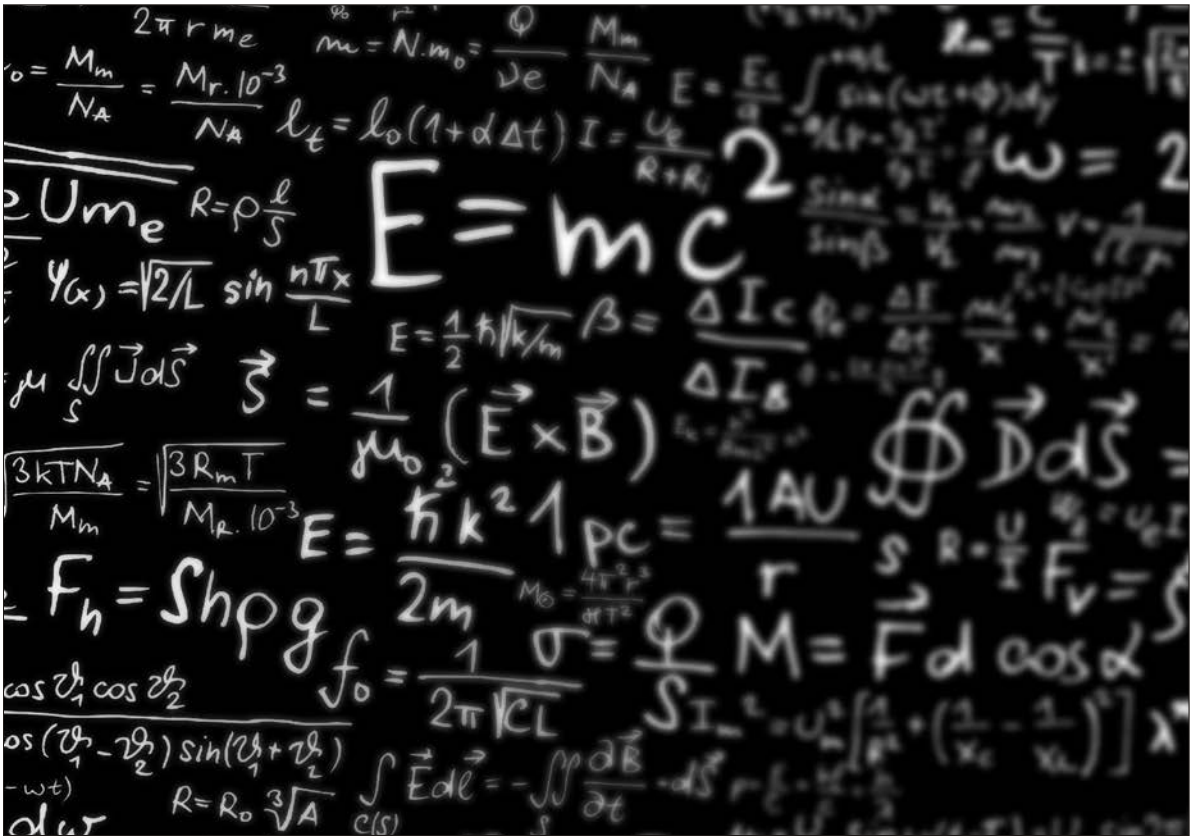

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On Approximate Solution of Second Order Differential Equation by Iterative Decomposition Method

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INTRODUCTION

This study deals with the study of second order differential equation. Such differential equation is often used to model phenomena in scientific and technological problems. Solutions to such models have been obtained by Adesanya *et al.* (2008), Aladeselu (2007), Awoyemi (1999), Awoyemi and Kayode (2005), Cash (2005) and Ramos and Vigo-Agular (2007) using various techniques. In this study, a numerical method based on the iterative decomposition is introduced for the approximate solution of such equations. The obtained results are presented where only a few terms are required to obtain a good approximation to the exact solution. Numerical examples are presented to illustrate the applicability, accuracy and efficiency of the new scheme.

This research has to do with the numerical solution of the second order differential equations of the form:

$$\begin{cases} y(t) = f(t, y, y) \\ y(\alpha) = \alpha \\ y(\alpha) = \beta \end{cases} \qquad (1)$$

where, $\alpha \le t \le b$, $\alpha = t_0 < t_1 < \cdots < t_{n-1} = b$, α, $\beta \in \mathbb{R}$,

$$h = \frac{b-\alpha}{n}, n = 1, 2, \ldots \text{ and } h$$

is the step length.

The conditions on the function f(t, y(t), y(t)) are such that existence and uniqueness of solution is guaranteed (Henrici, 1962). This class of problems is important for their applications in science and engineering especially in biological sciences and control theory. An active research work has been carried out in this area, with a number of numerical methods of solution, developed and it is still receiving attention due to its wider area of applicability in modeling real life problems (Aladeselu, 2007; Awoyemi, 1999; Awoyemi and Kayode, 2005; Cash, 2005; Lambert, 1973; Ramos and Vigo-Agular, 2007).

However, the decomposition methods are presently receiving more attention as efficient techniques for the solution of linear and non linear, ordinary, partial, deterministic or stochastic differential equations. Adomian (1993), Daftardar-Gejji and Jafari (2006), He and Wu (2007), Reid (1972) and Taiwo *et al.* (2009). These methods have been found to converge rapidly to the exact solution.

In this study, a new class of the decomposition method, is applied which offers further insight into convergence, minimizes the already reduced volume of calculations introduced by the Adomian's method without jettisoning its accuracy and efficiency.

METHOD OF SOLUTION

Consider a second order initial value problem:

$$y''+p(x)\,y'+q(x)\,y+N(y)=g(x) \tag{2}$$

$$y(\alpha)=\alpha \tag{3}$$

$$y(\alpha)=\beta \tag{4}$$

where, α and β are constants, N(y) is a nonlinear term and g(x) is the source term.

Equation 2 can be re-written in canonical form:

$$Ly=-p(x)y-q(x)y-N(y)+g(x) \tag{5}$$

where, the differential operator L is given by:

$$L^{-1}=\frac{d^2}{dx^2}(\cdot) \tag{6}$$

The inverse operator L^{-1} is thus a two-fold definite integral operator defined by:

$$L^{-1}(\cdot)=\int_0^x\int_0^x(\cdot)dsds \tag{7}$$

Operating the inverse operator Eq. 7 on 2 and using Eq. 3-4, it follows that:

$$y(x)=\alpha+\beta+L^{-1}g(x)-L^{-1}[p(x)y+q(x)y+N(y)] \tag{8}$$

The iterative decomposition method assumes that the unknown function y(x) can be expressed in terms of an infinite series of the form:

$$y(x)=\sum_{n=0}^{\infty}y_n(x) \tag{9}$$

So, that the component $y_n(x)$ can be determined iteratively. To convey the idea and for sake of completeness of the method Eq. 7, it is obvious that Eq. 8 is of the form:

$$y = N(y)+f \tag{10}$$

where, f is a constant and N(y) is the non linear term.

We then split the non linear team as:

$$\sum_{j=0}^{\infty} y_j = L^{-1}[y_0] + \sum_{j=0}^{\infty}\left\{L^{-1}\left(\sum_{j=0}^{\infty} y_j\right) - L^{-1}\left(\sum_{j=0}^{n-1} y_j\right)\right\} \tag{11}$$

On substituting Eq. 8 and 9 into Eq. 10. We have:

$$\sum_{n=0}^{\infty} y_n = f + L^{-1}(y_0) + \sum_{j=0}^{\infty}\left\{L^{-1}\left(\sum_{j=0}^{n} y_j\right) - L^{-1}\left(\sum_{j=0}^{n-1} y_j\right)\right\} \tag{12}$$

which yields the recurrence relation below:

$$\begin{cases} y_0 = f = \alpha + \beta + L \ g(x) \\ y_1 = L^{-1}(y_0) \\ y^2 = L^{-1}(y_0 + y_1) - L^{-1}(y_0) \\ \vdots \\ y_{n+1}(x) = L^{-1}(y_0 + y_1 + \cdots + y_n) - L^{-1}(y_0 + y_2 + \cdots + y_{n-1}); n \geq 1 \end{cases} \tag{13}$$

All of the y_{n+1} and

$$y = \sum_{i=0}^{n} y_i$$

are calculated. Since, the series converges and does so very rapidly, the n-term partial $y_0(x) = \sum_{i=0}^{n} y_i$ can serve as a practical solution (Adomian, 1993).

From where we obtained an n-term approximate solution:

$$y(x) = \sum_{i=0}^{n} y_i \tag{14}$$

as the exact solution in closed form or the approximate solution to Eq. 2:

NUMERICAL EXPERIMENT

To give a clear overview of our study and to illustrate the above discussed technique, we consider the following examples. In all cases considered, where the exact solution are known, we have defined our error as:

$$r_j(x) = \max_{a<x<b}\left|y_j(x) - y_j^*(x)\right|; j = 1, 2, \ldots, N$$

y^* (t) is the computed value and y(x) is the exact solution.

Example 1: Consider the second order initial value problem:

$$u''(x) + u(x) = x, \qquad 0 \le x \le 1$$
$$u(0) = u'(0) = 1$$

The iterative decomposition method gives:

$$u_0 = 1 + x + \frac{1}{3}x^3$$
$$u_1 = -\frac{1}{2!}x^2 - \frac{1}{3!}x^3 - \frac{1}{5!}x^5$$
$$u_2 = -\frac{1}{4!}x^4 - \frac{1}{5!}x^5 - \frac{1}{7!}x^7$$
$$\vdots$$
$$u(x) = x + 1 - \frac{1}{2!}x^2 - \frac{1}{4!}x^4 - \cdots$$

Hence, $u(x) = \lim_{n \to \infty} u_n(x) = x + \cos x$ which is the exact solution.

Example 2: Consider the problem of second order non-linear homogenous differential equation given as:

$$y''(t) - t(y')^2 = 0; 0 \le t \le 1$$
$$y(0) = 1$$
$$y'(0) = 1/2, \; h = 1/40$$

The theoretical solution of this example is:

$$y(t) = 1 + \frac{1}{2}\ln\left(\frac{2+t}{2-t}\right)$$

Applying the iterative decomposition algorithm to above example, we get

$$y_0 = 1 + \frac{1}{2}t$$
$$y_1 = \frac{1}{24}t^3$$
$$y_2 = \frac{1}{160}t^5 + \frac{1}{2688}t^7$$
$$y_3 = \frac{1}{1344}t^7 + \frac{1}{6912}t^9 + \frac{1}{67584}t^{11} + \frac{1}{958464}t^{13} + \frac{1}{30965760}t^{15}$$
$$\vdots$$

Hence, the series solution is:

Table 1: Errors for example 2

T	y-Exact	y-Computed	Error
0.0025	1.001250001	1.001250001	0
0.0050	1.002500005	1.002500005	0
0.0075	1.003750018	1.003750018	0
0.0100	1.005000042	1.005000042	0
0.0125	1.006250081	1.006250081	0
0.0150	1.007500141	1.007500141	0
0.0175	1.008750223	1.008750223	0
0.0200	1.010000333	1.010000333	0
0.0225	1.011250475	1.011250475	0
0.0250	1.012500651	1.012500651	0

$$y(t) = 1 + \frac{1}{2}t + \frac{1}{24}t^3 + \frac{1}{160}t^5 + \frac{1}{896}t^7 + \frac{1}{6912}t^9 + \frac{1}{67584}t^{11} + \frac{1}{958464}t^{13} + \frac{1}{30965760}t^{14} + 0(t^{16})$$

The comparison of the exact solution with the series solution of example two obtained using our algorithm is shown in Table 1. Also, the new scheme gives a better accuracy than the results obtained using block method proposed and used for the same example in Adesanya *et al*. (2008).

Example 3: Consider the non-linear oscillator problem:

$$\begin{aligned} &u''(x) - u(x) + u^2(x) + (u'(x))^2 - 1 = 0;\ 0 \le x \le 1 \\ &u(0) = 2 \\ &u'(0) = 0 \end{aligned}$$

Theoretical solution u(x) = 1+cos x.

Using the iterative decomposition method approach to example three, we obtain the approximate solution as:

$$\begin{aligned} u_0 &= 2 + \frac{1}{2}x^2 \\ u_1 &= -x^2 - \frac{s}{24}x^4 - \frac{1}{120}x^6 \\ u_2 &= \frac{1}{4}x^4 \frac{s}{144}x^6 - \frac{11}{630}x^8 - \frac{1}{2620800}x^{14} - \frac{43}{950400}x^{12} - \frac{389}{259200}x^{10} \end{aligned}$$

Following the same approach of example two, we obtained the approximate solution:

$$u_3 = \frac{1}{24}x^6 - \frac{29}{2688}x^8 - \frac{17}{30240}x^{10} - \frac{1068385573}{66691392768000}x^{18} - \frac{521496397}{476411925749760000}x^{24} - \frac{1021}{122398320844480000}x^{28} - \frac{739181}{5342783846400000}x^{26} - \frac{1456252001}{31070342983680000}x^{22} - \frac{16108627}{14041589760000}x^{20} - \frac{1}{5975675586800000}x^{30} - \frac{621526039}{5230697472000}x^{16} - \frac{335047}{871782912}x^{19} - \frac{97231}{2395006000}x^{12}$$

The comparison of the exact solution with the series solution of the example three, using our algorithm is shown in Table 2. It is obvious that the errors involved are quite small.

Table 2: Errors for example 3

X	u-Exact	u-Computed	Error
0.1	1.995004165	1.995004165	0
0.2	1.980066578	1.980066561	1.700E-9
0.3	1.955336489	1.955336037	4.520E-8
0.4	1.921060994	1.921056380	4.614E-7
0.5	1.877582562	1.877554254	2.831E-6
0.6	1.825335615	1.825209350	1.263E-5
0.7	1.764842187	1.764388610	4.536E-5
0.8	1.696706709	1.695310184	1.397E-5
0.9	1.621609968	1.617768156	3.842E-4
1.0	1.540302306	1.530591249	9.331E-4

$$\begin{aligned} u(x) = {} & \frac{1}{720}x^6 + \frac{269}{40320}x^8 + \frac{3743}{1814400}x^{10} \frac{1068385573}{66691392768000}x^{18} \\ & - \frac{521496397}{476411925749760000}x^{24} - \frac{1021}{12398320844480000}x^{28} \\ & - \frac{73919181}{53427838460000}x^{20} - \frac{1}{597567559680000}x^{30} \\ & - \frac{62156039}{52306974720000}x^{16} - \frac{108067}{239500800}x^{12} + 2 - \frac{1}{2}x^2 + \frac{1}{24}x^4 + 0(x^{31}) \end{aligned}$$

CONCLUSION

A numerical scheme of high accuracy has been proposed for the numerical solution of general second order differential equation. In the present study, the function representing the approximate solution proves to be a good estimate of the exact solution for the test examples. This suggests wider application of the method for more complicated problem since the method is implemented with less stress computer coding which makes it cheaper and cost effective in implementation. The fact that non-linear problems are solved by this method without using the so-called Adomian's polynomial is an added advantage.

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