(t) of those individuals who are recovered from the disease and gain complete immunity (at least for a short period of time which we consider of). Also we assume that the community of people in Haj has no natural mortality or birth. On the other hand, the Haj community is constituted within three steps:

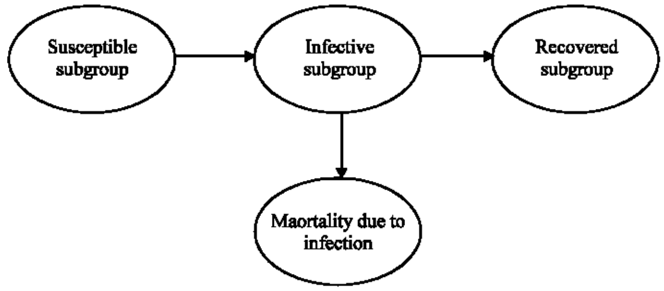

Fig. 1: A simple figuration of subgroups interacting in a SIR system

Step 1: With a period of 13 days in which the people receive with a constant rate to the place of the ceremony

Step 2: With a period of 4 days, in which the major traditions of the ceremony is accomplished

Step 3: With a period of 13 days, in which the people return with a constant rate and fixed schedule from the ceremony to the country

If N_0 is the number of the people contributing in the ceremony then we can say that N_0 is the potential size of the Haj community. Thus, we can define the corresponding SIR models for step 1-3, respectively by the following set of three ordinary differential equations.

$$\frac{dS_1}{dt}=\frac{N_0}{13}-\alpha S_1 I_1,\quad \frac{dI_1}{dt}=-(\mu+\varepsilon)I_1+\alpha S_1 I_1,\quad \frac{dR_1}{dt}=\varepsilon I_1 \tag{1}$$

$$\frac{dS_2}{dt}=-\alpha S_2 I_2,\quad \frac{dI_2}{dt}=-(\mu+\varepsilon)I_2+\alpha S_2 I_2,\quad \frac{dR_2}{dt}=\varepsilon I_2 \tag{2}$$

$$\frac{dS_3}{dt}=-\frac{S_3}{13}-\alpha S_3 I_3,\quad \frac{dI_3}{dt}=-(\mu+\varepsilon+\frac{1}{13})I_3+\alpha S_3 I_3,\quad \frac{dR_3}{dt}=\varepsilon I_3-\frac{R_3}{13} \tag{3}$$

Equation 1 explains that the susceptible hosts arrive to the ceremony place by constant rate $N_0/13$ and they are infected at rate $\alpha S_1 I_1$; then, the infective hosts are recovered at rate εI_1, die at rate μI_1nd are generated at rate $\alpha S_1 I_1$; also the recovered hosts are generated at rate εI_1. The Eq. 2 and 3 use the same structure depending on the stage of the ceremony that they simulate. These Eq. 1-3 are respectively subjected to the following initial conditions.

$$\begin{aligned}
&S_1(1) = (N_0/13)-1, I_1(1)=1, R_1(1)=0,\\
&S_2(13)=S_1(13), I_2(13)=I_1(13), R_2(13)=R_1(13),\\
&S_3(17)=S_2(17), I_3(17)=I_2(17), R_3(17)=R_2(17)
\end{aligned} \tag{4}$$

The condition $I_1(1) = 1$ is considered because of the fact that any pandemic begins by existence at least one infective individual and we assume that the first infection appears in the first day of the ceremony, also in the first day of the ceremony, the total size of the community is $N_0/13$. The parameter μ stands for the rate of mortality due to the infection and the parameter ε stands for the rate of recovering from the disease. Therefore, $\varepsilon+\mu$ shows the rate of individuals who leave the infective subgroup. Therefore, if the average number of days which a specific infective individual expends to be recovered or die is l, then

$\mu+\varepsilon = 1/l$ day^{-1} (Brauer and Castillo-Chavez, 2001; Hethcote, 2000; Andreasen *et al.*, 1997). Thus, $\varepsilon = (1/l)-\mu$. In this study, according to the statistical data (Castillo-Chavez *et al.*, 1989; Lin *et al.*, 1999; Andreasen *et al.*, 1997), we assume that the average time expended by the individuals in the infective subgroup is 4 days i.e., $l = 4$; also the mortality rate is considered 1% i.e., $\mu = 0.01/l = 0.0025$ and therefore $\varepsilon = 0.99/l = 0.2475$.

Computation of the parameter α is more difficult. This parameter is called effective contact rate and it must be measured as effective contacts per unit time. So, it defers depending on the size and social specifications of the given population. This may be expressed as the total contact rate (the total number of contacts, effective or not, per unit time, denoted γ), multiplied by the risk of infection, given contact between an infectious and a susceptible individual. This risk is called the transmission risk and is denoted by p; (Brauer and Castillo-Chavez, 2001). Thus: $\alpha = \gamma p$. On the other hand $p = x/N$, where N denotes the total size of the community and x denotes the number of secondary infected cases in the infective subgroup. Thus: $\alpha = \gamma x/N$, which means that α is of scale of $(1/N)$ day^{-1} and so it is usually considered a small parameter. The total size of the community at any time t is obtained from $N = S+I+R$. Thus, as time goes farther, the total size of the community for systems 1-3 holds, respectively for Eq. 5 below.

$$\frac{dN_1}{dt} = \frac{N_0}{13} - \mu I_1(t), \quad \frac{dN_2}{dt} = -\mu I_2(t), \quad \frac{dN_3}{dt} = -\frac{S_3(t)+I_3(t)+R_3(t)}{13} - \mu I_3(t) \tag{5}$$

In order to bring the systems Eq. 1-3 into dimensionless standard form, first we divide variables N, S, I, R to N_0. This makes the variables dimensionless. Then we rescale the time with $\tau = 2(\mu+\varepsilon)t/15 = t/30$. Thus τ is dimensionless. Therefore the dimensionless SIR systems are obtained, respectively as below:

$$\frac{dS_1}{d\tau} = \frac{30}{13} - (30N_0\alpha)\, S_1I_1, \quad \frac{dI_1}{d\tau} = -(\frac{15}{2})I_1 + (30N(0)\alpha)\, S_1I_1, \quad \frac{dR_1}{d\tau} = (7.425)I_1 \tag{6}$$

$$\frac{dS_2}{d\tau} = -(30N_0\alpha)\, S_2I_2, \quad \frac{dI_2}{d\tau} = -(\frac{15}{2})I_2 + (30N(0)\alpha)\, S_2I_2, \quad \frac{dR_2}{d\tau} = (7.425)I_2 \tag{7}$$

$$\begin{aligned} &\frac{dS_3}{d\tau} = \frac{30}{13}S_3 - (30N_0\alpha)\, S_3I_3, \quad \frac{dI_3}{d\tau} = -(\frac{15}{2}+\frac{30}{13})I_3 + (30N(0)\alpha)\, S_3I_3, \\ &\frac{dR_3}{d\tau} = (7.425)I_3 - (\frac{30}{13})R_3 \end{aligned} \tag{8}$$

These Eq. 6-8 are subjected, respectively to the following new initial conditions.

$$\begin{aligned} &S_1(0.033) = 1/13 - 1/N_0, I_1(0.033) = 1/N_0, R_1(0.033) = 0 \\ &S_2(13/30) = S_1(13/30), I_2(13/30) = I_1(13/30), R_2(13/30) = R_1(13/30) \\ &S_3(17/30) = S_2(17/30), I_3(17/30) = I_2(17/30), R_3(17/30) = R_2(17/30) \end{aligned} \tag{9}$$

These three sets of ordinary differential equations subjected to the corresponding initial conditions are considered as the standard dimensionless SIR models for simulating the patterns of infection by influenza A. Since, the total size of the community N(t) is always less than N_0, so in systems 6-8, we have $S(t)+I(t)+R(t)\leq 1$. Furthermore, according to the way we rescaled parameter t, when t varies from 0 to 30 days then τ varies from 0 to 1. This means that for systems Eq. 6-8, we have respectively $0 \le \tau \le 13/30$, $13/30 \le \tau \le 17/30$, $17/30 \le \tau \le 1$. Now we will study the patterns of infection predicted by these models and try to find critical values for effective contact rate α.

RESULTS

Estimating Patterns of Infection

In this study of a pandemic by the infectious disease, there are three matters of importance: First of all, the patterns by which the infection develops in the community; the second one is the total number (or total portion) of all people in the community who get infection at a certain time. This is especially important because it helps us to know how much of vaccines, antiviral drugs or other necessary medical cares we should be able to supply during this certain time. Finally, the third one is the number (or portion) of the people in the community who are infected at time t. This helps us to know how much of medical cares is needed at any time t. In this section we try to indicate these three matters of importance. We begin with the first one.

It is obvious that here the goal is to keep the number of people who get infection during the Haj ceremony as low as possible. Therefore, the patters which describe the number (or portion) of the individuals in each of susceptible, infective or recovered subgroup are interesting to us; especially, the portion of susceptible hosts in compare with the portion of infective and recovered hosts is an appropriate variable which describes the pattern by which infection develops during the Haj ceremony. Thus, we define two new variables $A(\tau) = S(\tau)/N(\tau)$ and $B(\tau) = (I(\tau)+R(\tau)/N(\tau))$. By this definition, the variable $A(\tau)$ shows the portion of non-infected people in the community and the variable $B(\tau)$ shows the portion of the people who belong to infective or recovered subgroups in the community. These variables are nonnegative and hold for the equation.

$$A(\tau) + B(\tau) = 1$$

Thus, the graphs of these two new variables can describe development of infection during the Haj ceremony. In the Fig. 2, the numerical solutions of systems 6-7, computed for different values of the parameters α and N_0, are applied for plotting the graph of $A(\tau)$ and $B(\tau)$. In the Fig. 2a-r, the green and black curves are, respectively stand for $A(\tau)$ and $B(\tau)$, where the horizontal axis stands for τ and the vertical axis stands for the values of $A(\tau)$ and $B(\tau)$; also the vertical dash lines separate three distinct parts of the Haj ceremony. The values of the parameters α and N_0 for each of the Fig. 2 are also indicated in its below. In the Fig. 2a-f, the parameter α is constantly 5×10^{-6} while the parameter N_0 varies from 5×10^4 to 10^5. This means that the average number of secondary infection by each infective individual is 5 person for per 1 million susceptibles. As it can be seen, the portion of non-infected individuals decays by growth of size of the community. However, as the Fig. 2 show the portion of infected people is very low. This means that, as long as the effective contact rate α is lower that 5×10^{-6}, the pandemic will be under control.

In the Fig. 2g-*l*, the effective contact rate α is constantly 1×10^{-5} and the parameter N_0 varies from 5×10^4 to 10^5. As it can be seen, the portion of infected individuals increases fastly by growth of size of the community. Especially, for $N_0 = 10^5$ the green curve intersects decreasingly the black curve which means that the portion of infected people will be more that the portion of the non-infected people after the intersection point. In this case, the value of τ which corresponds to the intersection point can be considered as the time of breaking out of infection.

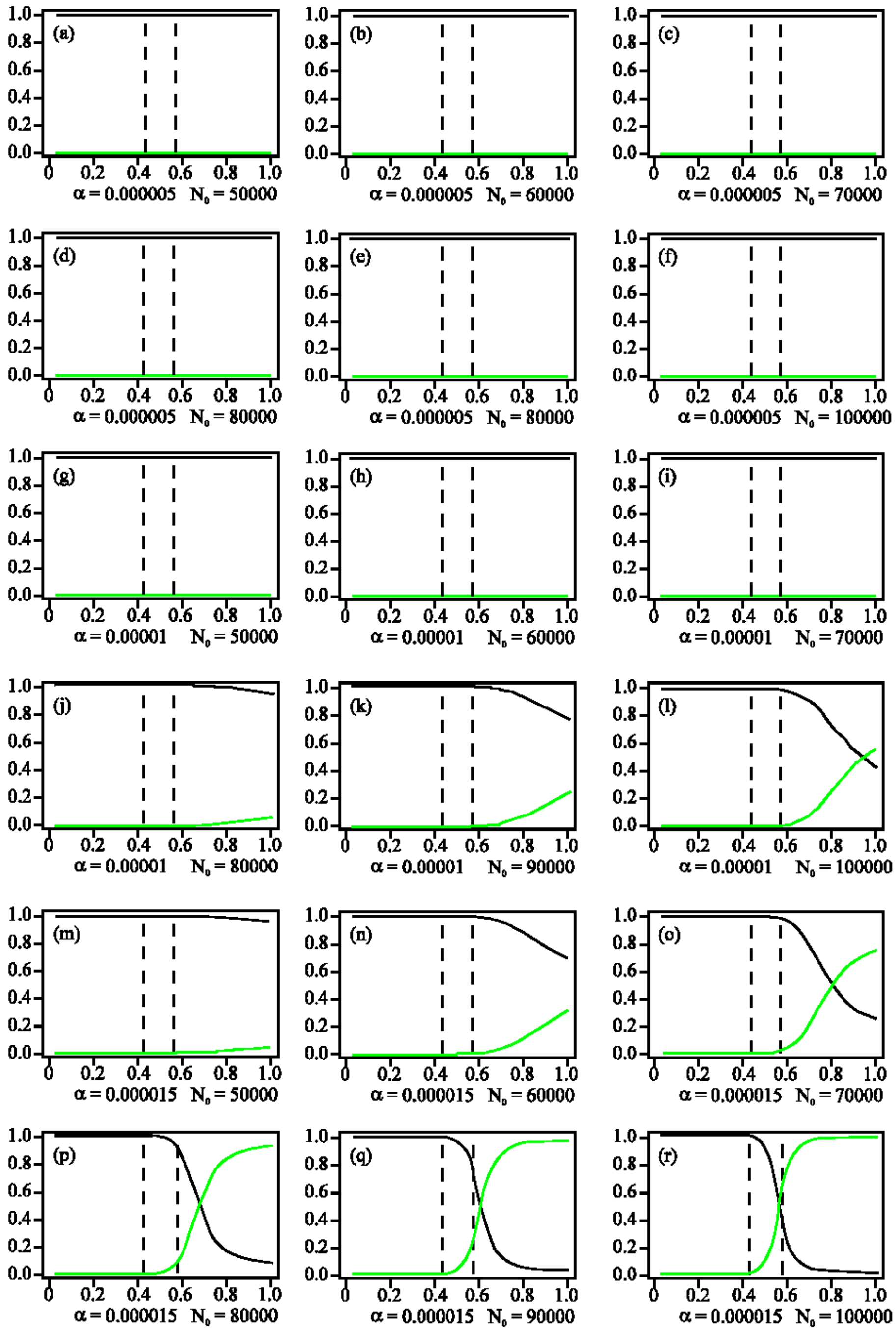

Fig. 2: (a-r) The graph of variables $A(\tau)$ and $B(\tau)$ for various range of the parameters α, N_0 according to the text. Here the green and the black curves are stand by respectively for $A(\tau)$ and $B(\tau)$ and the dashed lines indicate three separate parts of the Hajj ceremony

Finally, in the Fig. 2m-r, the effective contact rate is constantly 1.5×10^{-6} and the parameter N_0 varies from 5×10^4 to 10^5. As it can be seen, the pandemic occurs of $N_0\geq7\times10^4$. Figure 2a-r show that the time of breaking out decays by growth of size of the community. Moreover, for $N_0>9\times10^4$, the breaking out time may decay to the last days of the second part of the ceremony (Fig. 2q , r) which is the most important part. This means that increasing the value of effective contact rate to 1.5×10^{-5} and N_0 to more that 9×10^4 disturbs the ceremony.

Thus, the model estimates that, in order to have the pandemic under the control, the effective contact rate α must not exceed than 5×10^{-6} (Fig. 2a-f) and if it exceeds up to 1×10^{-5} then the total size of the community N_0 is better to be less than 9×10^4 (Fig. 2g-k). Furthermore, if α exceeds to 1.5×10^{-5} then, in order to control the pandemic, N_0 must be less that 6×10^4 (Fig. 2m, n).

The second important matter is the portion of total infected individuals during the ceremony. This is the portion of all individuals in the community who are infected or recovered at day 30 of the ceremony plus to the number of individuals who died due to the infection. To obtain this portion, firstly we have to compute the number of all infected people up to t-th day of the ceremony, then total number of all infected individuals during the Haj ceremony is obtained for t = 30 and the portion of total infection is obtained by rescaling the variables and putting $\tau = 1$.

From the Eq. 5 and the initial conditions Eq. 4, we have,

$$N_1(t)=\frac{t}{13}N_0-1-\mu\int_1^t I_1(s)ds, \qquad (1\leq t\leq 13)$$

$$N_2(t)=N_1(13)-\mu\int_{13}^t I_2(s)ds=(N_0-1)-\mu\int_1^{13} I_1(s)ds-\mu\int_{13}^t I_2(s)ds, \qquad (13\leq t\leq 17)$$

$$\begin{aligned}N_3(t)&=N_2(17)-\frac{1}{13}\int_{17}^t S_3(s)ds-\frac{1}{13}\int_{17}^t I_3(s)+R_3(s)ds-\mu\int_{17}^t I_3(s)ds\\ &=(N_0-1)-\mu\int_1^{13} I_1(s)ds-\mu\int_{13}^{17} I_2(s)ds-\frac{1}{13}\int_{17}^t S_3(s)ds,\\ &-\frac{1}{13}\int_{17}^t I_3(s)+R_3(s)ds-\mu\int_{17}^t I_3(s)ds. \qquad (17\leq t\leq 30)\end{aligned}$$

In each of the three equation above, $\mu\int I(s)ds$ shows the total mortality up to the t-th day of the corresponding part of the ceremony and I(t)+R(t) shows the number of the individuals who are currently in infective or recovered subgroups. Also in the third equation,

$$\frac{1}{13}\int_{17}^t I_3(s)+R_3(s)ds$$

shows the number of individuals in the infective and recovered subgroups who have left the ceremony (and returned to the country) up the t-th day of the third part of the ceremony. By the same way,

$$\frac{1}{13}\int_{17}^t S_3(s)ds$$

shows the number of individuals in the susceptible subgroup who have left the ceremony (and returned to the country) up the t-th day of the third part of the ceremony. This yields that the total number of infected individuals at each part of the ceremony, which we denote it by Γ, is obtained by the following equations:

$$\Gamma_1(t) = \frac{t}{13}N_0 - 1 - S_1(t), \quad \Gamma_2(t) = N_0 - 1 - S_2(t), \quad \Gamma_3(t) = N_0 - 1 - \frac{1}{13}\int_{17}^{t} S_3(s)ds - S_3(t)$$

Thus, the dimension less variable $\tilde{\Gamma} = \Gamma / N_0$ shows the portion of all infected people in the community. This variable can be computed through dimensionless standard systems Eq. 6-8 as below:

$$\tilde{\Gamma}_1(\tau) = \frac{30}{13}\tau - 1 - S_1(\tau), \quad \tilde{\Gamma}_2(\tau) = (1 - \frac{1}{N_0}) - S_2(\tau), \quad \tilde{\Gamma}_3(\tau) = (1 - \frac{1}{N_0}) - S_3(\tau) - \frac{30}{13}\int_{0.57}^{\tau} S_3(s)ds$$

Thus, the portion of total infection which we denote it by Ω is equal to $\tilde{\Gamma}_3(1)$. The Fig. 3a-f show this portion for various range of, where the horizontal axis stands for α and vertical axis stands for Ω.

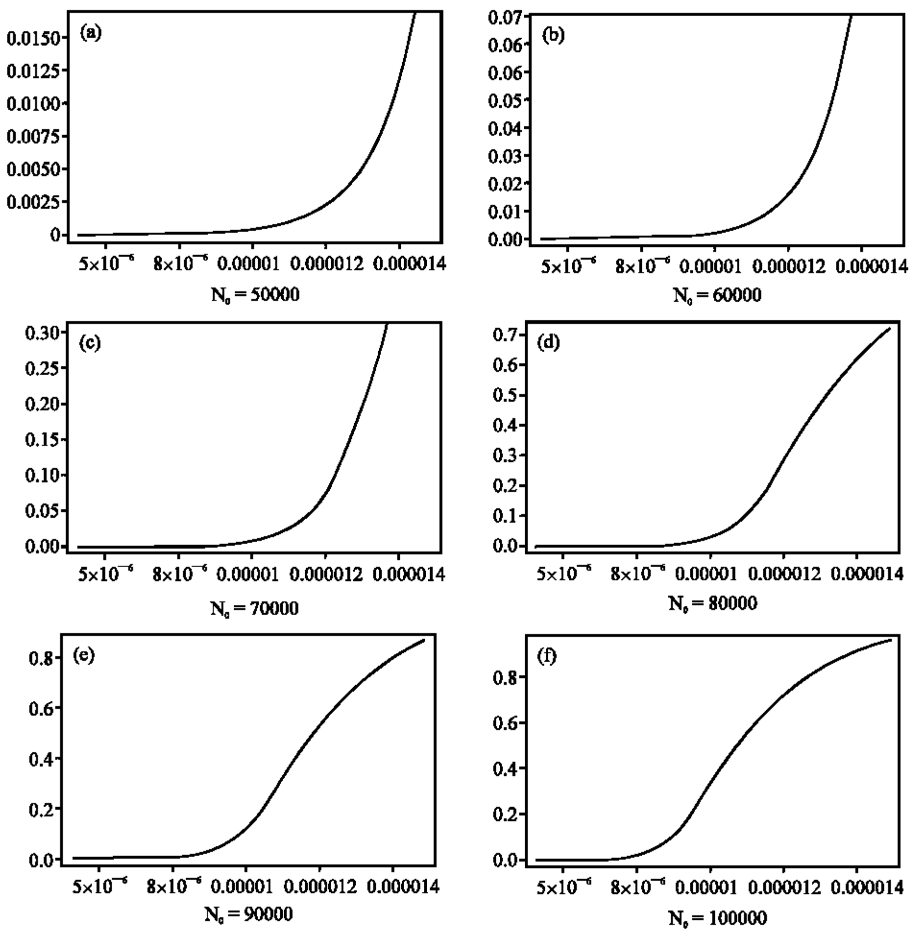

Fig. 3: (a-f) The total portion of infected individuals for various range of N_0 according to the text. Here, the vertical axis stands for Ω and the horizontal axis stands for α

As it can be seen through Fig. 3a-d, for the effective contact rate $\alpha \le 10^{-6}$and $N_0 \le 8\times10^4$ the portion of all infected individuals is less that 5% of the community however, this portion may suddenly rise up to 20% of the community for $N_0 \ge 8\times10^4$ (Fig. 3e) and even to 40% of the community for $N_0 \ge 9\times10^4$(Fig. 3f). This means that the critical value for the potential

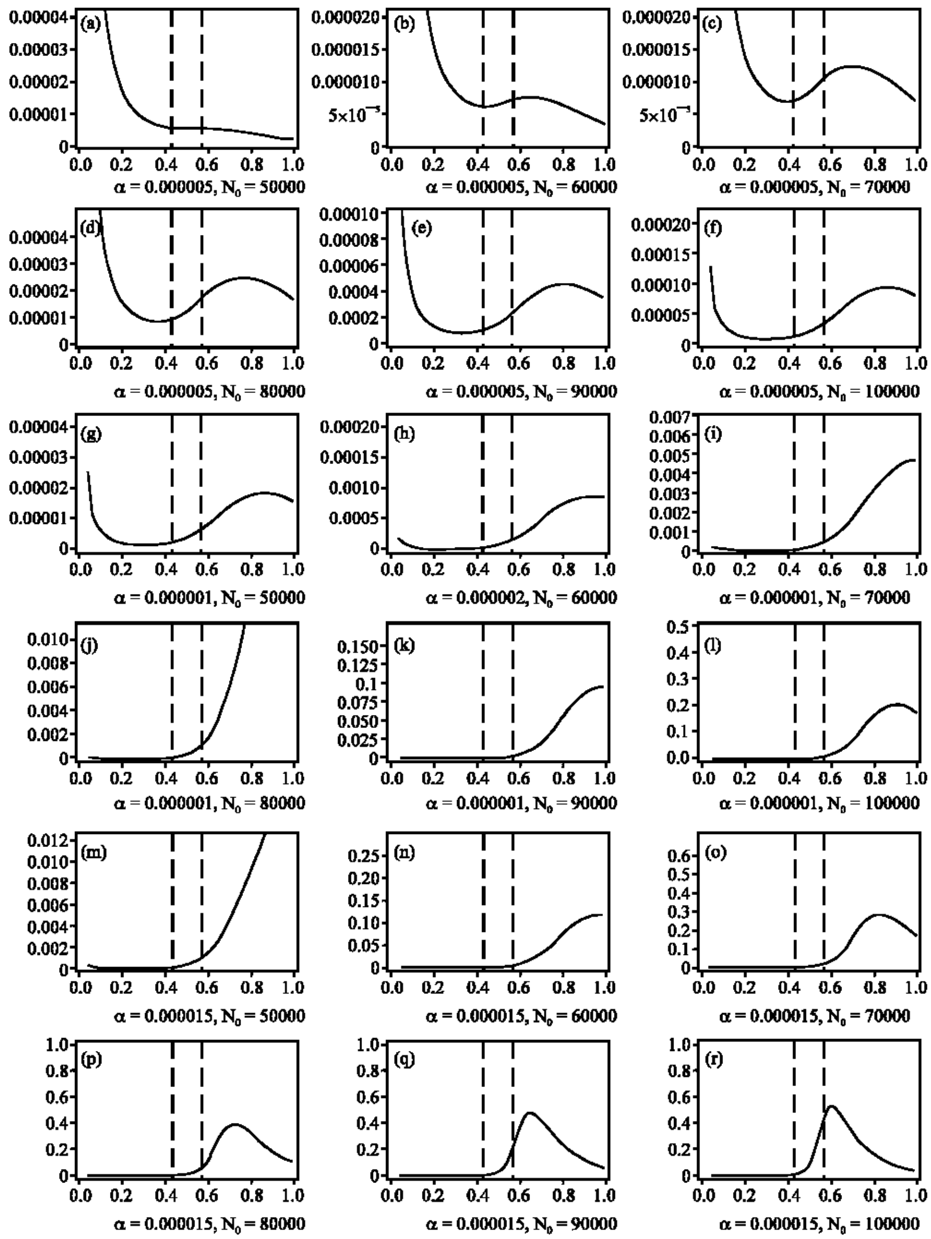

Fig. 4: (a-r) The graphs of portion of infective individuals according to the text. Here, the vertical axis stands for total portion of infected individuals and the horizontal axis stands for τ

community size N_0 is approximately 8×10^4. This is because of the fact that any exceeding from this value may cause a sudden breaking out of the infection. One more important fact which can be observed is the existence of a relatively sudden rise up in the portion of total infection by exceeding the values of α from 10^{-5}. This means that the critical value for effective contact rate is 10^{-5}. Therefore, in order to keep the portion of total infection lower than 25%, the secondary infections must be keep lower that one person in per 100000 susceptible individuals.

At the end of the third important matter i.e., the number of the people how are in infective subgroup at t-day of the ceremony. This is directly obtained from the values of the variable I(t) in Eq. 1-3. Similarly, the dimensionless variable portion of the infective individuals can be obtained through the values of $I(\tau)$ in Eq. 6-8. The Fig. 4a-r show this portion for various values of parameters α and N_0, where the horizontal axis stands for τ and the vertical axis stands for the portion of individuals in infective subgroup.

As it can be seen, probably except for the Fig. 4a, we have an increase in portion of infective individuals within the second part of the ceremony and this increase may continue even up to the last day of the ceremony (Fig. 4i-k and m, n).

The peak point in each of the Fig. 4 indicates the time in which the portion of the infective hosts is in its maximal portion. Since, accomplishing the traditions of the ceremony is hard for the individuals how is being infected by the disease, so it is better to keep the peak of the Fig. 4 far from the second part of the ceremony, especially for those figures which are according to a great portion of total infection in Fig. 3. For example, Fig. 3d-f with $\alpha = 1.5\times10^{-4}$ and $N_0\geq8\times10^4$ and Fig. 4p-r. Also, according to the Fig. 3, 4, we must expect a flash rise up in portion of infective individuals right after the second part of the ceremony. For $\alpha\leq10^{-4}$, this rise up is less that 2 percents of the community (Fig. 4a-l), but if α increases it may even reach to 50% of the community (Fig. 4q , r).

DISCUSSION

We developed a system of SIR models for simulating the probably pandemic by influenza A during the Haj ceremony. We developed SIR models to three sets of ordinary differential Eq. 1-3 and in order to avoid complexity due to scales, we rescaled these equations to dimensionless differential Eq. 6-8, which are subject to appropriate initial conditions (9). Then we considered three matter of importance: the portion of susceptible hosts in compare with the portion of infective and recovered hosts in the community; the total portion of infected individuals and the portion of infective hosts at any time.

First in the Fig. 2, we saw that in order to have the portion of infective and recovered hosts low in compare with the susceptible hosts, the effective contact rate α must be less that 5×10^{-6} (Fig. 2a-f). We also saw that of the effective contact rate exceeds up to 10^{-5} then it is necessary to keep the number of the people how join the ceremony N_0 less than 8×10^4 (Fig. 2g-j). Also, we saw that for $\alpha\approx1.5\times10^{-5}$ there exists a breaking out of the disease which may begin even from the last days of the second part of the ceremony (Fig. 2q, r).

In the Fig. 3, we plotted the total portion of infection and discussed about different cases which occurs. Especially we indicated the critical values for α and N_0; for example, we saw that if the value of α exceeds from 10^{-5}, then we expect a relatively sudden rise up in the total portion of infection.

Finally, in the Fig. 4, we plotted and discussed about the different cases of portion of infective hosts at time t of the ceremony. We distinguished an increase in portion of infective individuals within the second part of the ceremony and we saw that for $\alpha \geq 10^{-5}$ this increase may continue even up to the last day of the ceremony (Fig. 4i-k and m, n). Also, we found the cases for which the peak of infection occurs in earlier days of the third part of the ceremony (Fig. 4q, r).

From all discussion we conclude that the these SIR models predicts that the optimal situation for the Haj ceremony which is obtainable in real world can be occurred for N_0 at most up to 8×10^4 and medical cares which keep the effective contact rate α less than 10^{-5}. If it happens then, according to Fig. 2-4, we expect that the portion of infective or recovered individuals and the total portion of infection hold less that 10%. Also in this case the peak of infective individuals is less that 1.5% and it occurs in last days of the ceremony which is not much important because at that time much part of the community has left the ceremony and returned to the country.

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