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# A Mathematical Model for Vehicle Routing Problem in a Flexible Supply Network

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## ABSTRACT

The development of robust supply chains requires careful attention to both the location of individual supplier facilities and the opportunities for effective transportation among them. Here, we propose a supply chain which considers multiple depots, multiple vehicles, multiple products and multiple customers, with multi time periods. The supplier receives the order and forwards it to depots of multiple products. The depots investigate the capacity level and accept/refuse supplying the order. Considering the location of the customers, the depots decide upon sending the suitable vehicles. Each vehicle has its specific traveling time and cost. We present a mathematical model for the allocation of orders to depots and vehicles minimizing the total cost. We also provide a Lingo encoding and use it to solve an illustrating example to show the effectiveness of the proposed model.

**Key words:** Supply chain management, allocation problem, mathematical model, multi time period

## INTRODUCTION

The rapid industrialization and economic growth of many countries around the world have spurred the development of various supply chains reaching around the world. This has provided opportunities for manufacturers to cut costs and be closer to emerging and highly grown markets but it has also created new risks. As supply chains become increasingly dependent on the efficient movement of materials among geographically dispersed facilities, there is more opportunity for disruption.

Supply chain coordination has gained considerable notice lately from both practitioners and researchers. In some markets there is only one chain in which there is only one retailer being called perfect competing market, to maximize the profit we need to integrate supply chain vertically (Jeuland and Shugan, 1983; Cachon, 2003; Bernstein and Federgruen, 2005).

A market with two competing supply chains was investigated in the seminal work of McGuire and Staelin (1983). The researchers considered two suppliers price competition (i.e., Bertrand) each selling through an independent retailer. Coughlan (1985) applied this research to the electronics industry and Moorthy (1988) further explained why the decentralized chains could lead to higher profits for the manufacturer and the entire chains. Bonanno and Vickers (1988) considered a similar methodology to show that in some cases it is optimal to sell the products via an independent retailer.

In operation management of supply chains, Wu and Chen (2003) presented a newsvendor-based model for demand competition but they ignored the pricing decisions, which is a common ignorance amongst all researches in this field.

A few supply chain coordination mechanisms that induce the chain to act as if they were vertically integrated (VI) were investigated; e.g., buy back (Pasternack, 1985), quantity flexibility (Tsay, 1999) and revenue sharing (Cachon and Lariviere, 2005). Cachon (2003) for a survey of this literature. Two more reviews are found by Kouvelis *et al.* (2006) that focuses on supply chain coordination literature published in Production and Operations Management journals during 1992- 2006 and in Tang (2006) which covers much literature on supply chain coordination. Lin and Kong (2002) consider a duopoly that has no demand uncertainty and investigate a symmetric Nash Bargaining model. Similar to McGuire and Staelin (1983) they show that Nash Bargaining can lead to higher supply chain profits than a vertically integrated chain.

In a recent study, Baron *et al.* (2008) investigate the Nash Equilibrium of an industry with two supply chains by extending the seminal work of McGuire and Staelin (1983). Baron *et al.* (2008) show that both the traditional MS and the VI strategies are special cases of Nash Bargaining on the wholesale price when the demand is deterministic. They warn that the supply chain coordination mechanisms that focus on inducing supply chains to act as if they were vertically integrated, should be treated with caution.

The Vehicle Routing Problem (VRP) is recently being focused in SCM literature. The VRP problem considers a set of homogenous vehicles stationed at a depot to service the demands of customers in geographically scattered locations via the routes with the least cost. Each vehicle with a certain capacity starting and ending a tour in the depot, should find a rout which visit each customer only once. The VRP is considered as NP-hard problems which cannot be solved with analytical computational approaches and tackled by heuristics (Toth and Vigo, 2002; Laporte *et al.*, 2000; Cordeau *et al.*, 2002). The advantages of VRP are:

- Reducing the length of the delivery routes
- Decreasing the number of vehicles
- Providing better service to the customers
- Operating in a more efficient manner
- Increasing the market share

The transportation problem we tackle can be described as a multi-depot pickup and delivery problem with time windows and side constraints (Desrosiers *et al.*, 1995) and is regarded as one of the richest within the class of time constraint vehicle routing and scheduling problems in terms of scope and complexity. The earliest pickup time for shipments corresponds to one-sided time window constraints. In addition, operating time restrictions at some locations impose delivery time windows. The coexistence of consolidation (and of LTL shipments), relaying and trailer availability requirements in our problem context makes it a unique and even a more complicated problem than the ones studied before. Early major work on pickup and delivery problems with time windows has been reported by Savelsbergh and Sol (1995). Variants of the basic problem with context specific characteristics have been reported by Currie and Salhi (2003), Liu *et al.* (2003) and Sigurd *et al.* (2004), to name a few.

The concept of transportation network equilibrium has a longer history than supply chain networks and has been studied by Pigou (1920), with the first rigorous mathematical treatment given by Beckmann *et al.* (1956) in their classical book. Some other researches in modeling of transportation network equilibrium were related to Smith (1979), Dafermos (1980, 1982) and Boyce *et al.* (1983). Transportation network equilibrium was further studied by Florian and Hearn (1985) and the books by Patriksson (1994) and Nagurney (1999, 2000).

In supply chain modeling and analysis (Lee and Billington, 1993; Slats, 1995; Anupindi and Bassok, 1996), one typically associates the decision-makers with the nodes of the multi tiered supply chain network. In transportation networks, on the other hand, the nodes represent origins and destinations as well as intersections. Travelers or users of the transportation networks seek, in the case of user-optimization, to determine their cost-minimizing routes of travel.

Here, we propose a supply chain which considers multiple depots, multiple vehicles, multiple products, multiple customers with multi time periods.

## LITERATURE REVIEW

Zhang *et al.* (2011) proposed the design and implementation of a dynamic Radio Frequency Identification (RFID) data-driven supply chain management system with respect to the domains of supply chain management, simulation, Dynamic Data-Driven Application Systems (DDDAS) and Radio Frequency Identification (RFID) technology. Their model will be able to (1) model supply chain entities (2) simulate supply chain events and activities (3) use real-time RFID data to maintain a more accurate picture of the overall supply chain and (4) use the simulation results to control experiments.

Since, implementation of supply chain management requires integration of processes between supply chain members in all functional areas, including sourcing, manufacturing and distribution and the need for the successful implementation of information sharing being critical to effective innovation and development of supply chain management at an industry and enterprise level, Khurana *et al.* (2011) aimed to identify and measure the perceived importance of information sharing barriers in supply chain management. The barriers have been categorized into the six main different levels namely managerial, organizational, technological, individual, financial, social and cultural. Using a questionnaire and interview based research approach, they adopted to identify perceptions of the most significant barriers to information sharing. It was found that the data collected through questionnaires were sometimes very much ambiguous or vague and insufficient to interpret the significant results. Therefore, a fuzzy Analytic Hierarchy Process (AHP) approach has been used to overcome this kind of deficiency for modeling the rankings of the barriers of information sharing in supply chain management is used. The findings of their research can be used for developing an evidence based ranking of barriers of information sharing in supply chain.

Shafia *et al.* (2009) studied the causes and effects of the factors that determine the trend of employing Common Platforms (CP) in Supply Chain Management (SCM) of automotive industries. They also proposed a framework for analyzing Supply Chain Based on Common Platforms (SCBCP) in industries. The research methodology of their study was based on fact finding approach. Therefore, presenting the definitions and concepts of pertinent subjects, a conceptual model was developed for determining various aspects and finding facts regarding SCBCP in automotive industry. Critical factors and important facts in SCBCP have been identified by developing and analyzing the conceptual model. In addition, a triple performance criterion for the evaluation of SCBCP was developed.

Chong and Ooi (2008) studied the adoption status of Collaborative Commerce (C-Commerce) in the Malaysian Electrical and Electronic (E and E) organizations. Original research performed using a self-administered questionnaire that was distributed to 400 Malaysian E and E organizations. Data were analyzed employing descriptive statistics. In general, the adoption level of C-Commerce tools in the Malaysian E and E industry was still considered low with an average mean of 3.011. Based on the tools adopted, most organizations were utilizing C-Commerce for their supply chain execution. Among, tools with lower adoption, they were mainly supply chain planning tools such as capacity planning tool and business strategy tool.

Ghoseiri and Ghannadpour (2009) aimed to solve Vehicle Routing Problem with Time Windows (VRPTW), which has received considerable attention in recent years, using hybrid genetic algorithm. Vehicle Routing Problem with Time Windows was an extension of the well-known Vehicle Routing Problem (VRP) and involved a fleet of vehicles set-off from a depot to serve a number of customers at different geographic locations with various demands within specific time windows before returning to the depot eventually. To solve this problem, they suggested a hybrid genetic algorithm combined with Push Forward Insertion Heuristic (PFIH) to make an initial solution and λ-interchange mechanism to neighborhood search and improving method. The proposed genetic algorithm uses an integer representation in which a string of customer identifiers represents the sequence of deliveries covered by each of the vehicles. Part of initial population was initialized using Push Forward Insertion Heuristic (PFIH) and part was initialized randomly.

Ismail and Irhamah (2008) primarily studied to solve the Vehicle Routing Problem with Stochastic Demands (VRPSD) under restocking policy by using adaptive Genetic Algorithm (GA). The problem of VRPSD was one of the most important and studied combinatorial optimization problems, which finds its application on wide ranges of logistics and transportation area. It was a variant of a Vehicle Routing Problem (VRP). The algorithms for stochastic VRP were considerably more intricate than deterministic VRP and very time consuming. This has led the authors to explore the use of meta-heuristics focusing on the permutation-based GA. The GA was enhanced by automatically adapting the mutation probability to capture dynamic changing in population. The GA became a more effective optimizer where the adaptive schemes were depend on population diversity measure. The proposed algorithm was compared with standard GA on a set of randomly generated problems following some discrete probability distributions inspired by real case of VRPSD in solid waste collection in Malaysia. The performances of several types of adaptive mutation probability were also investigated.

Shahrabi *et al*. (2009) compared several time series methods to forecast supply chain demand. In this research, traditional time series forecasting methods including moving average, exponential smoothing, exponential smoothing with trend at the first stage and finally two machine learning techniques including Artificial Neural Networks (ANNs) and Support Vector Machines (SVMs), were used to forecast the long-term demand of supply chain. By using the data set of the component supplier of the biggest Iranian`s car company this research was then implemented. The comparison reveals that the results producing by machine learning techniques were more accurate and much closer to the actual data in contrast with traditional forecasting methods.

Neghab and Haji (2008) considered a two-level supply chain system consisting of one warehouse and a number of identical retailers. In this system, they incorporated transportation costs into inventory replenishment decisions. The transportation cost contained a fixed cost and a variable cost. The authors assumed that the demand rate at each retailer was known and the demand was confined to a single item. First, they derived the total cost which was the sum of the holding and ordering cost at the warehouse and retailers as well as the transportation cost from the warehouse to retailers. Then, they proposed a search algorithm to find the economic order quantities for the warehouse and retailers which minimize the total cost.

Ahmadi and Teimouri (2008) proposed a dynamic programming model which determines order penetration point in auto export supply chain. They also studied the characteristics and concepts relating to the Order Penetration Point (OPP). One of the most important characteristics of this supply chain was that, the product was packaged in different modules and after various stockings and passing long routs, was assembled in the target country. This modularized characteristic of the product was encouraging to explore the OPP of the chain from one point to several points in which the OPP of each module was located. Their proposed model tried to put the OPP of expensive modules (that have higher inventory holding cost) in the upstream section of the chain and puts the OPP of cheaper ones which created delay, in the downstream section of the chain.

## THE PROPOSED MODEL

We consider different customers being serviced with one supplier. The supplier provides various products and keeps them in different depots. Each depot uses different types of vehicles to carry out the orders. All depots are already stationed at the related locations. Here, we consider a multi echelon supply chain network (one supplier, multi depots and customers, multi commodity with deterministic demands). A set of vehicles exist at each depot. Each depot can store a set of products. The received order list from a customer can be handled by one or several depots at each time. Each selected vehicle for delivery can transfer only one product and after delivering the product, the vehicle returns to its corresponding depot. A penalty is assigned when a delivery time exceeds the predetermined time for transferring the products from depots to customers. A configuration of the proposed model is shown in Fig. 1.

## MATHEMATICAL MODEL

The mathematical model for this problem is as follows:

**Notations:**

P = Set of products
I = Set of depots stationed
J = Set of customers
T = Set of time periods
V = Set of vehicles

**Parameters:**

$D_{jpt}$ = Demand of customer j for product p at time t
$TH_{ipt}$ = Maximum throughput of depot i for product p at time t
$CA_i$ = Total capacity of depot i
$N_{ivt}$ = The number of existing vehicles v in depot i at time t
$VL_{vp}$ = Capacity of vehicle v for product p
$d_{ij}$ = Distance between depot i and customer j
$r_{ijpvt}$ = Number of return vehicles of type v from customer j to depot i at time t that have already received product p
M = A large number

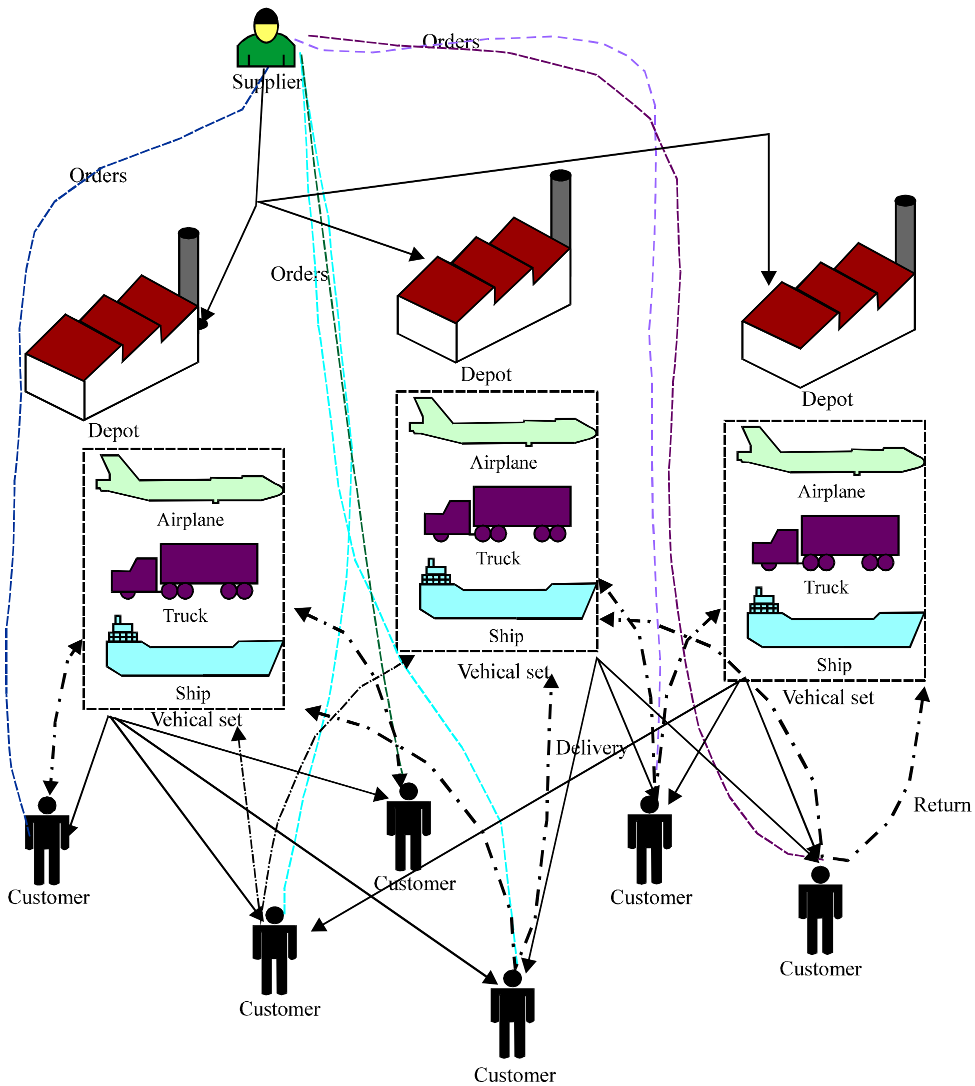


Fig. 1: A configuration of the proposed model

$Ct_{ijv}$ = Traveling fixed cost per mile from depot i to customer j using vehicle v
$TRT_{ijv}$ = Traveling time from depot i to customer j using vehicle v
C = The fixed cost for the whole planning horizon
Pen = The fixed cost as a penalty
|t| = The size of period t
$\alpha_{ijv}$ = 1, if the time of delivery from depot i to customer j using vehicle v exceeds a prespecified limit; 0, otherwise

**Decision variables:**

$$x_{ijpvt}: \begin{cases} 1, \text{ if depot i is selected to deliver product p to customer j by vehicle v at time t} \\ 0, \text{ otherwise} \end{cases}$$

$f_{ijpvt}$ : No. of transferred vehicle type v from depot i to customer j at time t
$QP_{ijpt}$ : Quantity of product p that can be satisfied by depot i to customer j at time t

**Objective function:**

$$\text{Minimize } F = \text{Min. } (f_1+f_2+f_3)$$

$$f_1 = \sum_{i\in I}\sum_{j\in J}\sum_{p\in P}\sum_{v\in V}\sum_{t\in T} f_{ijpvt} \cdot d_{ij} \cdot CT_{ijv} \cdot X_{ijpvt} \quad (1)$$

$$f_2 = \sum_{i\in I}\sum_{j\in J}\sum_{p\in P}\sum_{v\in V}\sum_{t\in T} TRT_{ijv} \cdot C \cdot x_{ijpvt} \quad (2)$$

$$f_3 = \sum_{i\in I}\sum_{j\in J}\sum_{p\in P}\sum_{v\in V}\sum_{t\in T} \alpha_{ijv} \cdot pen \cdot x_{ijpvt} \quad (3)$$

**Constraints:**

$$\sum_{i\in I} QP_{ijpt} = D_{jpt}, \quad \forall j\in J, \ \forall p\in P, \ \forall t\in T \quad (4)$$

$$QP_{ijpt} \cdot (1 - \sum_{v\in V} x_{ijpvt}) = 0, \quad \forall i\in I, \ \forall j\in J, \ \forall p\in P, \ \forall t\in T \quad (5)$$

$$QP_{ijpt} \ge \sum_{v\in V} x_{ijpvt}, \quad \forall i\in I, \ \forall j\in J, \ \forall p\in P, \ \forall t\in T \quad (6)$$

$$\left\lceil (\frac{QP_{ijpt}}{VL_{vp}}).x_{ijpvt} + 0.999 \right\rceil = f_{ijpvt}, \quad \forall i\in I, \ \forall j\in J, \ \forall p\in P, \ v\in V, \ t\in T \quad (7)$$

$$r_{ijpv(t-1+\left\lceil \frac{2 \cdot TRT_{ijv}}{|t|}+0.999 \right\rceil)} = f_{ijpvt}, \quad \forall i\in I, \ \forall j\in J, \ \forall p\in P, \ \forall v\in V, \ \forall t\in T \quad (8)$$

$$\sum_{t\in T}\sum_{i\in I}\sum_{j\in J}\sum_{p\in P}\sum_{v\in V} r_{ijpvt} - \sum_{t\in T}\sum_{i\in I}\sum_{j\in J}\sum_{p\in P}\sum_{v\in V} r_{ijpv(t-1+\left\lceil \frac{2 \cdot TRT_{i,j,v}}{|t|}+0.999 \right\rceil)} = 0 \quad (9)$$

$$N_{ivt-1} - \sum_{j\in J}\sum_{p\in P} f_{ijpvt} + \sum_{j\in J}\sum_{p\in P} r_{ijpvt} = N_{ivt}, \quad \forall i\in I, \ \forall v\in V, \ \forall t\in T \quad (10)$$

$$\sum_{j\in J}\sum_{p\in P} f_{ijpvt} \le N_{ivt-1}, \quad \forall i\in I, \ \forall v\in V, \ \forall t\in T \quad (11)$$

$$TH_{ipt-1} - \sum_{j\in J} QP_{ijpt} = TH_{ipt}, \quad \forall i\in I, \ \forall p\in P, \ \forall t\in T \quad (12)$$

$$\sum_{j\in J} QP_{ijpt} \le TH_{ipt-1}, \quad \forall i\in I, \ \forall p\in P, \ \forall t\in T \quad (13)$$

$$x_{ijpvt} \in \{0,1\}, \quad \forall i\in I, \ \forall j\in J, \ \forall p\in P, \ \forall v\in V, \ \forall t\in T \quad (14)$$

$$f_{ijpvt} \geq 0, \text{ Integer } \forall i \in I, \quad \forall j \in J, \quad \forall p \in P, \quad \forall v \in V, \quad \forall t \in T \tag{15}$$

$$QP_{ijpt} \geq 0, \text{ Integer}, \quad \forall i \in I, \quad \forall j \in J, \quad \forall p \in P, \quad t \in T \tag{16}$$

Formulas (1) and (2) are the objective functions which minimize the total cost and time, respectively. Formula (3) considers penalty for delivery times exceeding a pre-specified time limit. The constraints (4) guarantee that all customer demands are met for all products required at each period. The constraints (5) and (6) ensure that delivery is accomplished by only one vehicle. The number of each traveling vehicle between the depots and customers is shown by constraints (7). The constraints (8) and (9) identify return times of only the remaining vehicles. The constraints (10) represent the numbers of remaining vehicles at the end of the period. The constraints (11) ensure that the number of traveled vehicles from depot would not exceed the existing vehicles. The amounts of remaining product in depots at the end of the period are shown by constraints (12). The constraints (13) represent the capacity constraint of each depot for each product at the corresponding time. They must receive enough products from supplier in order to meet all the demands. The constraints (14) impose that the variables be binary. The last constraints (15) and (16) show the non-negativity requirements for all the other variables.

## NUMERICAL ILLUSTRATIONS

We present a numerical example to show the effectiveness of the proposed mathematical model. The number of customers is three, number of products is three, number of depots is two and number of vehicles is two. We consider a six period supply chain which receives order list in periods one, two and three with the size of time period $|t| = 10$. The orders for products in different periods are given in Table 1.

The distance from depots to customers, the capacity of vehicles for different products and the capacity of depots for different products are given in Table 2, 3 and 4, respectively.

The maximum capacity of both depots 1 and 2 are equal to 600. The transferring cost per unit of distance for vehicles 1 and 2 are 50 and 30, respectively. The transferring times for vehicles from depots to customers are given in Table 5.

The number of vehicle 1 in both depots 1 and 2 are 14 and the number of vehicle 2 in both depots 1 and 2 are 12.

Table 1: The orders for products in different periods

| Order | Product 1 | Product 2 | Product 3 |
|---|---|---|---|
| **First period** | | | |
| Customer 1 | 40 | 45 | 60 |
| Customer 2 | 70 | 30 | 50 |
| Customer 3 | 0 | 20 | 30 |
| **Second period** | | | |
| Customer 1 | 19 | 0 | 18 |
| Customer 2 | 0 | 0 | 13 |
| Customer 3 | 13 | 15 | 17 |
| **Third period** | | | |
| Customer 1 | 30 | 25 | 17 |
| Customer 2 | 16 | 20 | 18 |
| Customer 3 | 26 | 25 | 20 |

To facilitate the computation, LINGO 8 package is applied. The output for the decision variables are summarized in Table 6 and 7. The quantity of products (Qp) that can be satisfied by depots to customers at different time periods, selected route (X), type of vehicle and number of transferred vehicle (F) are presented in Table 6.

The number of return vehicles from customers to depots at different time periods (r) is shown in Table 7.

The number of remaining vehicles and capacity at the end of each period is given in Table 8 and 9, respectively.

Table 2: The distance from depots to customers

| Distance | Customer 1 | Customer 2 | Customer 3 |
|---|---|---|---|
| Depot 1 | 20 | 25 | 10 |
| Depot 2 | 10 | 15 | 17 |

Table 3: The capacity of vehicles for different products

| Capacity of vehicle | Product 1 | Product 2 | Product 3 |
|---|---|---|---|
| Vehicle 1 | 15 | 9 | 12 |
| Vehicle 2 | 10 | 15 | 8 |

Table 4: The capacity of depots for different products

| Depot capacity | Product 1 | Product 2 | Product 3 |
|---|---|---|---|
| Depot 1 | 100 | 200 | 100 |
| Depot 2 | 200 | 100 | 200 |

Table 5: The transferring time for vehicles from depots to customers

| Transferring time | Vehicle 1 | Vehicle 2 |
|---|---|---|
| **Depot 1** | | |
| Customer 1 | 10 | 15 |
| Customer 2 | 12 | 17 |
| Customer 3 | 5 | 10 |
| **Depot 2** | | |
| Customer 1 | 5 | 10 |
| Customer 2 | 7 | 12 |
| Customer 3 | 9 | 14 |

Table 6: The quantity of products that can be satisfied by depots to customers at different time periods

| Qp | Depot (i) | Customer (j) | Product (p) | Time period (t) | X | Vehicle | F |
|---|---|---|---|---|---|---|---|
| 10 | 1 | 2 | 1 | 1 | 1 | 2 | 1 |
| 15 | 1 | 2 | 2 | 1 | 1 | 2 | 1 |
| 20 | 1 | 3 | 2 | 1 | 1 | 2 | 2 |
| 30 | 1 | 3 | 3 | 1 | 1 | 2 | 4 |
| 40 | 2 | 1 | 1 | 1 | 1 | 2 | 4 |
| 45 | 2 | 1 | 2 | 1 | 1 | 2 | 3 |
| 60 | 2 | 1 | 3 | 1 | 1 | 1 | 5 |
| 60 | 2 | 2 | 1 | 1 | 1 | 1 | 4 |
| 15 | 2 | 2 | 2 | 1 | 1 | 2 | 1 |
| 50 | 2 | 2 | 3 | 1 | 1 | 1 | 5 |
| 5 | 1 | 3 | 1 | 2 | 1 | 2 | 1 |
| 17 | 1 | 3 | 3 | 2 | 1 | 2 | 3 |

Table 6: Continued

| Qp | Depot (i) | Customer (j) | Product (p) | Time period (t) | X | Vehicle | F |
|---|---|---|---|---|---|---|---|
| 19 | 2 | 1 | 1 | 2 | 1 | 1 | 2 |
| 18 | 2 | 1 | 3 | 2 | 1 | 1 | 2 |
| 13 | 2 | 2 | 3 | 2 | 1 | 2 | 2 |
| 8 | 2 | 3 | 1 | 2 | 1 | 2 | 1 |
| 15 | 2 | 3 | 2 | 2 | 1 | 2 | 1 |
| 15 | 1 | 1 | 2 | 3 | 1 | 2 | 1 |
| 20 | 1 | 2 | 2 | 3 | 1 | 2 | 2 |
| 25 | 1 | 3 | 2 | 3 | 1 | 2 | 2 |
| 20 | 1 | 3 | 3 | 3 | 1 | 1 | 2 |
| 30 | 2 | 1 | 1 | 3 | 1 | 1 | 2 |
| 10 | 2 | 1 | 2 | 3 | 1 | 2 | 1 |
| 17 | 2 | 1 | 3 | 3 | 1 | 1 | 2 |
| 16 | 2 | 2 | 1 | 3 | 1 | 1 | 2 |
| 18 | 2 | 2 | 3 | 3 | 1 | 2 | 3 |
| 26 | 2 | 3 | 1 | 3 | 1 | 2 | 3 |

Table 7: The number of return vehicles from customers to depots at different time periods

| r | Depot (i) | Customer (j) | Product (p) | Vehicle (v) | Time period (t) |
|---|---|---|---|---|---|
| 1 | 1 | 2 | 1 | 2 | 4 |
| 1 | 1 | 2 | 2 | 2 | 4 |
| 2 | 1 | 3 | 2 | 2 | 2 |
| 4 | 1 | 3 | 3 | 2 | 2 |
| 4 | 2 | 1 | 1 | 2 | 2 |
| 3 | 2 | 1 | 2 | 2 | 2 |
| 5 | 2 | 1 | 3 | 1 | 1 |
| 4 | 2 | 2 | 1 | 1 | 2 |
| 1 | 2 | 2 | 2 | 2 | 3 |
| 5 | 2 | 2 | 3 | 1 | 2 |
| 1 | 1 | 3 | 1 | 2 | 3 |
| 3 | 1 | 3 | 3 | 2 | 3 |
| 2 | 2 | 1 | 1 | 1 | 2 |
| 2 | 2 | 1 | 3 | 1 | 2 |
| 2 | 2 | 2 | 3 | 2 | 4 |
| 1 | 2 | 3 | 1 | 2 | 4 |
| 1 | 2 | 3 | 2 | 2 | 4 |
| 1 | 1 | 1 | 2 | 2 | 5 |
| 2 | 1 | 2 | 2 | 2 | 6 |
| 2 | 1 | 3 | 2 | 2 | 4 |
| 2 | 1 | 3 | 3 | 1 | 3 |
| 2 | 2 | 1 | 1 | 1 | 3 |
| 1 | 2 | 1 | 2 | 2 | 4 |
| 2 | 2 | 1 | 3 | 1 | 3 |
| 2 | 2 | 2 | 1 | 1 | 4 |
| 3 | 2 | 2 | 3 | 2 | 5 |
| 3 | 2 | 3 | 1 | 2 | 5 |

Table 8: The number of remaining vehicles at the end of the periods

| No. of remaining vehicles | Vehicle 1 | Vehicle 2 |
|---|---|---|
| **At the end of period 1** | | |
| Depot 1 | 14 | 4 |
| Depot 2 | 5 | 4 |
| **At the end of period 2** | | |
| Depot 1 | 14 | 6 |
| Depot 2 | 14 | 7 |
| **At the end of period 3** | | |
| Depot 1 | 14 | 5 |
| Depot 2 | 12 | 1 |
| **At the end of period 4** | | |
| Depot 1 | 14 | 9 |
| Depot 2 | 14 | 6 |
| **At the end of period 5** | | |
| Depot 1 | 14 | 10 |
| Depot 2 | 14 | 12 |
| **At the end of period 6** | | |
| Depot 1 | 14 | 12 |
| Depot 2 | 14 | 12 |

Table 9: The amount of remaining capacity at the end of the periods

| Depot capacity | Product 1 | Product 2 | Product 3 |
|---|---|---|---|
| **At the end of period 1** | | | |
| Depot 1 | 90 | 165 | 70 |
| Depot 2 | 100 | 40 | 90 |
| **At the end of period 2** | | | |
| Depot 1 | 85 | 165 | 53 |
| Depot 2 | 73 | 25 | 59 |
| **At the end of period 3** | | | |
| Depot 1 | 85 | 105 | 33 |
| Depot 2 | 1 | 15 | 24 |

The best objective value for the problem is 34350

## CONCLUSIONS

We proposed a supply network model in which one supplier would provide various products for customers in different time periods. The contribution of the proposed model is in its flexibility with respect to vehicles and depots. The aim was to minimize the total cost and time of the orders' delivery process. Furthermore, deliveries needing times longer than the pre-specified limits are penalized. The effectiveness and validity of the proposed mathematical model are illustrated by working out a numerical example using our presented Lingo encoding for the solution of the model.

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