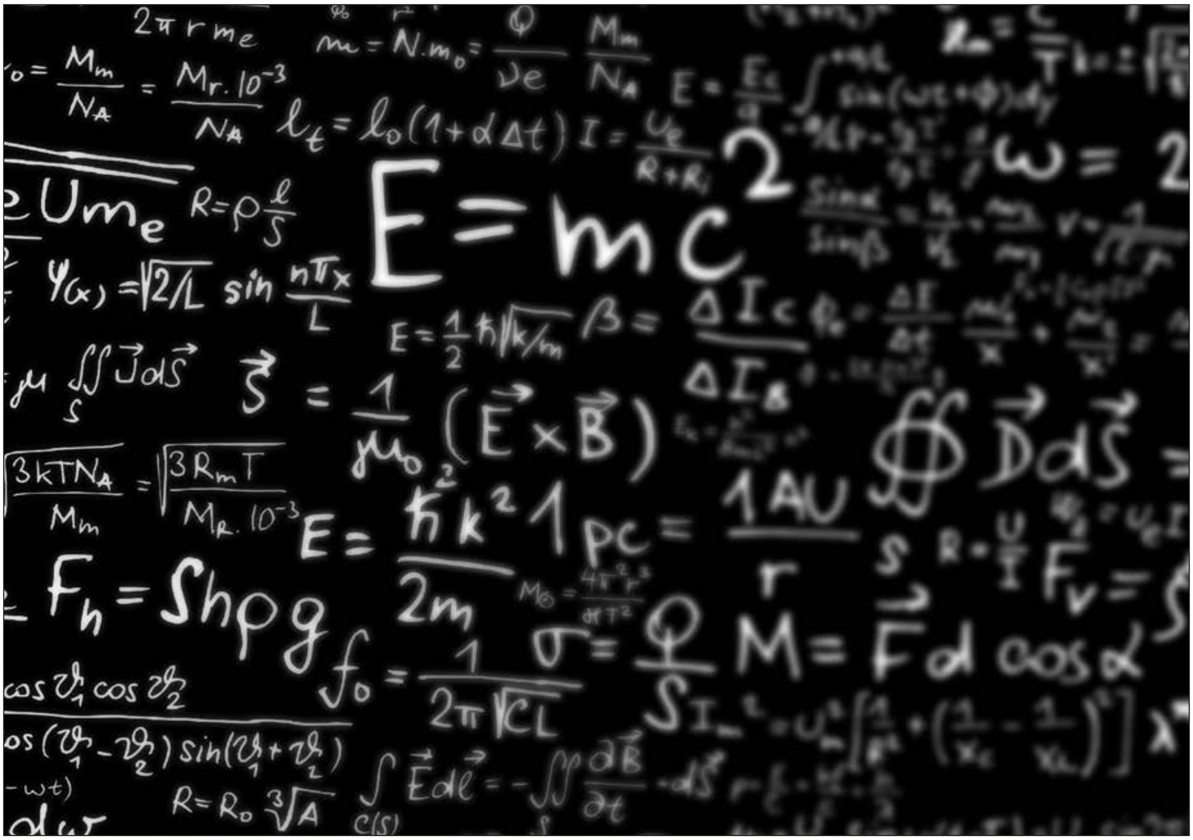

Asian Journal of Mathematics & Statistics

ISSN 1994-5418

science alert

ANSI*net*
an open access publisher
http://ansinet.com

Asian Journal of Mathematics and Statistics 1 (1): 50-56, 2008
ISSN 1994-5418

Bilinear Autoregressive Vector Models and Their Application to Estimation of Revenue Series

[1]A.E. Usoro and [2]C.O. Omekara
[1]Department of Statistics, Akwa Ibom State Polytechnic, Ikot Osurua,
P.M.B. 1200, Ikot Ekpene, Akwa Ibom State, Nigeria
[2]Michael Okpara University of Agriculture, Umudike, Abia State, Nigeria

Abstract: This study was motivated by the need to establish multivariate time series models for pure autoregressive vector series which assume both linear and nonlinear components. General Bilinear Autoregressive Vector (BARV) models were established. The three vector series namely, a response vector (X_{1t}) and predictor vectors (X_{2t}) and (X_{3t}) used for the modelling called for trivariate time series models as a special case of multivariate time series models and estimates obtained from the models. The finding in this study is the isolation of multivariate bilinear models for a pure autoregressive process based on the distribution of autocorrelation and partial autocorrelation functions of the series from mixed models. This has been achieved as the models were used for the estimation of the vector series. These prove reality of the BARV models established.

Key words: Response and predictor vectors, multivariate time series, bilinear autoregressive vectors

INTRODUCTION

Most time series analysts assume linearity and stationarity, for technical convenience, when analyzing macroeconomic and financial time series data (Franses, 1998). However, evidence of nonlinearity which is usually found in the dynamic behaviour of such data implies that classical linear models are not appropriate for modelling these series (Subba Rao and Gabr, 1984). In most cases, nonlinear forecast is more superior to linear forecast. Maravall (1983) used a bilinear model to forecast Spanish monetary data and reported a near 10% improvement in one-step ahead mean square forecast errors over several ARMA alternatives. There is no-gainsaying the fact that most of the economic or financial data assume fluctuations due to certain factors. That is why the use of nonlinear models in forecast gives higher precision than linear models.

According to Granger and Anderson (1978), the general Bilinear Autoregressive Moving Average model of order (p,q,P,Q), denoted by BARMA (p,q,P,Q) takes the form

$$X_t = \sum_{j=1}^{p} \alpha_j X_{t-j} + \sum_{i=1}^{q} b_i \varepsilon_{t-i} + \sum_{k=0}^{Q} \sum_{\ell=1}^{P} \beta_{k\ell} \varepsilon_{t-k} X_{t-\ell}$$

where, ε_t is strict white noise. The model is thus linear in the X's and also in the ε's separately, but not in both. It obviously includes all the ARMA (p,q) models as a special case. At this point, it is convenient to give names to several subclasses of the general model.

Corresponding Author: A.E. Usoro, Department of Statistics, Akwa Ibom State Polytechnic, Ikot Osurua, P.M.B. 1200, Ikot Ekpene, Akwa Ibom State, Nigeria

The complete bilinear model with p = q = 0 is

$$X_t = \sum_{k=1}^{Q} \sum_{\ell=1}^{P} \beta_{k\ell} \varepsilon_{t-k} X_{t-\ell} + \varepsilon_t \quad (1)$$

This can be written in matrix form as

$$X_t = (\varepsilon_{t-i}, \ldots, \varepsilon_{t-Q}) \underline{\beta} (X_{t-1}, X_{t-2}, \ldots, X_{t-p})' + \varepsilon_t$$

Where, $\underline{\beta}$ is the (Q×P) matrix of coefficients

$$\underline{\beta} = \{\beta_{k\ell}\},\ k = 1,\ldots,Q;\ \ell = 1,\ldots,P$$

If $\beta_{k\ell} = 0$, for all $k > \ell$, the model is called superdiagonal.

Bibi and Oyet (1991) defined a process $(X_t)_{t\in z}$ on a probability space (Ω,ξ,P) as a time varying bilinear process of order (p,q,P,Q) and denoted by BL(p,q,P,Q), if it satisfies the following stochastic difference Equation:

$$X_t = \sum_{j=1}^{P} a_{i,t}(a) X_{t-i} + \sum_{i=1}^{q} c_{j,t}(c) \varepsilon_{t-i} + \sum_{i=1}^{P} \sum_{j=1}^{Q} b_{ij,t}(b) X_{t-i} \varepsilon_{t-j} + \varepsilon_t$$

where, $(a_{i,t}(a))_{1\le i\le p}$, $(c_{j,t}(c))_{1\le j\le q}$, $(b_{ij,t}(b))_{1\le i\le P,\ 1\le j\le Q}$ are time-varying coefficients which depend on finite dimensional unknown parameter vectors a, c and b, respectively. The sequence $(\varepsilon_t)_{t\in z}$ is a heteroscedastic white noise process. That is, $(\varepsilon_t)_{t\in z}$ is a sequence of independent random variables, not necessarily identically distributed, with mean zero and variance σ_t^2. Moreover ε_t is independent of past X_t. The initial values X_t, $t < 1$, and $\varepsilon_{t,\ t<1}$ are assumed to be equal to zero.

Boonchai and Eivind (2005) stated the general form of a multivariate bilinear time series model as:

$$X_t = \Sigma_{Ai} X_{t-i} + \Sigma M_j e_{t-j} + \Sigma\Sigma\Sigma B_{dij} X_{t-i} e_{dt-j} + e_t$$

Here the state X_t and noise e_t are n-vectors and the coefficients A_i, M_j and $B_{dij} = 0$, we have the class of well-known vector ARMA models. The bilinear models include additional product terms $B_{dij} X_{t-i} e_{dt-j}$; as the name indicates these models are linear in state X_t and in noise e_t separately, but not jointly. From a theoretical point of view, it is therefore natural to consider bilinear models in the process of extending linear theory to non-linear cases. According to Boonchai and Eivind (2005) a particular reason for introducing bilinear time series in population dynamics, is that they are suitable for modeling environmental noise. One may start with a deterministic system with (constant) parameters that describe conditions that depend on a fluctuating environment. The idea is to replace them by stochastic parameters. Boonchai and Eivind (2005) made extension first to univariate and then to multivariate bilinear models. The main results give conditions for stationarity, ergodicity, invertibility and consistency of least square estimates.

In this research, we are interested in estimation of Bilinear Autoregressive Vector (BARV) models. We consider three vectors, which consist of a response and two predictor vectors. The data source is a monthly generated revenue (for a period of ten years) of a Local Government Area in Nigeria.

METHODS OF ESTIMATION

Linear Model

The general multivariate analogue to the univariate Autoregressive Moving Average (ARMA) model for the vectors is:

$$X_{it} = \sum_{a=1}^{p} \sum_{i=1}^{n}\sum_{f=1}^{k} \gamma_{a.if} X_{it-a} + \in_{jt} + \sum_{b=1}^{q} \sum_{j=1}^{v}\sum_{h=1}^{m} \lambda_{b.if} \in_{it-b} \quad (2)$$

where, $X_{it} = (X_{1t}, X_{2t}, \ldots, X_{nt})$ are vectors, γ_{aif} are matrices of coefficients of the autoregressive parameters, ϵ_{jt} are the vector white noise, $\lambda_{b.jh}$ are matrices of coefficients of the moving average parameters, (r = n).

Non-Linear Model

The non-linear models for X_{1t}, X_{2t}, $X_{3t}, \ldots, X_{nt}$ is:

$$X_{it} = \sum_{i=1}^{n} \sum_{j=1}^{v}\sum_{a=1}^{p} \sum_{b=0}^{q} \beta_{ab.ij} X_{it-a} \in_{jt-b} \quad (3)$$

Where,
$X_{it} = (X_{1t}, X_{2t}, \ldots, X_{nt})$, $\beta_{ab.ij}$ are the matrices of coefficients of the respective vector product series.

Bilinear Autoregressive Vector Model

The general BARV model may be written in the form:

$$X_{it} = \sum_{a=1}^{p} \sum_{i=1}^{n}\sum_{f=1}^{k} \gamma_{a.if} X_{it-a} + \in_{jt} + \sum_{i=1}^{n} \sum_{j=1}^{v}\sum_{a=1}^{p} \beta_{aij} X_{it-a} \in_{it} \quad (4)$$

Where,
Vectors and coefficients are as described above.

RESULTS AND DISCUSSION

Estimates for BARV Models

The distribution of autocorrelation and partial autocorrelation functions of the non stationary series suggested pure autoregressive process of order 3 for X_{1t}, autoregressive process of order 2 for X_{2t} and autoregressive process of order 1 for X_{3t}. The vector autoregressive bilinear process is a process which consists of two parts. The first part is a pure autoregressive process of the vector series, while the second part is the product of the vector series and white noise. The regression estimates obtained provides the following models for the three vector series:

$$\begin{aligned} X_{1t} = {} & 0.661X_{1t-1} - 0.184X_{2t-1} + 0.0148X_{1t-2} + 0.386X_{2t-2} + 0.205X_{1t-3} \\ & + \epsilon_{1t} + 0.00165X_{1t-1}\epsilon_{1t-0} + 0.00130X_{1t-2}\epsilon_{1t-0} + 0.000546X_{1t-3}\epsilon_{1t-0} \\ & -0.00372X_{1t-1}\epsilon_{2t-0} - 0.00080X_{1t-2}\epsilon_{2t-0} - 0.000225X_{1t-3}\epsilon_{2t-0} \\ & -0.00088X_{2t-1}\epsilon_{1t-0} + 0.00067X_{2t-2}\epsilon_{1t-0} + 0.00410X_{2t-1}\epsilon_{2t-0} \\ & + 0.00228X_{2t-2}\epsilon_{2t-0} \end{aligned} \quad (5)$$

From model (5)

$\gamma_{1.11} = 0.661$, $\gamma_{1.12} = 0.184$, $\gamma_{2.11} = 0.0148$, $\gamma_{2.12} = 0.386$, $\gamma_{3.11} = 0.205$
$\beta_{10.11} = 0.00165$, $\beta_{20.11} = 0.00130$, $\beta_{30.11} = 0.000546$, $\beta_{10.12} = -0.00372$
$\beta_{20.12} = -0.00080$, $\beta_{30.12} = -0.000225$, $\beta_{10.21} = -0.00088$,
$\beta_{20.21} = 0.00067$ and $\beta_{10.22} = 0.00410$, $\beta_{20.22} = 0.00228$.

$$\begin{aligned} X_{2t} &= 0.194X_{1t-1} + 0.202X_{2t-1} + 0.0824X_{1t-2} + 0.290X_{2t-2} + 0.120X_{1t-3} \\ &+ \epsilon_{2t} + 0.00027X_{1t-1}\epsilon_{1t-0} + 0.00124X_{1t-2}\epsilon_{1t-0} - 0.000171X_{1t-3}\epsilon_{1t-0} \\ &- 0.00161X_{1t-1}\epsilon_{2t-0} - 0.00148X_{1t-2}\epsilon_{2t-0} + 0.000557X_{1t-3}\epsilon_{2t-0} \\ &+ 0.00047X_{2t-1}\epsilon_{1t-0} - 0.00283X_{2t-2}\epsilon_{1t-0} + 0.00189X_{2t-1}\epsilon_{2t-0} \\ &+ 0.00667X_{2t-2}\epsilon_{2t-0} \end{aligned} \quad (6)$$

From model (6)

$\gamma_{1.21} = 0.194$, $\gamma_{1.22} = 0.202$, $\gamma_{2.21} = 0.0824$, $\gamma_{2.22} = 0.290$, $\gamma_{3.21} = 0.120$
$\beta_{10.11} = 0.00027$, $\beta_{20.11} = 0.00124$, $\beta_{30.11} = -0.000171$, $\beta_{10.12} = -0.00161$
$\beta_{20.12} = -0.00148$, $\beta_{30.12} = 0.000227$, $\beta_{10.21} = 0.00047$,
$\beta_{20.21} = -0.00283$ and $\beta_{10.22} = 0.00189$, $\beta_{20.22} = 0.00667$.

$$\begin{aligned} X_{3t} &= 0.466X_{1t-1} - 0.385X_{2t-1} - 0.0676X_{1t-2} + 0.0965X_{2t-2} + 0.0851X_{1t-3} \\ &+ \epsilon_{3t} + 0.00138X_{1t-1}\epsilon_{1t-0} + 0.000058X_{1t-2}\epsilon_{1t-0} + 0.000717X_{1t-3}\epsilon_{1t-0} \\ &- 0.00211X_{1t-1}\epsilon_{2t-0} + 0.00069X_{1t-2}\epsilon_{2t-0} - 0.000782X_{1t-3}\epsilon_{2t-0} \\ &- 0.00135X_{2t-1}\epsilon_{1t-0} + 0.00350X_{2t-2}\epsilon_{1t-0} + 0.00220X_{2t-1}\epsilon_{2t-0} \\ &+ 0.00439X_{2t-2}\epsilon_{2t-0}. \end{aligned} \quad (7)$$

From model (7)

$\gamma_{1.31} = 0.466$, $\gamma_{1.32} = -0.385$, $\gamma_{2.31} = -0.0676$, $\gamma_{2.32} = 0.0965$, $\gamma_{3.31} = 0.0851$
$\beta_{10.11} = 0.00138$, $\beta_{20.11} = 0.000058$, $\beta_{30.11} = 0.000717$, $\beta_{10.12} = -0.00211$
$\beta_{20.12} = 0.00069$, $\beta_{30.12} = -0.000782$, $\beta_{10.21} = -0.00135$
$\beta_{20.21} = 0.00350$ and $\beta_{10.22} = 0.00220$, $\beta_{20.22} = -0.00439$

The first set of estimates in models 5-7 forms the parameter estimates of the linear part, while the second set are the parameter estimates of interactive products of vectors.

The vector models for X_{1t}, X_{2t} and X_{3t} are used to obtain estimates, which are shown in Appendix 2. The actual and estimated values in Appendices 1 and 2 are for each vector in Fig. 1-3.

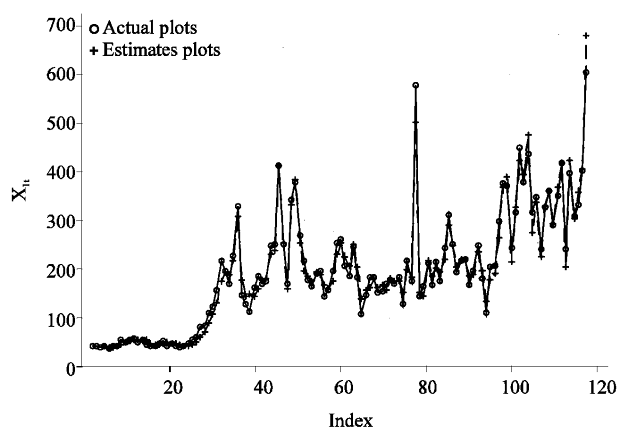

Fig. 1: Plots of actual and estimates of a response vector X_{1t}

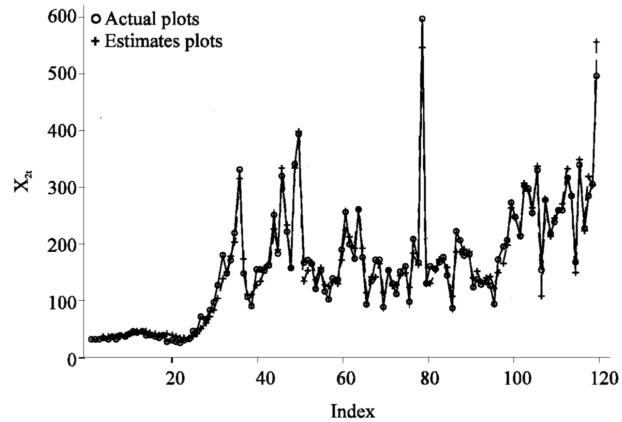

Fig. 2: Plots of actual and estimates of a predictor vector X_{2t}

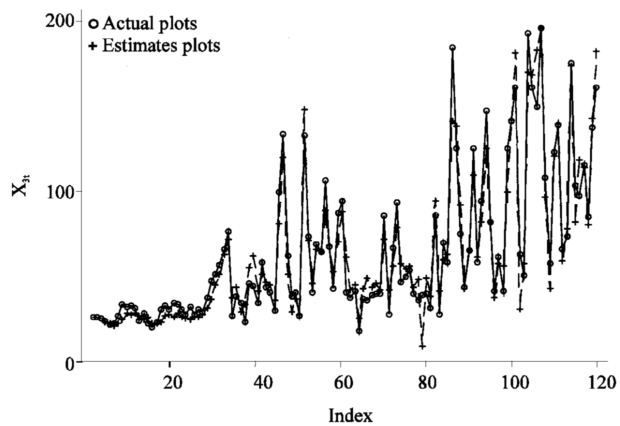

Fig. 3: Plots of actual and estimates of the second predictor vector X_{3t}

CONCLUSIONS

There is no gainsaying the fact some series, especially, revenue series assume not only linear component, but both linear and nonlinear components. This is so because of the random nature of observations assume by certain processes. It is in this regards that bilinear multivariate time series models were developed. The Bilinear Autoregressive Vector Models established in this paper provide better estimates for most non-stationary revenue series than pure linear models.

Appendix 1: Actual internally generated revenue series represented by three vectors

S/n	X_{1t}	X_{2t}	X_{3t}	s/n	X_{1t}	X_{2t}	X_{3t}	s/n	X_{1t}	X_{2t}	X_{3t}
1	30.87	17.01	13.86	41	186.82	139.41	47.41	81	164.91	145.21	19.70
2	31.26	17.31	13.95	42	169.89	137.98	31.91	82	215.65	139.52	76.13
3	29.35	16.10	13.25	43	176.91	147.73	29.18	83	167.03	151.33	15.70
4	30.05	18.68	11.37	44	256.21	238.38	17.83	84	219.36	160.19	59.17
5	25.96	17.46	8.50	45	260.00	169.12	90.88	85	176.06	129.01	47.05
6	30.31	20.55	9.76	46	434.75	308.15	126.60	86	251.51	70.66	180.85
7	31.54	17.04	14.50	47	258.23	207.11	51.12	87	325.11	207.01	118.10
8	45.20	23.85	21.35	48	169.79	143.58	26.21	88	257.86	192.54	65.32

Appendix 1: Continued

S/n	X_{1t}	X_{2t}	X_{3t}	s/n	X_{1t}	X_{2t}	X_{3t}	s/n	X_{1t}	X_{2t}	X_{3t}
9	41.07	20.57	20.50	49	358.15	328.97	29.18	89	195.03	162.92	32.11
10	45.46	24.86	20.60	50	397.26	383.01	14.25	90	220.52	165.52	55.00
11	48.68	29.65	19.03	51	279.01	152.71	126.30	91	225.77	107.42	118.35
12	40.17	28.67	11.50	52	220.75	157.39	63.36	92	167.89	120.52	47.37
13	45.79	29.76	16.03	53	178.99	149.68	29.31	93	198.30	112.85	85.45
14	32.76	22.89	9.87	54	164.50	105.69	58.81	94	257.08	115.70	141.38
15	30.77	23.25	7.52	55	192.33	138.53	53.80	95	183.01	110.86	72.15
16	32.07	21.97	10.10	56	198.54	100.29	98.25	96	106.12	76.75	29.37
17	37.83	19.64	18.19	57	143.54	86.21	57.33	97	207.17	156.60	50.57
18	43.85	22.60	21.25	58	155.90	124.20	31.70	98	209.36	179.21	30.15
19	30.77	12.60	18.17	59	198.51	120.68	77.83	99	309.66	191.79	117.87
20	37.06	14.53	22.53	60	260.93	175.79	85.14	100	394.27	258.99	135.28
21	31.96	10.61	21.35	61	299.44	270.84	28.60	101	388.93	232.97	155.96
22	29.00	10.30	18.70	62	211.02	185.08	25.94	102	250.32	198.14	52.18
23	30.36	15.04	15.32	63	188.06	158.68	29.38	103	328.70	289.35	39.15
24	36.63	16.90	19.73	64	252.71	247.66	5.05	104	475.41	285.73	189.68
25	45.77	30.45	15.32	65	185.72	160.47	25.25	105	396.98	241.31	155.67
26	50.00	31.50	18.50	66	101.75	77.25	24.50	106	461.13	317.68	143.45
27	72.50	55.20	17.30	67	145.56	118.56	27.00	107	331.10	138.69	192.41
28	77.18	51.73	25.45	68	184.41	156.59	27.83	108	363.17	263.85	99.32
29	104.08	67.58	36.50	69	184.41	156.59	27.82	109	248.50	202.20	46.30
30	120.70	80.90	39.80	70	149.33	73.20	76.13	110	339.98	224.38	115.60
31	157.34	111.47	45.87	71	153.39	138.19	15.70	111	377.75	245.45	132.30
32	220.45	164.79	55.66	72	171.38	115.46	55.92	112	300.42	244.67	55.75
33	198.76	132.35	66.41	73	180.48	96.33	84.15	113	366.28	303.08	63.20
34	171.03	156.70	14.33	74	170.13	135.03	35.10	114	441.37	270.02	171.35
35	231.97	205.76	26.21	75	184.16	145.96	38.20	115	246.69	151.69	95.00
36	343.58	321.12	22.46	76	124.36	81.71	42.65	116	416.48	327.73	88.75
37	143.73	132.88	10.85	77	222.96	194.45	28.51	117	320.97	213.59	107.35
38	126.16	91.21	34.95	78	175.75	151.25	24.50	118	347.35	272.14	75.21
39	107.93	74.75	33.18	79	614.93	587.93	27.00	119	422.91	291.66	131.25
40	162.04	139.41	22.63	80	142.32	114.50	27.82	120	641.23	485.56	155.67

Appendix 2: Regression estimates from bilinear autoregressive vector models

s/n	X_{1t}	X_{2t}	X_{3t}	s/n	X_{1t}	X_{2t}	X_{3t}	s/n	X_{1t}	X_{2t}	X_{3t}
1				41	159.45	121.75	37.71	81	124.85	106.15	18.70
2				42	180.38	144.39	36.00	82	202.73	129.08	73.64
3				43	182.68	149.67	33.01	83	185.88	158.55	27.33
4	29.94	20.07	9.88	44	238.27	213.93	24.34	84	208.85	158.24	50.60
5	29.17	20.17	9.00	45	255.14	181.76	73.38	85	194.96	144.50	50.45
6	27.87	20.00	7.87	46	434.58	310.78	123.79	86	235.50	84.91	150.60
7	29.78	20.50	9.28	47	266.13	216.11	50.02	87	273.78	160.53	113.26
8	32.80	21.25	11.55	48	157.78	140.80	16.98	88	240.24	169.25	70.99
9	39.25	24.49	14.76	49	357.08	333.91	23.17	89	206.50	173.88	32.61
10	40.63	26.36	14.27	50	402.15	380.72	21.43	90	224.53	169.37	55.17
11	43.87	28.58	15.30	51	266.82	123.39	143.43	91	229.70	118.54	111.16
12	44.69	30.94	13.75	52	198.53	149.29	49.24	92	194.00	138.21	55.79
13	43.27	31.35	11.92	53	181.97	150.24	31.72	93	191.13	121.28	69.85
14	44.65	31.18	13.47	54	175.07	118.46	56.60	94	241.50	119.49	122.01
15	37.03	27.62	9.38	55	195.09	139.98	55.10	95	207.82	128.01	79.81
16	34.42	25.04	9.38	56	193.74	115.46	78.28	96	140.05	103.25	36.79
17	33.88	23.36	10.52	57	174.67	110.56	64.11	97	179.03	131.37	47.66
18	37.58	23.80	13.78	58	154.18	115.17	39.00	98	187.15	148.07	39.07
19	39.17	24.73	14.44	59	175.48	117.01	58.48	99	281.37	183.43	92.94
20	35.83	22.36	13.47	60	227.91	153.55	74.36	100	371.58	244.47	127.10
21	36.45	21.05	15.41	61	243.17	203.53	39.64	101	399.43	241.60	157.84
22	32.01	18.92	13.09	62	227.90	199.26	28.64	102	239.20	203.89	35.31
23	29.69	17.73	11.96	63	206.45	178.29	28.16	103	331.25	293.47	37.77
24	29.08	18.16	10.92	64	261.90	254.83	7.07	104	476.25	286.57	189.68
25	34.04	21.31	12.72	65	205.61	178.96	26.65	105	420.82	251.94	168.88
26	39.18	27.07	12.11	66	126.72	98.54	28.17	106	487.72	323.63	164.09

Appendix 2: Continued

s/n	X_{1t}	X_{2t}	X_{3t}	s/n	X_{1t}	X_{2t}	X_{3t}	s/n	X_{1t}	X_{2t}	X_{3t}
27	50.52	35.94	14.57	67	156.37	124.81	31.56	107	326.08	106.70	219.38
28	63.14	45.54	17.60	68	159.62	127.35	32.27	108	354.32	264.11	90.21
29	81.88	56.99	24.89	69	182.19	151.61	30.58	109	230.63	188.85	41.78
30	99.46	67.99	31.47	70	164.33	98.30	66.03	110	354.23	224.04	130.56
31	124.12	87.92	36.20	71	168.03	142.13	25.90	111	375.55	241.99	49.14
32	170.15	124.82	45.33	72	155.42	114.54	40.88	112	303.06	253.91	63.87
33	196.64	138.24	58.40	73	184.10	109.29	74.80	113	383.94	320.07	186.48
34	188.17	164.42	23.75	74	172.27	127.84	44.43	114	459.24	272.77	91.32
35	221.06	190.11	30.95	75	173.18	134.18	39.00	115	235.64	144.32	100.02
36	331.14	307.04	24.11	76	151.35	107.19	44.15	116	432.11	332.09	101.09
37	168.96	158.25	10.71	77	201.13	169.03	32.10	117	313.74	212.65	72.49
38	128.67	87.25	41.42	78	179.09	145.93	33.16	118	378.19	305.70	135.68
39	135.69	94.68	41.01	79	570.28	546.89	23.39	119	427.32	291.64	157.54
40	139.25	110.99	28.26	80	125.21	103.00	22.21	120	694.70	537.16	

REFERENCES

Bibi, A. and A.J. Oyet, 1991. J. Am. Mathem. Soc.

Boonchai, K.S. and E. Stensholt, 2005. Geophys. Res. Abst., 7: 02219.

Franses, P.H., 1998. Time series models for business and economic forecast. J. Time Series. Annal, 16: 509-529.

Granger, C.W.J. and A.P. Anderson, 1978. Introduction to Bilinear Time Series Models. Vandenhoeck and Ruprecht.

Maravall, A., 1983. An application of nonlinear time series forecasting. J. Bus. Econ. Stat., 1: 66-74.

Subba Rao, T. and M.M. Gabr, 1984. An Introduction to Bispectral Analysis and Bilinear Time Series Models. Lecture Notes in Statistics No. 24. Springer Verlag.