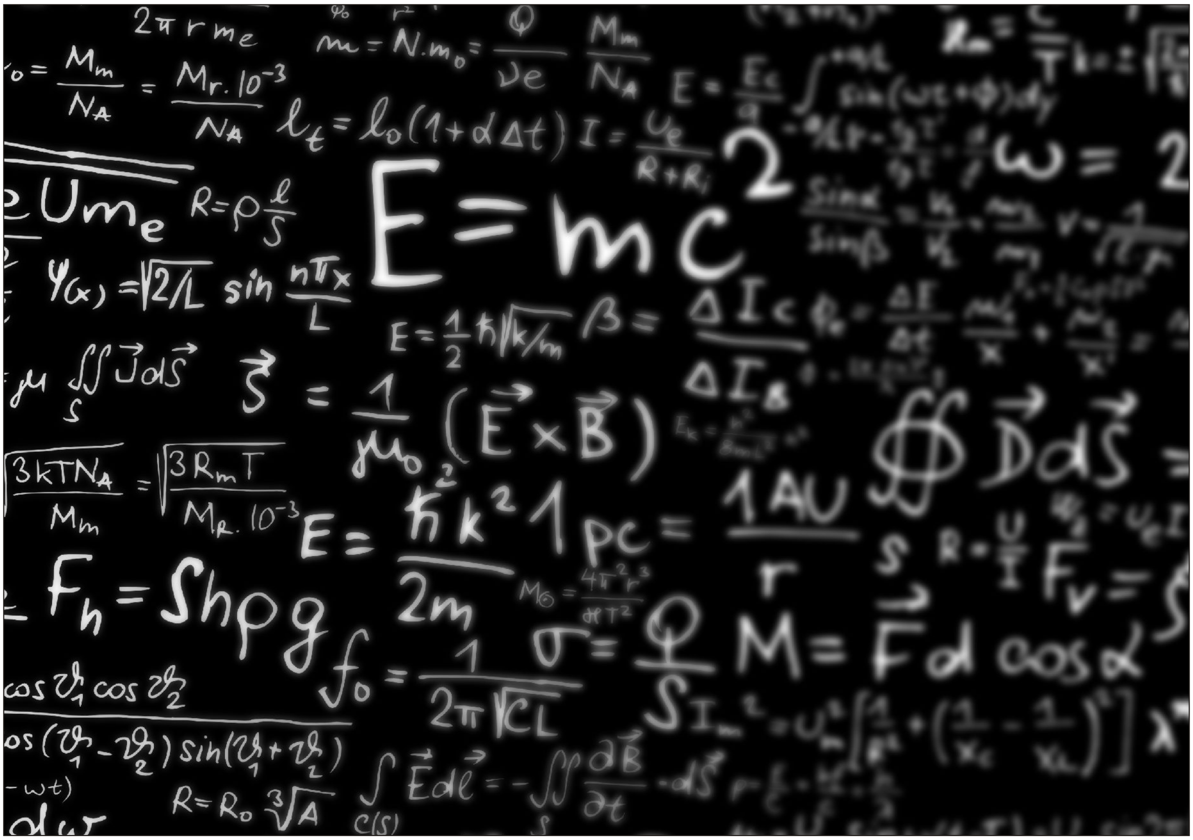

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Pseudo-Additive (Mixed) Fourier Series Model of Time Series

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Abstract: This study improves on the Additive Fourier Series and traditional model of discrete periodic time series. It seeks to formulate a mixed (multiplicative-additive) Fourier Series model which decomposes a time series into multiplicative trend, seasonal components and additive error component together with additive trend. It is discovered that for time series with strongly marked and obviously fluctuating seasonal effects a multiplicative-additive (mixed) Fourier series model is suitable. The relevance of the new model is shown by analyzing the rainfall data of Uyo metropolis with the use of the model. The resulting model gives $Y_t = 210.1\ (1\text{-}0.984 \cos \omega t)$ which fits well to the original data and can be used in forecasting future values of the rainfall data.

Key words: Multiplicative-additive (mixed) models, fourier series seasonal effects, forecasting, pseudo-additive model

INTRODUCTION

Most periodic time series have been assumed to be modeled by the use of additive Fourier Series model or any other seasonal models, but not much have been done on the modeling of a special case of such time series with a strongly marked and obviously fluctuating seasonal effects. The researcher discovered that for such time series a pseudo-additive (mixed) Fourier series model is very suitable. The development is as a result of the work of Hernmann *et al*. (2006), who demonstrated that a series with sharply pronounced seasonal fluctuations and trend-cycles movement, which is extremely weather-dependent could be modeled suitably using a multiplicative-additive (Mixed) model given by $Y_t = D\ (C_t + S_t + I_t)$. The further said that this can be implemented in X-12-ARIMA. In a similar development, Findley *et al*. (1998) earlier suggested a Pseudo-additive decomposition with a relative working day factor (D) as $Y_t = C_t D_t\ (S_t + I_t^{-1})$, which made it possible to estimate relative calendar factors based on a logarithmic REGARIMA model. Traditionally, time series are decomposed into four basic components: the trend, seasonal, cyclical and irregular components. Nkpodot and Usoro (2005) stated three traditional models of time series as completely additive, completely multiplicative and mixed models. They outlined the models mathematically as:

- $Y_t = C_t + S_t + I_t$-completely additive model
- $Y_t = C_t\ S_t I_t$-completely multiplicative model
- $Y_t = C_t S_t + I_t$-Mixed model.

Where:

Y_t = Observation at time t
C_t = Current mean or trend-cycle effect
S_t = Seasonal effect
I_t = Residual or random error.

Also in time series analysis, it is assumed that an unadjusted series (Y) may be decomposed into four unobservable components. The first of those is the trend cycle component (C), which includes not just the long-term trend but also cyclical fluctuations. Then comes the calendar component (D), derived from the effects of working-day variations, for example. There is additionally the seasonal component (S), which includes annual fluctuations that recur to almost the same degree in the same period (Anonymous, 2001).

According to Priestly (1981), one of the most important advantages of Fourier Series Analysis is its simple way of modeling a series with seasonality or cyclicalness. He further said that Fourier series method can be used to model seasonal effects using several seasonal peaks per year.

Chatfield (1975), stated that a seasonal component of a time series can be decomposed into underlying sine and cosine functions of different frequencies known as Fourier series. One way of doing this is to cast the issue as a linear multiple regression problem, where the dependent variable is the detrended time series and the independent variable are the sine and cosine functions of all possible (discrete) frequencies (Priestly, 1981).

Roerink *et al*. (2000) used Fourier Series Analysis or Harmanic Analysis of Time Series to screen and remove cloud affected observations and temporarily interpolate the remaining observations to reconstruct gapless images at a prescribed time.

METHODS OF ANALYSIS

The analysis of this study is done through the help of a statistical package called MINITAB. The model is given by:

$$Y_t = (a_0 + b_0 t) \sum_{i=1}^{k} \left[a_i \cos i\omega t + b_i \sin i\omega t \right] + (a_0 + b_0 t) + e_t \tag{1}$$

$$Y_t = (a_0 + b_0 t) \left[\sum_{i=1}^{k} [a_i \cos i\omega t + \sin i\omega t] + 1 \right] + e_t \tag{2}$$

The estimated model is given by:

$$\hat{Y}_t = (a_0 + b_0 t) \left[\sum_{i=1}^{k} [a_i \cos i\omega t + \sin i\omega t] + 1 \right] \tag{3}$$

Where:

Y_t	=	The observation at time t
b_0	=	The trend parameter estimate
a_0	=	The constant used to set the level of the series
a_i	=	The parameter estimates.
e_t	=	The error term
ω	=	2π f/n

Where:

f	=	The fourier frequency
k	=	The highest harmonic

It is noteworthy here that the highest harmonic, k in Fourier Series Analysis model is the number of observations per season divided by two (2) for an even number of observation and n-1/2 for an odd number of observations.

Hence; The model given above can be represented traditionally as

$$Y_t = T_t (S_t + 1) I_t = T_t S_t I_t + T_t I_t$$

$T = a_0 + b_0 t$ is used in estimating the trend, while the expressions containing the sine and cosine terms gives the estimated model for seasonality. The method used in estimating the parameters of the model is the method of least squares in Multiple Regression Analysis. Firstly the trend is removed from the series by multiplicative procedure:

$$\frac{Y_t}{a_0 + b_0 t} = \sum_{i=1}^{k} (a_i \cos i\omega t + b_i \sin i\omega t) + e_t = DT \tag{5}$$

Then DT (the detrended series) is then estimated by method of ordinary least squares

$$DT = \sum_{i=1}^{k} (a_i \cos i\, \omega t + b \sin i\, \omega t) \tag{6}$$

A statistical test of significance of the general model and that of the parameter estimates are done using a computer package known as MINITAB.

It is noted here that if the trend parameter b_0 is not significant (i.e., $b_o = o$), then the estimated model becomes.

$$\hat{Y}_t = a_0 \left[\sum_{i=1}^{k} (a_i \cos i\omega t + b_i \sin i\omega t + 1 \right] \tag{7}$$

DATA ANALYSIS

From the trend analysis,

b_0 = 0

a_o = 210.1

Therefore $T = a_0 = 210.1$

Also, since

$k = 6$, $I = 1,2,3,\ldots,6$ $I = 1$ and $\omega = \pi/6$

Therefore,

$$DT = \sum_{i=1}^{6} (a_i \cos i \frac{\pi}{6} t + b_i \sin i \frac{\pi}{6} t)$$

is modeled using in Table 1 and 2:

From Table 1, the estimated model with the significant parameter estimates is; DT = -0.984 cos ω.

Table 1: Results for testing the significance of the detrended series model

Predictor noconstant	Coefficient	SD	T	p
coswt	-0.98370	0.13350	-7.37	0.000
sinwt	-0.21840	0.13350	-1.64	0.105
cos2wt	-0.15270	0.13350	-1.14	0.255
sin2wt	0.09850	0.13350	0.74	0.462
cos3wt	0.09670	0.13350	0.72	0.470
sin3wt	0.00520	0.13350	0.04	0.969
cos4wt	0.19540	0.13350	1.46	0.146
sin4wt	0.01570	0.13350	0.12	0.907
cos5wt	-0.13060	0.13350	-0.98	0.330
sin5wt	-0.02700	0.13350	-0.20	0.840
cos6wt	0.03203	0.09440	0.34	0.735

Table 2: Result for testing the significance of general detrended series model

Source	df	SS	MS	F	p
Regression	11	66.961	6.087	5.69	0.000
Error	109	116.567	1.069		
Total	120	183.527			

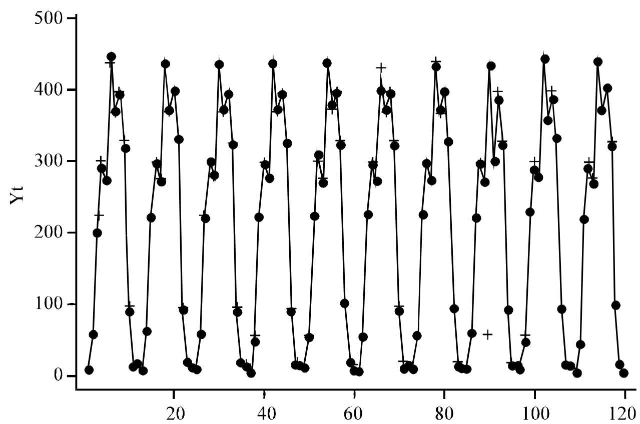

Fig. 1: Plot of actual and estimated values

RESULTS AND DISCUSSION

From the above analysis, the general estimated model is obtained as $Y_t = 210.1$ (-0.984 cos ωt+1) or $Y_t = 210.1$-206.728cosùt. The plot of the actual and fitted values of the rainfall data is given in Fig. 1. The plot indicates a good fit of the model.

It is a known fact that there are many models used in modeling a periodic time series. In this study, developments are made in the use of Fourier Series model to fit a time series. It is discovered that a pseudo-additive Fourier series model so developed is quite suitable for the modeling and forecasting of a time series with strongly marked and obviously fluctuating seasonal effects. This is shown in Appendix 1 and 2, where the estimated values fit well to the actual values.

Appendix 1: Actual values (Y_t)

S. No.	Y_t	S. No.	Y_t	S. No.	Y_t	S. No.	Y_t	S. No.	Y_t
1	6.2	25	6.2	49	8.8	73	5.3	97	4.8
2	56.0	26	55.0	50	50.9	74	55.0	98	45.0
3	200.4	27	219.0	51	224.1	75	225.3	99	230.0
4	290.0	28	300.1	52	310.0	76	299.0	100	287.4
5	273.0	29	280.0	53	268.2	77	273.0	101	277.0
6	450.0	30	438.0	54	440.0	78	433.0	102	444.2
7	371.0	31	372.0	55	382.0	79	372.0	103	356.0
8	396.0	32	396.6	56	396.2	80	400.7	104	389.0
9	320.5	33	323.3	57	325.0	81	328.4	105	333.3
10	89.1	34	87.4	58	100.0	82	92.6	106	92.1
11	10.9	35	14.0	59	15.0	83	10.0	107	12.4
12	14.0	36	9.9	60	3.5	84	5.8	108	10.0
13	4.5	37	0.0	61	2.9	85	5.7	109	0.2
14	60.5	38	45.0	62	53.8	86	57.0	110	41.8
15	221.3	39	222.0	63	225.1	87	222.0	111	216.8
16	299.0	40	297.0	64	295.7	88	296.5	112	290.0
17	271.0	41	277.0	65	273.2	89	272.0	113	268.0
18	439.0	42	439.5	66	400.0	90	436.0	114	442.0
19	372.0	43	372.1	67	372.0	91	300.0	115	372.0
20	400.0	44	397.0	68	395.0	92	388.9	116	403.0
21	331.0	45	328.0	69	323.6	93	323.1	117	321.4
22	90.1	46	88.0	70	88.0	94	90.3	118	97.0
23	15.0	47	12.0	71	9.0	95	11.6	119	12.7
24	10.0	48	10.0	72	11.2	96	12.5	120	1.1

Appendix 2: Fitted values (Y)

S. No.	Y_t	S. No.	Y_t	S. No.	Y_t	S. No.	Y_t	S. No.	Y_t
1	7.663	25	7.664	49	7.665	73	7.665	97	7.666
2	55.189	26	55.195	50	55.200	74	55.206	98	55.211
3	223.775	27	223.786	51	223.797	75	223.808	99	223.819
4	299.682	28	299.678	52	299.675	76	299.671	100	299.667
5	276.425	29	276.432	53	276.438	77	276.445	101	276.452
6	439.364	30	439.368	54	439.371	78	439.375	102	439.379
7	367.324	31	367.319	55	367.314	79	367.309	103	367.304
8	399.429	32	399.434	56	399.438	80	399.443	104	399.447
9	328.992	33	328.979	57	328.967	81	328.954	105	328.941
10	94.682	34	94.672	58	94.663	82	94.653	106	94.643
11	15.463	35	15.462	59	15.462	83	15.461	107	15.460
12	12.006	36	12.005	60	12.005	84	12.004	108	12.004
13	7.664	37	7.664	61	7.665	85	7.666	109	7.666
14	55.192	38	55.197	62	55.203	86	55.209	110	55.214
15	223.781	39	223.792	63	223.803	87	223.814	111	223.825
16	299.680	40	299.676	64	299.673	88	299.669	112	299.665
17	276.428	41	276.435	65	276.442	89	276.448	113	276.455
18	439.366	42	439.369	66	439.373	90	439.377	114	439.381
19	367.321	43	367.316	67	367.311	91	367.306	115	367.301
20	399.431	44	399.436	68	399.440	92	399.445	116	399.449
21	328.986	45	328.973	69	328.960	93	328.948	117	328.935
22	94.677	46	94.667	70	94.658	94	94.648	118	94.639
23	15.463	47	15.462	71	15.461	95	15.461	119	15.460
24	12.006	48	12.005	72	12.005	96	12.004	120	12.008

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