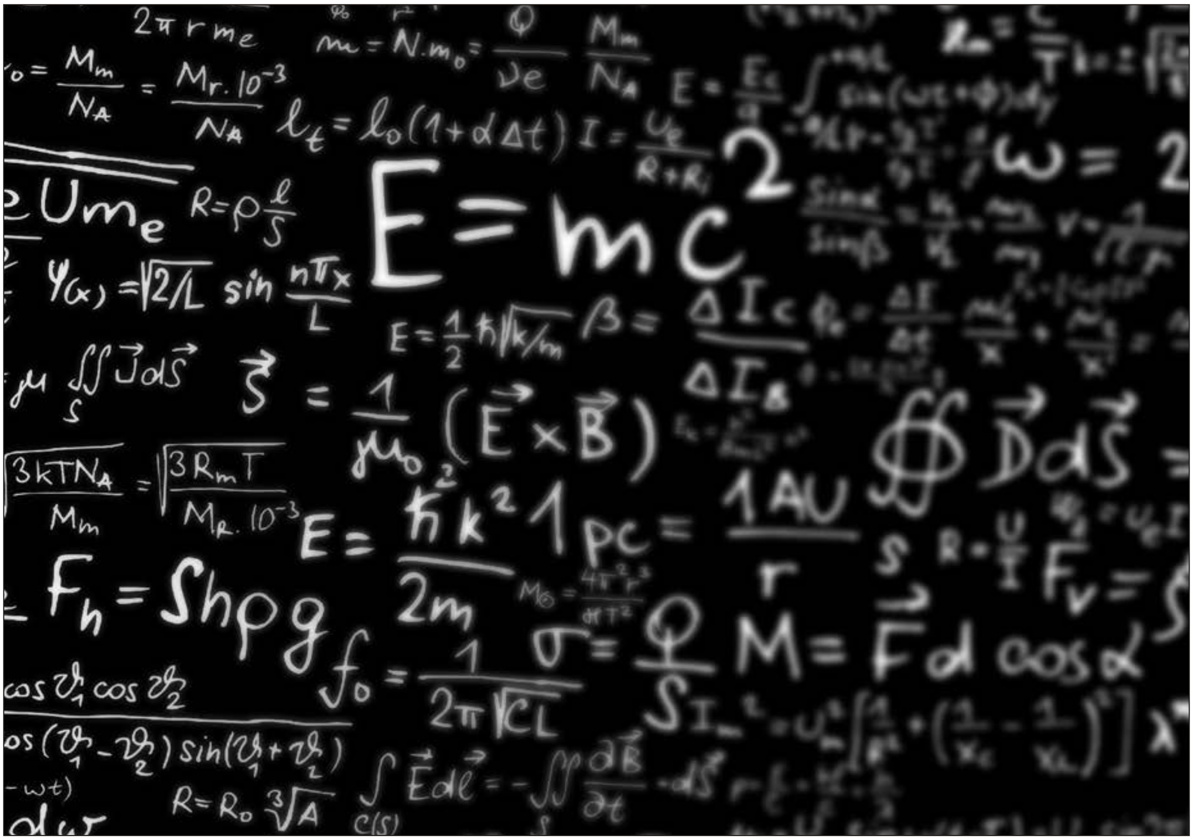

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Seasonal Analysis of Transformations of the Multiplicative Time Series Model

[1]Iheanyi S. Iwueze, [1]Anthony C. Akpanta and [2]Hycinth C. Iwu
[1]Department of Statistics, Abia State University, P.M.B. 2000, Uturu, Abia State, Nigeria
[2]Department of Mathematics and Computer Science,
Federal University of Technology, P.M.B. 1526, Owerri, Imo State, Nigeria

Abstract: Transformations of the purely multiplicative time series model will either remain the purely multiplicative model or the additive time series model. These transformations are required to meet the variant assumptions on either the multiplicative or the additive models with respect to the seasonal component. When these assumptions are met, a transformation is regarded as being successful with respect to the seasonal component. This study deals with the methods required in the determination of intervals for the seasonal indices for successful transformations. Intervals derived are found to lie in the neighbourhood of 1.0. Numerical examples are used to illustrate the results obtained.

Key words: Transformations, seasonal indices, multiplicative model, additive model

INTRODUCTION

The general time series is always considered as a mixture of four components (Kendall and Ord, 1990): (a) A trend, or long-term movement, (b) A seasonal component, (c) A cyclical or fluctuations about the trend of greater or less regularity and (d) A residual, irregular or random effect. If short period of time are involved, the cyclical component is superimposed into the trend (Chatfield, 1980) and we obtain a trend-cycle component.

We shall consider two types of model, depending on whether the seasonal effect is additive or multiplicative. If M_t is the smooth component of the series (trend and cyclical effects), S_t is the seasonal component and e_t the error term, we may have

$$Y_t = M_t \ S_t \ e_t \tag{1}$$

$$Y_t = M_t + S_t + e_t \tag{2}$$

We shall assume that the seasonal effect when it exists has period s, that is, it repeats after s time periods. In effect:

$$S_{is+j} = S_j, \text{ for all } i \text{ and } j \tag{3}$$

For the purely multiplicative model (1), we impose the condition

$$\sum_{j=1}^{s} S_{t+j} = s \tag{4}$$

and the invariant assumption for the additive model (2) is

Corresponding Author: Iheanyi S. Iwueze, Department of Statistics, Abia State University, P.M.B. 2000, Uturu, Abia State, Nigeria

Table 1: Transformations of the purely multiplicative model

Y_t*	M_t *	S_t *	e_t *	Model for Y_t *	Assumption on S_t *
$\log_e Y_t$	$\log_e M_t$	$\log_e S_t$	$\log_e e_t$	Additive	$\sum_{j=1}^{s} S_t^* = 0$
$\sqrt{Y_t}$	$\sqrt{M_t}$	$\sqrt{S_t}$	$\sqrt{e_t}$	Multiplicative	$\sum_{j=1}^{s} S_t^* = s$
$1/Y_t$	$1/M_t$	$1/S_t$	$1/r_t$	Multiplicative	$\sum_{j=1}^{s} S_t^* = s$
Y_t^2	M_t^2	S_t^2	e_t^2	Multiplicative	$\sum_{j=1}^{s} S_t^* = s$
$1/\sqrt{Y_t}$	$1/\sqrt{M_t}$	$1/\sqrt{S_t}$	$1/\sqrt{e_t}$	Multiplicative	$\sum_{j=1}^{s} S_t^* = s$
$1/Y_t^2$	$1/M_t^2$	$1/S_t^2$	$1/e_t^2$	Multiplicative	$\sum_{j=1}^{s} S_t^* = s$

$$\sum_{j=1}^{s} S_{t+j} = 0 \qquad (5)$$

We shall now concentrate upon the transformations of the purely multiplicative model (1). Transformation is a mathematical operation that changes the measurement scale of a variable and is usually done to make a set of variables useable with a particular statistical test or method. Reasons for transforming data can be found in Dolby (1963), Bond and Fox (2001), Bland and Altman (1996) and Osborne (2002). Transformations often used in statistical practice are:

$$\log_e Y_t,\ \sqrt{Y_t},\ 1/Y_t,\ Y_t^2,\ 1/\sqrt{Y_t},\ 1/Y_t^2$$

The logarithmic transformation converts the purely multiplicative model (1) to the additive model (2), while the other transformations listed leave the transformed model still multiplicative. For the logarithmic transformation,

$$Y_t^* = \log_e Y_t = \log_e M_t + \log_e S_t + \log_e e_t = M_t^* + S_t^* + e_t^* \qquad (6)$$

The result of transformations of model (1) is given in Table 1. In what follows, we will use the following notations:

Y_t^* = Transformed trend-cycle component
M_t^* = Transformed seasonal component
e_t^* = Transformed error term

It is clear from Table 1 that only the logarithmic transformation alters the assumption concerning the seasonal component. Therefore we must be interested in the values the seasonal indices of the multiplicative model must take to realize the assumptions on the seasonal indices of the resultant additive model of the transformed series. Similarly, the other transformations where there is no change in model structure must also be investigated to make sure that the transformed seasonal indices add up to s over a complete period.

If we are interested in seasonal variations, we should take several observations (quarterly, monthly) per year. In this case the frequency with which data are recorded determines the value assigned to s ($s \geq 1$), the length of the periodic interval. In seasonal time series analysis, as we shall often find that there are equality of seasonal indices which must help us to aggregate the series over shorter time periods. In our investigation of the effect of these transformations on the seasonal component of the purely multiplicative model, we must not loose sight of the cases where some of the indices are equal.

The methods available for analysis of seasonal time series data in the time domain approach include the descriptive method and the fitting of probability models (Box *et al*., 1994; Chatfield, 1980). Traditionally, seasonal effects determination in the descriptive method is not done without some prior adjustments for the trend. The problem of de-trending a series before computing the estimates of the seasonal effects can be avoided by use of Buys-Ballot procedure for time series decomposition. Since computational procedure is not part of this study, details of the Buys-Ballot procedure is given in Iwueze and Nwogu (2004, 2005) and Iwueze and Ohakwe (2004). Our interest is only on the seasonal indices.

The purpose of this study is to study the effect of transformations on the seasonal component of the purely multiplicative model with a view to achieving the desired value for the sum of the seasonal components of the transformed series.

$$\left(\sum_{j=1}^{s} S_j{}^* = 0 \ \text{ or } \ \sum_{j=1}^{s} S_j{}^* = s \right)$$

We must remember that the common values of s for variation within a year are: s = 1 for yearly data; s = 2 for data collected two times in a year; s = 3 for data collected three times in a year; s = 4 for quarterly data showing seasonal effects within years; s = 6 for bi-monthly data and s = 12 for monthly data showing seasonal effects within years.

EQUALITY OF ALL INDICES FOR ALL LENGHTS OF PERIODIC INTERVAL

This section examines the case where all the seasonal indices are equal and observations have been taken s times per year. That is, $S_j = S$, j = 1,2,...,s. Hence,

$$\sum_{j=1}^{s} S_j = sS = s \Rightarrow S = S_j = 1, \ j = 1, 2, ..., s \tag{7}$$

Even though observations have been taken s times per year, there will be no seasonal effect since variation is the same at the length of time between observations or recording frequency. For such series, it is clear that $S_j{}^* = 0$, j = 1,2,...,s (for the logarithmic transformation), $S_j{}^* = 1$, j = 1,2,...,s (for the other transformations listed in Table 1) and

$$\sum_{j=1}^{s} S_j{}^* = \begin{cases} \sum_{j=1}^{s} \log_e 1 = 0, \text{ for logarithmic transformation} \\ \sum_{j=1}^{s} 1 = s, \text{ other transformations listed in Table 1} \end{cases} \tag{8}$$

An example of this situation will be illustrated with the data on monthly unemployed (in thousands) females between ages 16 and 19 in the United States from January 1961 to December 1985, listed as Series W4 in Wei (1989). As noted by Wei (1989), the series is non-stationary with a marked linear trend and he fitted the IMA(1,1) model

$$\begin{aligned} (1 - B)Y_t &= (1 - 0.51B)a_t \\ & \qquad\ (\pm 0.05) \end{aligned} \tag{9}$$

with $\hat{\sigma}_a^2 = 1397.269$ and a_t is a zero mean white noise process.

Table 2: Seasonal indices for transformations of the monthly unemployed females (in thousands) between ages 16 and 19 in the United States from January 1961 to December 1985

S/No.	Y_t	$\log_e Y_t$	$\sqrt{Y_t}$	$1/Y_t$	Y_t^2	$1/\sqrt{Y_t}$	$1/Y_t^2$
1	1.00	0.00	1.00	1.00	1.02	1.00	0.92
2	1.01	0.01	1.01	0.99	1.02	0.99	0.91
3	1.01	0.01	1.00	0.99	1.02	0.99	0.91
4	1.01	0.01	1.01	0.98	1.01	0.99	0.93
5	1.01	0.01	1.01	0.98	1.02	0.99	0.93
6	0.99	-0.01	0.99	1.01	0.97	1.01	1.01
7	1.00	0.00	1.00	1.01	1.00	1.00	1.01
8	0.99	-0.01	1.00	1.01	1.00	1.01	1.09
9	1.00	0.01	1.00	1.00	1.00	1.00	1.03
10	1.01	0.01	1.00	0.99	1.01	1.00	1.03
11	0.99	-0.01	0.99	1.02	0.97	1.01	1.12
12	0.98	-0.02	0.99	1.02	0.96	1.01	1.11
$\sum_{j=1}^{12} S_j \Big/ \sum_{j=1}^{12} S_j*$	12.00	0.00	12.00	12.00	12.00	12.00	12.00

Computing the seasonal indices (all computations in this study were performed using MINITAB) for the original series and the transformed series, we obtain the results shown in Table 2 which helps to confirm the results of Eq. 8.

WHEN LENGTH OF PERIODIC INTERVAL IS TWO: (s = 2)

When s = 2, there are only two possibilities. The first is when $S_1 = S_2 = 1$ which was treated. The other is when $S_1 \neq S_2$ and $S_1 + S_2 = 2$. Results obtained here are applicable to other values of s when the s indices are partitioned into two equal groups where the basis of classification is equality of indices Eq. 10.

$$\left.\begin{array}{l} s = 2: \; S_1 + S_2 = 2 \\ s = 4: \; 2S_1 + 2S_2 = 4 \\ s = 6: \; 3S_1 + 3S_2 = 6 \\ s = 8: \; 4S_1 + 4S_2 = 8 \\ s = 10: \; 5S_1 + 5S_2 = 10 \\ s = 12: \; 6S_1 + 6S_2 = 12 \end{array}\right\} \Rightarrow S_1 + S_2 = 2 \qquad (10)$$

We now determine the values of S_1 and S_2 that will make $S_1* + S_2* = 0$ for the logarithmic transformation and $S_1* + S_2* = 2$ for the other transformations. For want of space, we give an extract of the computations for the logarithmic transformation in Table 3. We would accept values of S_1 and S_2 for which $S_1* + S_2* \approx 0$ to one number of decimal places. If we let $S_1 = \theta_1$ and $S_2 = \theta_2$ we see that $0.78 \le \theta_1 \le 1.22$ and $0.78 \le \theta_2 \le 1.22$. This result can be presented using set notation as $A_1 = \{(\theta_1, \theta_2): 0.78 \le \theta_1 \le 1.22, 0.78 \le \theta_2 \le 1.22, \theta_1 + \theta_2 = 2\}$. Results for all transformations are similarly determined and shown in Table 4.

These intervals of Table 4 can geometrically be represented as is shown in Fig. 1. It is clear from Table 4 that $A_2 \supset A_5 \supset A_1 \supset A_3 = A_4 \supset A_6$. In effect, square root ($\sqrt{Y_t}$) transformation gives a wider interval for a successful transformation, while the inverse of the squares ($1/Y_t^2$) transformation gives the smallest interval. As will be noted in all cases considered, the inverse transformation ($1/Y_t$) will always give an approximate interval to that of the square (Y_t^2) transformation.

An example of this situation will be illustrated with the 32 consecutive quarters of U.S beer production, in millions of barrels, from the first quarter of 1975 to the fourth quarter of 1982, listed as Series W10 in Wei (1989). This series is clearly seasonal with a slight upward trend. Wei (1989),

Table 3: Values of S_1 and S_2 that satisfy $S_1^* + S_2^* = 0$ when $S_1 + S_2 = 2$ under the logarithmic transformation

S_1	S_2	$\sum_{j=1}^{2} S_j$	S_1^*	S_2^*	$\sum_{j=1}^{2} S_j^*$	$\left\lvert \sum_{j=1}^{2} S_j^* \right\rvert$
0.78	1.22	2	-0.2485	0.1989	-0.0496	0.0496
0.79	1.21	2	-0.2357	0.1906	-0.0451	0.0451
0.80	1.20	2	-0.2231	0.1823	-0.0408	0.0408
0.81	1.19	2	-0.2107	0.1740	-0.0368	0.0368
0.82	1.18	2	-0.1985	0.1655	-0.0329	0.0329
0.83	1.17	2	-0.1863	0.1570	-0.0293	0.0293
0.84	1.16	2	-0.1744	0.1484	-0.0259	0.0259
0.85	1.15	2	-0.1625	0.1398	-0.0228	0.0228
0.86	1.14	2	-0.1508	0.1310	-0.0198	0.0198
0.87	1.13	2	-0.1393	0.1222	-0.0170	0.0170
0.88	1.12	2	-0.1278	0.1133	-0.0145	0.0145
0.89	1.11	2	-0.1165	0.1044	-0.0122	0.0122
0.90	1.10	2	-0.1054	0.0953	-0.0101	0.0101
0.91	1.09	2	-0.0943	0.0862	-0.0081	0.0081
0.92	1.08	2	-0.0834	0.0770	-0.0064	0.0064
0.93	1.07	2	-0.0726	0.0677	-0.0049	0.0049
0.94	1.06	2	-0.0619	0.0583	-0.0036	0.0036
0.95	1.05	2	-0.0513	0.0488	-0.0025	0.0025
0.96	1.04	2	-0.0408	0.0392	-0.0016	0.0016
0.97	1.03	2	-0.0305	0.0296	-0.0009	0.0009
0.98	1.02	2	-0.0202	0.0198	-0.0004	0.0004
0.99	1.01	2	-0.0101	0.0100	-0.0001	0.0001
1.00	1.00	2	0.0000	0.0000	0.0000	0.0000
1.01	0.99	2	0.0100	-0.0101	-0.0001	0.0001
1.02	0.98	2	0.0198	-0.0202	-0.0004	0.0004
1.03	0.97	2	0.0296	-0.0305	-0.0009	0.0009
1.04	0.96	2	0.0392	-0.0408	-0.0016	0.0016
1.05	0.95	2	0.0488	-0.0513	-0.0025	0.0025
1.06	0.94	2	0.0583	-0.0619	-0.0036	0.0036
1.07	0.93	2	0.0677	-0.0726	-0.0049	0.0049
1.08	0.92	2	0.0770	-0.0834	-0.0064	0.0064
1.09	0.91	2	0.0862	-0.0943	-0.0081	0.0081
1.10	0.90	2	0.0953	-0.1054	-0.0101	0.0101
1.11	0.89	2	0.1044	-0.1165	-0.0122	0.0122
1.12	0.88	2	0.1133	-0.1278	-0.0145	0.0145
1.13	0.87	2	0.1222	-0.1393	-0.0170	0.0170
1.14	0.86	2	0.1310	-0.1508	-0.0198	0.0198
1.15	0.85	2	0.1398	-0.1625	-0.0228	0.0228
1.16	0.84	2	0.1484	-0.1744	-0.0259	0.0259
1.17	0.83	2	0.1570	-0.1863	-0.0293	0.0293
1.18	0.82	2	0.1655	-0.1985	-0.0329	0.0329
1.19	0.81	2	0.1740	-0.2107	-0.0368	0.0368
1.20	0.80	2	0.1823	-0.2231	-0.0408	0.0408
1.21	0.79	2	0.1906	-0.2357	-0.0451	0.0451
1.22	0.78	2	0.1989	-0.2485	-0.0496	0.0496

Table 4: Intervals for transformations when $\theta_1 + \theta_2 = 2$

Transformations	Intervals
$\log_e Y_t$	$A_1 = \{(\theta_1, \theta_2): 0.78 \le \theta_1 \le 1.22,\ 0.78 \le \theta_2 \le 1.22,\ \theta_1 + \theta_2 = 2\}$
$\sqrt{Y_t}$	$A_2 = \{(\theta_1, \theta_2): 0.57 \le \theta_1 \le 1.43,\ 0.57 \le \theta_2 \le 1.43,\ \theta_1 + \theta_2 = 2\}$
$1/Y_t$	$A_3 = \{(\theta_1, \theta_2): 0.85 \le \theta_1 \le 1.15,\ 0.85 \le \theta_2 \le 1.15,\ \theta_1 + \theta_2 = 2\}$
Y_t^2	$A_4 = \{(\theta_1, \theta_2): 0.85 \le \theta_1 \le 1.15,\ 0.85 \le \theta_2 \le 1.15,\ \theta_1 + \theta_2 = 2\}$
$\sqrt{Y_t}$	$A_5 = \{(\theta_1, \theta_2): 0.75 \le \theta_1 \le 1.25,\ 0.75 \le \theta_2 \le 1.25,\ \theta_1 + \theta_2 = 2\}$
$1/Y_t^2$	$A_6 = \{(\theta_1, \theta_2): 0.91 \le \theta_1 \le 1.09,\ 0.91 \le \theta_2 \le 1.09,\ \theta_1 + \theta_2 = 2\}$

ignoring the stochastic trend in the series, used 30 observations of the original series for ARIMA model construction. Based on the forecasting performance of his models, he settled on the seasonal ARIMA model.

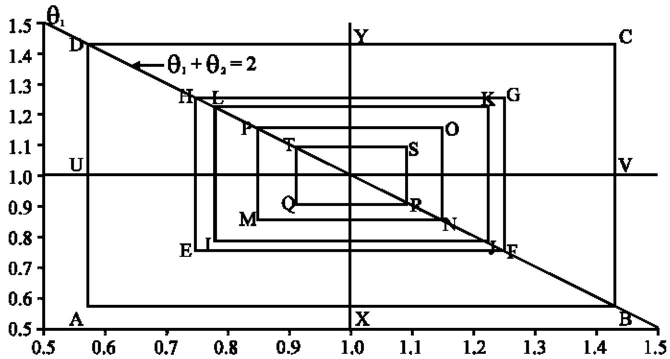

Fig. 1: Regions for successful transformations for two distinct values with equal number of same indices in each partition ($\theta_1 + \theta_2 = 2$)

Table 5: Seasonal indices for transformations of the quarterly US beer production data, in millions of barrels, from the first quarter of 1975 to the fourth quarter of 1982

S/No.	Y_t	$\log_e Y_t$	$\sqrt{Y_t}$	$1/Y_t$	Y_t^2	$\sqrt{Y_t}$	$1/Y_t^2$
1	0.94	-0.06	0.97	1.05	0.88	1.03	1.10
2	1.11	0.11	1.06	0.89	1.23	0.94	0.78
3	1.08	0.08	1.04	0.92	1.15	0.96	0.83
4	0.87	-0.13	0.93	1.14	0.74	1.07	1.29
$\sum_{j=1}^{4} S_j \Big/ \sum_{j=1}^{4} S_j{*}$	4.00	0.00	4.00	4.00	4.00	4.00	4.00

$$(1-B^4)Y_1 = 1.49 + (1-0.87B^4)a_t \quad (11)$$
$$(\pm 0.09) \quad (\pm 0.16)$$

with $\hat{\sigma}_a^2 = 2.39$ and a_t is a zero mean white noise process.

Seasonal indices for the original series and the transformations of Table 1 are given in Table 5. Based on one number of decimal place, it is clear that $S_1 = S_4 = \theta_1 = 0.9$ and $S_2 = S_3 = \theta_2 = 1.1$. Here we see that $\theta_1 + \theta_2 = 2$, meaning that we can obtain the appropriate seasonal indices of the transformed variables by taking the equivalent transformation of θ_1 and θ_2.

WHEN LENGTH OF PERIODIC INTERVAL IS THREE: (s = 3)

When s = 3, there are only three possibilities. The first is when $S_1 = S_2 = S_3 = 1$ which was treated. The next is when the three seasonal indices are partition into two groups at the ratio of 1 : 2. If we let the group containing only one index to have a value of θ_1 for its unique index and the group containing two equal indices to have a value of θ_2 for each of the equal indices; we obtain $\theta_1 \neq \theta_2$ and $\theta_1 + 2\theta_2 = 3$. Applications to values of $s \geq 3$ are given in Eq. 12.

$$\left.\begin{array}{ll} s = 3: & \theta_1 + 2\theta_2 = 3 \\ s = 6: & 2\theta_1 + 4\theta_2 = 6 \\ s = 9: & 3\theta_1 + 6\theta_2 = 9 \\ s = 12: & 4\theta_1 + 8\theta_2 = 12 \end{array}\right\} \Rightarrow \theta_1 + 2\theta_2 = 3 \quad (12)$$

Determination of θ_1 and θ_2 are described earlier and the intervals obtained are given in Table 6. Unlike Table 4, these intervals in Table 6 are not symmetrical from the point 1.0 and the minimum and maximum values of θ_1 and θ_2 are not similar. Again, it is clear that $B_2 \supset B_5 \supset B_1 \supset B_3 = B_4 \supset B_6$.

Table 6: Intervals for transformations when $\theta_1 + 2\theta_2 = 3$

Transformations	Intervals
$\log_e Y_t$	$B_1 = \{(\theta_1, \theta_2): 0.76 \le \theta_1 \le 1.26, 0.87 \le \theta_2 \le 1.12, \theta_1 + 2\theta_2 = 3\}$
$\sqrt{Y_t}$	$B_2 = \{(\theta_1, \theta_2): 0.53 \le \theta_1 \le 1.53, 0.74 \le \theta_2 \le 1.24, \theta_1 + 2\theta_2 = 3\}$
$1/Y_t$	$B_3 = \{(\theta_1, \theta_2): 0.83 \le \theta_1 \le 1.18, 0.91 \le \theta_2 \le 1.09, \theta_1 + 2\theta_2 = 3\}$
Y_t^2	$B_4 = \{(\theta_1, \theta_2): 0.83 \le \theta_1 \le 1.18, 0.91 \le \theta_2 \le 1.09, \theta_1 + 2\theta_2 = 3\}$
$\sqrt{Y_t}$	$B_5 = \{(\theta_1, \theta_2): 0.73 \le \theta_1 \le 1.31, 0.85 \le \theta_2 \le 1.14, \theta_1 + 2\theta_2 = 3\}$
$1/Y_t^2$	$B_6 = \{(\theta_1, \theta_2): 0.90 \le \theta_1 \le 1.10, 0.95 \le \theta_2 \le 1.05, \theta_1 + 2\theta_2 = 3\}$

Table 7: Intervals for transformations when $\theta_1 + \theta_2 + \theta_3 = 3$

Transformations	Intervals
$\log_e Y_t$	$C_1 = \{(\theta_1, \theta_2, \theta_3): 0.76 \le \theta_1 \le 1.26, 0.76 \le \theta_2 \le 1.26, 0.76 \le \theta_3 \le 1.26, \theta_1 + \theta_2 + \theta_3 = 3\}$
$\sqrt{Y_t}$	$C_2 = \{(\theta_1, \theta_2, \theta_3): 0.53 \le \theta_1 \le 1.53, 0.53 \le \theta_2 \le 1.53, 0.56 \le \theta_3 \le 1.45, \theta_1 + \theta_2 + \theta_3 = 3\}$
$1/Y_t$	$C_3 = \{(\theta_1, \theta_2, \theta_3): 0.83 \le \theta_1 \le 1.17, 0.83 \le \theta_2 \le 1.17, 0.85 \le \theta_3 \le 1.15, \theta_1 + \theta_2 + \theta_3 = 3\}$
Y_t^2	$C_4 = \{(\theta_1, \theta_2, \theta_3): 0.83 \le \theta_1 \le 1.17, 0.83 \le \theta_2 \le 1.17, 0.85 \le \theta_3 \le 1.15, \theta_1 + \theta_2 + \theta_3 = 3\}$
$\sqrt{Y_t}$	$C_5 = \{(\theta_1, \theta_2, \theta_3): 0.73 \le \theta_1 \le 1.30, 0.73 \le \theta_2 \le 1.30, 0.73 \le \theta_3 \le 1.30, \theta_1 + \theta_2 + \theta_3 = 3\}$
$1/Y_t^2$	$C_6 = \{(\theta_1, \theta_2, \theta_3): 0.90 \le \theta_1 \le 1.10, 0.90 \le \theta_2 \le 1.10, 0.95 \le \theta_3 \le 1.06, \theta_1 + \theta_2 + \theta_3 = 3\}$

Table 8: Seasonal indices for transformations of the miles flown by British airlines for the 96 months January 1963 to December 1970

S/No.	1	2	3	4	5	6	7	8	9	10	11	12	$\sum_{j=1}^{12} S_j$ or $\sum_{j=1}^{12} S_j$ *
Y_t	0.8	0.8	1.0	1.0	1.0	1.2	1.2	1.2	1.2	1.0	0.8	0.8	12.0
$\text{Log}_e Y_t$	-0.2	-0.2	0.0	0.0	0.0	0.2	0.2	0.2	0.2	0.0	-0.2	-0.2	0.0
$\sqrt{Y_t}$	0.9	0.9	1.0	1.0	1.0	1.1	1.1	1.1	1.1	1.0	0.9	0.9	12.0

The last possibility for $s = 3$ is when the seasonal indices are equally partitioned into three groups. If we let θ_1 represent the common index of the first group; θ_2 represent the common index of the second group; θ_3 represent the common index of the third group, then $\theta_1 \neq \theta_2 \neq \theta_3$ and $\theta_1 + \theta_2 + \theta_3 = 3$. Applications to values of $s \geq 3$ are given in Eq. 13.

$$\left.\begin{array}{lllllll} s = 3: & \theta_1 & + & \theta_2 & + & \theta_3 & = 3 \\ s = 6: & 2\theta_1 & + & 2\theta_2 & + & 2\theta_3 & = 6 \\ s = 9: & 3\theta_1 & + & 3\theta_2 & + & 3\theta_3 & = 9 \\ s = 12: & 4\theta_1 & + & 4\theta_2 & + & 4\theta_3 & = 12 \end{array}\right\} \Rightarrow \theta_1 + \theta_2 + \theta_3 = 3 \tag{13}$$

Determination of θ_1, θ_2 and θ_3 are as presented and the intervals obtained are given in Table 7. Unlike Table 4, these intervals in Table 7 are not symmetrical from the point 1.0 and the minimum and maximum values of θ_1, θ_2 and θ_3 are not similar, except for the logarithmic transformation. Again, it is clear that $C_2 \supset C_5 \supset C_1 \supset C_3 = C_4 \supset C_6$, while for the logarithmic transformation θ_1, θ_2 and θ_3 have the same intervals.

An example of this situation will be illustrated with the data on the miles flown by British airlines for the 96 months (January 1963 to December 1970). The plot (not shown) of the series shows a marked seasonal pattern and a linear trend. Various seasonal ARIMA models were fitted to the original series and logarithmic transformed series by Kendall and Ord (1990).

Seasonal indices for the original series, logarithmic and square root (other transformations are meaningless because of the large values of the original series) transformations are given in Table 8. Based on one number of decimal place, it is clear that $S_1 = S_2 = S_{11} = S_{12} = \theta_1 = 0.8$, $S_3 = S_4 = S_5 = S_{10} = \theta_2 = 1.0$ and $S_6 = S_7 = S_8 = S_9 = \theta_3 = 1.02$. Here we note that $\theta_1 + \theta_2 + \theta_3 = 3$, meaning that we can obtain the appropriate seasonal indices of the transformed variables by taking the equivalent transformation of θ_1, θ_2 and θ_3. These indices can help to aggregate the data into three distinct groups of months based on the equality of seasonal indices which is different from the natural ordering of the months.

WHEN LENGTH OF PERIODIC INTERVAL IS FOUR: (s = 4)

When s = 4, there are only five possibilities. The first is when $S_1 = S_2 = S_3 = S_4 = 1$ which was treated. The second is when the four seasonal indices are partition into two groups at the ratio of 2:2 = 1:1 which has also been treated earlier.

The third is when the four seasonal indices are partition into two groups at the ratio of 1:3. If we let the group containing only one index to have a value of θ_1 for its unique index and the group containing three equal indices to have a value of θ_2 for each of the equal indices; we obtain $\theta_1 \neq \theta_2$ and $\theta_1 + 3\theta_2 = 4$. Applications to values of $s \geq 4$ are given in Eq. 14.

$$\left.\begin{array}{l} s = 4:\ \theta_1 + 3\theta_2 = 4 \\ s = 8:\ 2\theta_1 + 6\theta_2 = 8 \\ s = 12:\ 4\theta_1 + 8\theta_2 = 12 \end{array}\right\} \Rightarrow \theta_1 + 3\theta_2 = 4 \qquad (14)$$

Determination of θ_1 and θ_2 are described earlier and the intervals obtained are given in Table 9. Unlike Table 4, these intervals in Table 9 are not symmetrical from the point 1.0 and the minimum and maximum values of θ_1 and θ_2 are not similar. Again, it is clear that $D_2 \supset D_5 \supset D_1 \supset D_3 = D_4 \supset D_6$.

The fourth is when the four seasonal indices are partition into three groups at the ratio of 1: 1: 2. If we let the first group containing only one index to have a value of θ_1 for its unique index, the second group containing only one index to have a value of θ_2 for its unique index and the group containing two equal indices to have a value of θ_3 for each of the equal indices; we obtain $\theta_1 \neq \theta_2 \neq \theta_3$ and $\theta_1 + \theta_2 + 2\theta_3 = 4$. Applications to values of $s \geq 4$ are given in Eq. 15.

$$\left.\begin{array}{l} s = 4:\ \theta_1 + \theta_2 + 2\theta_3 = 4 \\ s = 8:\ 2\theta_1 + 2\theta_2 + 4\theta_3 = 8 \\ s = 12:\ 3\theta_1 + 3\theta_2 + 6\theta_3 = 12 \end{array}\right\} \Rightarrow \theta_1 + \theta_2 + 2\theta_3 = 4 \qquad (15)$$

Determination of θ_1, θ_2 and θ_3 and the intervals obtained are shown in Table 10. Unlike Table 4, these intervals in Table 10 are not symmetrical from the point 1.0 and the minimum and maximum values of θ_1, θ_2 and θ_3 are not similar. Again, it is clear that $E_2 \supset E_5 \supset E_1 \supset E_3 = E_4 \supset D_6$.

The last possibility for s = 4 is when the seasonal indices are equally partitioned into four groups. If we let θ_1 represent the common index of the first group; θ_2 represent the common index of the second group; θ_3 represent the common index of the third group and θ_4 represent the common index of the fourth group, then $\theta_1 \neq \theta_2 \neq \theta_3 \neq \theta_4$ and $\theta_1 + \theta_2 + \theta_3 + \theta_4 = 4$. Applications to values of $s \geq 4$ are given in Eq. 16.

$$\left.\begin{array}{l} s = 4:\ \theta_1 + \theta_2 + \theta_3 + \theta_4 = 4 \\ s = 8:\ 2\theta_1 + 2\theta_2 + 2\theta_3 + 2\theta_4 = 8 \\ s = 12:\ 3\theta_1 + 3\theta_2 + 3\theta_3 + 3\theta_4 = 12 \end{array}\right\} \Rightarrow \theta_1 + \theta_2 + \theta_3 + \theta_4 = 4 \qquad (16)$$

Table 9: Intervals for transformations when $\theta_1 + 3\theta_2 = 4$

Transformations	Intervals
$\log_e Y_t$	$D_1 = \{(\theta_1, \theta_2): 0.75 \leq \theta_1 \leq 1.28, 0.91 \leq \theta_2 \leq 1.08, \theta_1 + 3\theta_2 = 4\}$
$\sqrt{Y_t}$	$D_2 = \{(\theta_1, \theta_2): 0.52 \leq \theta_1 \leq 1.58, 0.81 \leq \theta_2 \leq 1.16, \theta_1 + 3\theta_2 = 4\}$
$1/Y_t$	$D_3 = \{(\theta_1, \theta_2): 0.82 \leq \theta_1 \leq 1.20, 0.93 \leq \theta_2 \leq 1.06, \theta_1 + 3\theta_2 = 4\}$
Y_t^2	$D_4 = \{(\theta_1, \theta_2): 0.82 \leq \theta_1 \leq 1.20, 0.93 \leq \theta_2 \leq 1.06, \theta_1 + 3\theta_2 = 4\}$
$\sqrt{Y_t}$	$D_5 = \{(\theta_1, \theta_2): 0.72 \leq \theta_1 \leq 1.34, 0.89 \leq \theta_2 \leq 1.08, \theta_1 + 3\theta_2 = 4\}$
$1/Y_t^2$	$D_6 = \{(\theta_1, \theta_2): 0.90 \leq \theta_1 \leq 1.11, 0.96 \leq \theta_2 \leq 1.03, \theta_1 + 3\theta_2 = 4\}$

Table 10: Intervals for transformations when $\theta_1 + \theta_2 + 2\theta_3 = 4$

Transformations	Intervals
$\log_e Y_t$	$E_1 = \{(\theta_1, \theta_2, \theta_3): 0.76\le\theta_1\le1.26, 0.75\le\theta_2\le1.28, 0.85\le\theta_3\le1.16, \theta_1 + \theta_2 + 2\theta_3 = 4\}$
$\sqrt{Y_t}$	$E_2 = \{(\theta_1, \theta_2, \theta_3): 0.55\le\theta_1\le1.55, 0.52\le\theta_2\le1.58, 0.69\le\theta_3\le1.31, \theta_1 + \theta_2 + 2\theta_3 = 4\}$
$1/Y_t$	$E_3 = \{(\theta_1, \theta_2, \theta_3): 0.85\le\theta_1\le1.16, 0.82\le\theta_2\le1.20, 0.90\le\theta_3\le1.10, \theta_1 + \theta_2 + 2\theta_3 = 4\}$
Y_t^2	$E_4 = \{(\theta_1, \theta_2, \theta_3): 0.85\le\theta_1\le1.16, 0.82\le\theta_2\le1.20, 0.90\le\theta_3\le1.10, \theta_1 + \theta_2 + 2\theta_3 = 4\}$
$\sqrt{Y_t}$	$E_5 = \{(\theta_1, \theta_2, \theta_3): 0.75\le\theta_1\le1.26, 0.72\le\theta_2\le1.33, 0.83\le\theta_3\le1.17, \theta_1 + \theta_2 + 2\theta_3 = 4\}$
$1/Y_t^2$	$E_6 = \{(\theta_1, \theta_2, \theta_3): 0.95\le\theta_1\le1.06, 0.90\le\theta_2\le1.11, 0.94\le\theta_3\le1.06, \theta_1 + \theta_2 + 2\theta_3 = 4\}$

Table 11: Intervals for transformations when $\theta_1 + \theta_2 + \theta_3 + \theta_4 = 4$

Transformations	Intervals
$\log_e Y_t$	$F_1 = \{(\theta_1, \theta_2, \theta_3, \theta_4): 0.76\le\theta_1\le1.26, 0.76\le\theta_2\le1.26, 0.85\le\theta_3\le1.14, 0.85\le\theta_3\le1.14, \theta_1 + \theta_2 + \theta_3 + \theta_4 = 4\}$
$\sqrt{Y_t}$	$F_2 = \{(\theta_1, \theta_2, \theta_3, \theta_4): 0.52\le\theta_1\le1.58, 0.58\le\theta_2\le1.58, 0.75\le\theta_3\le1.25, 0.75\le\theta_3\le1.25, \theta_1 + \theta_2 + \theta_3 + \theta_4 = 4\}$
$1/Y_t$	$F_3 = \{(\theta_1, \theta_2, \theta_3, \theta_4): 0.83\le\theta_1\le1.20, 0.83\le\theta_2\le1.20, 0.95\le\theta_3\le1.06, 0.95\le\theta_3\le1.06, \theta_1 + \theta_2 + \theta_3 + \theta_4 = 4\}$
Y_t^2	$F_4 = \{(\theta_1, \theta_2, \theta_3, \theta_4): 0.83\le\theta_1\le1.20, 0.83\le\theta_2\le1.20, 0.95\le\theta_3\le1.06, 0.95\le\theta_3\le1.06, \theta_1 + \theta_2 + \theta_3 + \theta_4 = 4\}$
$\sqrt{Y_t}$	$F_5 = \{(\theta_1, \theta_2, \theta_3, \theta_4): 0.73\le\theta_1\le1.30, 0.73\le\theta_2\le1.30, 0.86\le\theta_3\le1.15, 0.86\le\theta_3\le1.15, \theta_1 + \theta_2 + \theta_3 + \theta_4 = 4\}$
$1/Y_t^2$	$F_6 = \{(\theta_1, \theta_2, \theta_3, \theta_4): 0.90\le\theta_1\le1.08, 0.90\le\theta_2\le1.08, 0.96\le\theta_3\le1.05, 0.96\le\theta_3\le1.05, \theta_1 + \theta_2 + \theta_3 + \theta_4 = 4\}$

Determination of θ_1, θ_2, θ_3 and θ_4 are described earlier and the intervals obtained are given in Table 11. Unlike Table 4, these intervals in Table 11 are not symmetrical from the point 1.0 and while the minimum and maximum values of θ_1 and θ_2 are similar, those of θ_3 and θ_4 are also similar. Again, it is clear that $F_2 \supset F_5 \supset F_1 \supset F_3 = F_4 \supset F_6$.

For an illustration, we return to the 32 consecutive quarters of US beer production discussed in Section 3. Based on two number of decimal places, we can regard the four indices as being different with $S_1, = \theta_1 = 0.87$, $S_2 = \theta_2 = 0.94$, $S_3 = \theta_3 = 1.08$, $S_4 = \theta_4 = 1.11$ and $\theta_1 + \theta_2 + \theta_3 + \theta_4 = 4$. As before, to obtain approximate estimates of the seasonal indices of a transformation, we merely take the equivalent transformation of θ_1, θ_2, θ_3 and θ_4.

CONCLUSION

We have constructed intervals for the seasonal indices of the purely multiplicative time series model (S_j, j = 1,2,...,s) required for successful data transformations. By successful transformation we mean the ability to obtain the seasonal indices of the transformed series (S_j^*, j = 1,2,...,s) directly from those of the original series by merely taking the equivalent transformation of S_j, j = 1,2,...,s. We investigated this problem for all distinct indices for all s values and for the equality of some indices when s = 2, 3 and 4. Results obtained are shown to be applicable to given patterns of equality of indices for s = 6, 8, 9, 10 and 12.

We must bear in mind that for a given data set only one of these transformations will be used. Reasons for taking transformations differ and selecting the best transformation can be a complex issue. If you are unsure about the use of a transformation then take the advice contained in the references listed in this work. Once you have decided on a transformation, the intervals derived in this study for the seasonal indices of the original series will be of great help in the determination of the seasonal indices of the transformed series.

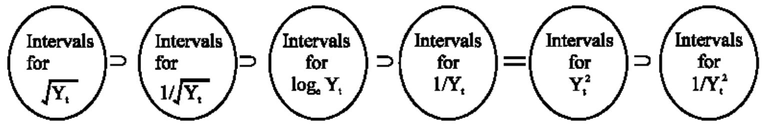

Fig. 2: Rules for successive transformations with respect to the seasonal effects

Finally, we must remember that this study has led us into accepting the following rule for successive transformation of the purely multiplicative time series model. Intervals of the seasonal indices of the original series must obey the rule given in Fig. 2.

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