Asian Journal of Mathematics and Statistics 4 (3): 128-139, 2011
ISSN 1994-5418 / DOI: 10.3923/ajms.2011.128.139

The Efficiency of Some Alternative Ridge Estimators for Seemingly Unrelated Regressions

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ABSTRACT

Parametric Seemingly Unrelated Regression (SUR) models are used for multivariate regression analysis. However, statistical literature has revealed that, multicollinearity often affects the efficiency of SUR estimators. One of the popular methods for coping with Multicollinearity problem is ridge regression estimation. In this study, some alternative ridge estimators for SUR parameters are proposed when the explanatory variables are affected by multicollinearity. The efficiency of the proposed estimators is evaluated and compared through simulation study, in terms of the Trace Mean Squared Error (TMSE) and the Proportion of Replications, (PR) criterion, under a variety of data conditions. The empirical results indicated that, under certain conditions, the performance of the multivariate regression estimators based on some SUR ridge parameters are superior to other estimators in terms of TMSE and PR criterion. In large samples and when the collinearity between the explanatory variables is not high, the unbiased SUR estimator produces a smaller TMSEs.

Key words: Multicollinearity, SUR regression, Monte carlo simulations, biased estimators, generalized least squares

INTRODUCTION

The Seemingly Unrelated Regressions (SUR) model proposed by Zellner (1962) is considered as one of the most successful and efficient methods for estimating seemingly unrelated regressions and tests of aggregation bias. The resulting (SUR) model has simulated a countless theoretical and empirical results in econometrics and other areas, (Zellner, 1962; Brown and Zidek, 1980; Srivastava and Giles, 1987; Saleh and Kibria, 1993; Fiebig and Kim, 2000). For example the methodology is applicable to allocation models, demand functions for a number of commodities; investment functions for a number of firms, income or consumption functions for subsets of populations or different regions, to mention some.

In most of the empirical studies, researchers are often concerned about problems with the specification of the model or problems with the data. In the sequel, our interest lies in data type problem, namely multicollinearity which arises in situations when the explanatory variables are highly inter-correlated. Then, it becomes difficult to determine the separate effects of each of the explanatory variables on the explained variable. As a result, the estimated parameters may be statistically insignificant and/or have (unexpectedly) different signs. Thus, to conduct meaningful statistical inference would be difficult for the researcher. One such remedial estimation technique is ridge regression. The class of ridge regression estimators was originally developed to deal with the problem of Multicollinearity in the linear regression model and contains estimators which although may be biased may have smaller MSE_s than their unbiased counterparts. Much of the discussions on ridge regression concern the problem of multicollinearity have been proposed by

Gujerati (2002), Kibria (2003), Alkhamisi *et al.* (2006), Alabi *et al.* (2008), Agunbiade and Iyaniwura (2010), D'Ambra and Sarnacchiaro (2010), Bagheri *et al.* (2010), Batah *et al.* (2009), Camminatiello and Lucademo (2010) and Oluwayemisi *et al.* (2010).

The objective of the study is to investigate the properties of some alternative proposed estimators associated with system-wise ridge estimation using different multivariate ridge parameters, since this topic is often only briefly mentioned in the literature and to make a comparison among them with other known existing estimators based on terms of TMSE and PR criterion.

Alternative SUR-Ridge type estimators: Consider a system of M equations given by:

$$Y_i = X_i \boldsymbol{\beta}_i + e_i, \quad i = 1, 2, \ldots\ldots, M \tag{1}$$

where, Y_i is a (T×1) vector of observations on the dependent variable, e_i is a (T×1) vector of random errors with $E(e_i) = 0$, X_i is a $(T\times n_i)$ matrix of observations on n_i explanatory variables including a constant term and $\boldsymbol{\beta}_i$ is a $(n_i\times 1)$ dimensional vector of unknown location parameters. M is the number of equations in the system, T is the number of observation per equation and n_i is the number of rows in the vector $\boldsymbol{\beta}_i$.

Let $Y=(Y_1', Y_2', \ldots\ldots\ldots, Y_M'), X=\text{diag}(X_1, X_2, \ldots\ldots\ldots, X_M)$ and similarly e and $\boldsymbol{\beta}$ are defined. Then the M equations in (1) can be written in the compact form as:

$$Y = X\boldsymbol{\beta} + e \tag{2}$$

where, Y and e are each of dimension (TM×1), X is of dimension (TM×n), $n=\sum_{i=1}^{M} n_i$ and $\boldsymbol{\beta}$ is a n-dimensional vector of location parameters.

Furthermore, consider the following assumptions:

- X_i is fixed with rank n_i
- Plim $\frac{1}{T}(X_i' X_j)=Q_{ii}$ is non-singular with finite and fixed elements, i.e., invertable
- Assume that Plim $\frac{1}{T}(X_i' X_j)=Q_{ii}$ is non-singular with finite and fixed elements
- E $(e_i e_j')\, \sigma_{ij} I_T$ where σ_{ij} designate the covariance between the ith and jth Equations for each observation in the sample. The above expression can be written as:

E (e) =0 and E (ee'), $\Sigma\otimes I_T$, where:

$$\Sigma=\begin{bmatrix} \sigma_{11} & \sigma_{12},\cdots & \sigma_{1M} \\ \sigma_{21} & \sigma_{22},\cdots & \sigma_{2M} \\ \vdots & \vdots & \vdots \\ \sigma_{M1} & \sigma_{M2},\cdots & \sigma_{MM} \end{bmatrix}$$

is an (M×M) positive definite symmetric matrix and represent the kronecker product.

Thus, the errors at each equation are assumed to be homoscedastic and not autocorrelated but that there is contemporaneous correlation between corresponding errors in different equations.

The OLS estimator of $\boldsymbol{\beta}$ in Eq. 2 is:

$$\hat{\beta}=(X'Y)^{-1}X,Y \text{ with } Var(\hat{\beta})=(X'X)^{-1}X'(\Sigma^{-1}\otimes I_T)X(X')^{-1} \tag{3}$$

In the context of SUR model, the general ridge regression estimator of $\boldsymbol{\beta}$ is :

$$\hat{\beta}_{DR}=(X'X+R)^{-1}X'Y \tag{4}$$

where, R is an (n×n) matrix of nonnegative elements. However, because the ridge estimator in Eq. 4 does not include the cross equation correlation among errors (Srivastava and Giles,1987), thus, the following transformation is suggested:

$$Y^* = (\boldsymbol{\Sigma}^{-1/2}\otimes I_T)Y,\ X^* = (\boldsymbol{\Sigma}^{-1/2}\otimes I_T)\ X \text{ and } e^* = (\boldsymbol{\Sigma}^{-1/2}\otimes I_T)\ e$$

Using the above transformation, the model in Eq. 2 can be expressed as:

$$Y^* = X^*\ \boldsymbol{\beta}+e \tag{5}$$

where, Y^* and e^* are (MT×1) vectors, X^* is an (MT×n) matrix, E $(e^{*\prime})=I_{TM}$ and E $(e^*)=0$.

Accordingly, the Generalized Least Squares (GLS) estimator of $\boldsymbol{\beta}$ in (5) and its ridge version are, respectively given by:

$$\hat{\beta}_{DR}=(X^{*\prime}X^*)^{-1}X^{*\prime}Y^* \tag{6}$$

$$\hat{\beta}_{GR}=(X^{*\prime}X^*+R)^{-1}X^{*\prime}Y^{*\prime} \text{with MSE } (\hat{\beta}_{GR})=(X^{*\prime}X^*+R)^{1}(R\beta\beta'R'+X^{*\prime}X^*)(X^{*\prime}X^*+R)^{1} \tag{7}$$

Set R = 0 in the expression for MSE ($\hat{\beta}_{GR}$) to obtain an expression for MSE ($\hat{\beta}_G$). let A and $\boldsymbol{\Psi}$ designate the eigen values and eigenvectors of, respectively. Then $\boldsymbol{\Psi}'$ $(X^{*\prime}X^*)$ $\boldsymbol{\Psi}$ = A and the canonical version of model (5) is given by:

$$Y^* = Z\alpha + e^* \tag{8}$$

where, $Z = X^*\boldsymbol{\Psi}$, $\alpha = \boldsymbol{\Psi}'\boldsymbol{\beta}$ and $Z'Z = (\boldsymbol{\Psi}'X^{*\prime}X^*\boldsymbol{\Psi}) = A$.

The corresponding GLS estimator of α is:

$$\hat{\alpha}=(Z'Z+R)^{-1}Z'Y^* \tag{9}$$

and the expression for the corresponding SUR-ridge regression parameter is:

$$\hat{\alpha}_{SUR}=(Z'Z+R)^{-1}Z'Y^* \tag{10}$$

where, $R = diag(R_1, R_2, ..., R_M)$, $R_i = diag(r_{i1}, r_{i2}, ..., r_{in})$ and $r_{ij} > 0$ for i = 1, 2, ..., M. Moreover, the bias vector, the mean squared error matrix of and trace of MSE ($\hat{\alpha}_{SUR}$) (TMSE) are, respectively given by:

$$E(\hat{\alpha}_{SUR}) - \alpha = -(Z'Z+R)^{-1} R\alpha, \quad MSE(\hat{\alpha}_{SUR}) = E(\hat{\alpha}_{SUR}-\alpha)(\hat{\alpha}_{SUR}-\alpha)' \tag{11}$$

$$\begin{aligned} MSE(\hat{\alpha}_{SUR}) &= E(\hat{\alpha}_{SUR}-\alpha)(\hat{\alpha}_{SUR}-\alpha)' = [(A+R)^{-1} A-I]\, \alpha\alpha' \,[(A+R)^{-1} A-I]'+(A+R)^{-1} A (A+R)^{-1} \\ &= [(A+R)^{-1} (A+R\, \alpha\alpha'\, R') (A+R)^{-1} \end{aligned} \tag{12}$$

and

$$\underset{i=1}{\overset{M}{\text{TMSE}}}(\hat{\alpha}_{\underset{j=1}{\overset{ni}{SUR}}}(R)) = \Sigma\Sigma \frac{\lambda_{ij} + r_{ij}^2\, r\alpha_{ij}^2}{(\lambda_{ij} + r_{ij})^2} \tag{13}$$

Now, set $\frac{\partial \text{TMSE}(\hat{\alpha}_{SUR})}{\partial_{rij}} = 0$, to determine the optimum values of r_{ij} as:

$$r_{ij} = \frac{1}{\hat{\alpha}_{ij}^2} \tag{14}$$

Moreover, conditions to ensure the superiority of $\hat{\alpha}_{SUR}$ (R) over $\hat{\alpha}$ with respect to the MSE criterion are given in the following result.

Result 1:

- MSE ($\hat{\alpha}$)-MSE ($\hat{\alpha}_{SUR}$ (R)) is a positive semidefinite matrix if:

$$\alpha' (A^{-1} + 2R^{-1})^{-1} \alpha \le 1 \tag{15}$$

- Sufficient conditions for Eq. 15 to hold are:

$$\text{(i) } \alpha' A \alpha \le 1 \text{ (ii) } \alpha' R \alpha \le 2 \tag{16}$$

Set R = rI in Eq. 10, to show that MSE ($\hat{\alpha}$)-MSE ($\hat{\alpha}_{SUR}$ (R) is a positive semidefinite matrix if:

$$r \le \frac{2}{\alpha'\alpha} \tag{17}$$

The following result presents some alternative methods for constructing SUR-ridge parameters:

Result 2: For j = 1, ..., n_i ; i = 1, ..., M, assume Eq. 14 holds, then:

- R_{SK}. The ij-th component of this matrix is given by Eq. 14
- R_{SHK}. Denotes the SUR version of ordinary ridge parameter as:

$$r_{ij\,(SHK)} = \frac{1}{\max(\hat{\alpha}_{ij}^2)} \tag{18}$$

- R_{Sharm}. Designates the SUR version to the harmonic mean as:

$$r_{ij\ (Sharm)} = \frac{n}{\sum_{i=1}^{M}\sum_{j=1}^{ni}} = \frac{n}{\sum_{i=1}^{M}\sum_{j=1}^{ni}\frac{1}{\hat{\alpha}_{ij}^{2}}} \tag{19}$$

- R_{Sarith}. A SUR extension to the single equation arithmetic mean is:

$$r_{ij\ (Sarith)} = \frac{1}{n}\sum_{i=1}\sum_{j=1}\frac{1}{\hat{\alpha}_{ij}^{2}} \tag{20}$$

- R_{Sgeom}. A generalization to the single equation geometric mean is:

$$r_{ij\ (Sgeom)} = \frac{1}{(\prod_{i=1}^{M}\prod_{I=1}^{ni}\hat{\alpha}_{ij}^{2})^{\frac{1}{n}}} \tag{21}$$

- R_{skmed}. The median of r_{ij} in Eq. 14 is used to define this parameter (Kibria, 2003) for a single equation version as:

$$r_{ij\ (Skmed)} = \text{median}\left(\frac{1}{\hat{\alpha}_{ij}^{2}}\right) \tag{22}$$

- $R_{Sqarith}$. The arithmetic mean of $\sqrt{r_{ij}}$, with r_{ij} as defined in Eq. 14 is used to define this parameter as:

$$r_{ij\ (Sqarith)} = \text{mean}\left(\frac{1}{\sqrt{\hat{\alpha}_{ij}^{2}}}\right) \tag{23}$$

- R_{Sqmax}. Based on the maximization of, with r_{ij} as defined in Eq. 14 this parameter is:

$$r_{ij\ (Sqmax)} = \max\left(\frac{1}{\sqrt{\hat{\alpha}_{ij}^{2}}}\right) \tag{24}$$

- R_{Smax}. A generalization to the single equation ridge parameter is:

$$r_{ij\ (Smax)} = \max\left(\frac{1}{\hat{\alpha}_{ij}^{2}}\right) \tag{25}$$

The last three ridge parameters are considered the proposed alternative estimators presented in this study and all of the ridge estimators defined by Eq. 18-22 are identical to R_{SHK} when $\hat{\alpha}_{ij}^2$ is replaced by max ($\hat{\alpha}_{ij}^2$). The estimators in Eq. 18-20 have already been considered by Firinguetti (1997).

In order to assess the performance of multivariate ridge regression estimators defined in terms of the above proposed multivariate ridge estimators, a Monte Carlo experiment can be used to compare them in terms of TMSE with the GLS estimator, Eq. 6 and the general ridge regression estimator defined by Eq. 7 and 14.

The Monte Carlo experiment: In order to evaluate the performance of the alternative different SUR ridge type estimators of the unknown vector parameter $\boldsymbol{\beta}$, the TMSE and the PR criteria were considered to measure the goodness of an estimator of $\boldsymbol{\beta}$, say $\hat{\beta}$.

The total mean square error (TMSE) is defined as:

$$\text{TMSE}(\tilde{\beta}) = \text{Trace}\,[E(\tilde{\beta}-\beta)(\tilde{\beta}-\beta)']$$

The PR criterion counts the proportion of replications, (out of 1000), for which the SUR version of generalized least square estimator (SGLS) produces a smaller TMSE than the remaining multivariate ridge estimators. In Table 1-4, these numbers are placed in parenthesis.The performance of the different SUR ridge estimators, under consideration, are examined via Monte Carlo simulations. The Monte Carlo experiment has been performed by generating data in accordance with the following Equation:

$$Y_{ti} = \sum_{j=1}^{5} x_{tij}\beta_{ij} + e_{ti},\ t=1,2,..,T;\ i=1,2,...,M \tag{26}$$

where, x_{ti1}=1. The explanatory variables are generated from MVN_4 (0, $\boldsymbol{\Sigma}_x$) and multivariate T(6), respectively. The random errors were generated from MVN_m (0, $\boldsymbol{\Sigma}_e$), m = 3, 10.

Algorithm: The simulation algorithm is based on the following steps:

- Generate the explanatory variables from MVN_4 (0, $\boldsymbol{\Sigma}_x$) or T (6)
- Set initial value of $\boldsymbol{\beta}$ either to (1, 1, 1, 1, 1)' or $(1, 2, 3, 4, 5)^t$
- Simulate the vector random error e from MVN_m (0, $\boldsymbol{\Sigma}_e$), m= 3, 10
- As outlined earlier, for a given X structure, transform the original model (2) to an orthogonal form given by Eq. 8 and calculate the SGLS estimator along with $\hat{\alpha}_{SUR}$ (R), R = R_{SK}, R_{SHK}, R_{Sharm}, R_{sarith}, R_{Sgeom}, R_{Skmed}, $R_{Sqarith}$, R_{Sqmax} and R_{Smax}. Then compute the corresponding total mean squared error for the above cases, respectively
- Repeat this process 1000 times and then calculate the average of the mean squared error and the (PR) for each ridge parameter R, under consideration. Values of total mean squared error and PR criterion are given in Table 1-4

Factors: In order to evaluate the performance of the proposed SUR ridge regression estimators $\hat{\alpha}_{SUR}$ (R) and compare them with the SGLS estimator $\hat{\alpha}$, some factors are used for designing the

Table 1: Estimated TMSEs and PRS for different methods, M = 3 equations (normal)

ρ_X	SGLS	SK	SHK	Sharm	Sarith	Sgeom	Skamed	Sqarith	Sqmax	Smax
T = 30, ρ_Σ = 0.35										
0.75	25.31	53.15	54.22	50.18	33.29	93.82	41.73	33.09	37.87	33.49
		(99.8)	(99.8)	(99.7)	(79.5)	(96.2)	(97.1)	(79.2)	(91.6)	(79.5)
0.90	49.89	95.4	98.32	86.95	37.81	55.13	60.05	39.29	45.88	37.39
		(100.0)	(100.00)	(99.9)	(33.0)	(70.3)	(77.6)	(31.40)	(49.2)	(33.6)
0.97	145.00	261.42	271.18	228.71	40.63	88.76	100.55	39.47	50.79	40.07
		(98.9)	(99.2)	(96.9)	(2.6)	(6.2)	(17.5)	(2.5)	(3.1)	(2.6)
0.99	416.54	737.08	766.95	634.85	41.44	143.92	182.27	40.30	49.45	41.53
		(96.7)	(98.4)	(88.6)	(0.0)	(0.1)	(3.3)	(0.0)	(0.0)	(0.0)
T = 100, ρ_Σ = 0.35										
0.75	14.49	40.93	41.32	40.45	37.84	38.34	38.83	37.19	38.47	39.18
		(99.9)	(99.9)	(99.9)	(99.9)	(99.9)	(99.9)	(99.9)	(99.9)	(99.9)
0.90	20.89	54.20	55.10	52.18	40.68	44.82	46.47	40.21	43.83	41.66
		(100.0)	(100.0)	(100.0)	(97.9)	(99.9)	(99.9)	(98.0)	(99.8)	(98.0)
0.97	45.67	96.66	99.61	88.65	42.84	57.39	63.25	42.20	49.23	43.43
		(100.0)	(100.0)	(100.0)	(49.7)	(83.6)	(90.2)	(48.6)	(64.1)	(50.9)
0.99	116.42	216.51	225.48	190.01	43.49	77.67	93.15	42.62	50.42	44.25
		(99.8)	(99.9)	(99.3)	(5.2)	(10.2)	(27.6)	(4.6)	(5.3)	(5.9)
T = 30, ρ_Σ = 0.75										
0.75	26.39	55.05	55.73	52.89	36.33	43.49	44.44	36.69	42.09	35.43
		(99.8)	(99.8)	(99.8)	(84.6)	(97.9)	(98.2)	(85.9)	(95.3)	(82.4)
0.90	50.19	99.16	100.94	93.09	41.13	62.23	65.12	41.30	52.77	39.34
		(99.8)	(99.8)	(99.6)	(37.7)	(81.6)	(83.2)	(38.0)	(62.2)	(34.8)
0.97	142.20	262.65	268.87	240.51	44.13	107.26	121.26	43.36	61.40	42.07
		(99.0)	(99.3)	(98.1)	(2.5)	(21.8)	(36.2)	(2.4)	(4.6)	(2.8)
0.99	404.89	720.38	739.31	654.43	44.55	188.39	244.52	43.35	59.96	46.23
		(96.7)	(97.4)	(93.2)	(0.2)	(0.8)	(13.0)	(0.2)	(0.2)	(0.2)
T = 100, ρ_Σ = 0.75										
0.75	16.45	43.62	43.82	43.31	40.57	41.75	42.17	40.46	41.80	4094.0
		(100.0)	(100.0)	(100.0)	(99.8)	(99.9)	(99.9)	(99.8)	(99.9)	(99.9)
0.90	22.93	58.90	59.41	57.68	45.00	51.46	52.66	45.34	50.31	44.73
		(99.8)	(99.8)	(99.8)	(99.7)	(99.8)	(99.7)	(99.1)	(99.7)	(99.7)
0.97	47.98	107.07	108.81	101.93	47.60	71.01	76.54	47.70	59.64	46.75
		(100.0)	(100.0)	(100.0)	(54.3)	(95.5)	(96.2)	(54.6)	(78.1)	(52.1)
0.99	119.50	237.40	242.70	221.25	48.09	107.56	127.15	47.62	64.29	47.34
		(99.7)	(99.7)	(99.5)	(5.9)	(37.7)	(60.9)	(5.7)	(8.8)	(6.1)

The PR values (in percentage) are placed in parenthesis below the values of the corresponding estimators

Monte Carlo experiment. Details of these factors and values over which these factors were allowed to vary, are as follows.

Number of equations: It is important that the number of equation, M, to be estimated is of central importance for the analysis of system-wise estimation. Therefore, as M increases, the computation time becomes larger and larger. Thus, M = 10 and 3 can be used to designate large and small models, respectively.

Number of observations per equation: To investigate the effect of sample size, T, on the properties of the suggested SUR-ridge parameters, T = 30 and 100, as the number of observations per equation.

Table 2: Estimated TMSEs and PRS for different methods, M = 10 equations (normal)

ρ_X	SGLS	SK	SHK	Sharm	Sarith	Sgeom	Skamed	Sqarith	Sqmax	Smax
T = 30, ρ_Σ = 0.35										
0.75	132.56	290.16	295.09	271.28	133.44	198.44	209.72	133.91	171.70	131.48
		(100.0)	(100.0)	(100.0)	(52.6)	(99.5)	(99.8)	(53.2)	(89.1)	(49.5)
0.90	283.01	599..89	612.44	551.99	142.29	311.63	341.25	141.99	206.04	139.20
		(100.0)	(100.0)	(100.0)	(0.8)	(72.9)	(85.3)	(0.9)	(11.9)	(0.7)
0.97	865.72	185.65	1849.85	1681.28	160.57	252.78	745.39	158.69	238.05	158.69
		(100.0)	(100.0)	(99.8)	(0.0)	(4.9)	(25.7)	(0.0)	(0.0)	(0.0)
0.99	2529.78	5494.19	525.06	4954.89	165.89	1197.87	1629.72	164.15	228.02	166.69
		(99.9)	(99.9)	(99.7)	(0.0)	(0.0)	(7.30)	(0.0)	(0.0)	(0.0)
T = 100, ρ_Σ = 0.35										
0.75	61.87	177.66	179.26	173.09	153.04	158.01	160.52	151.19	157.49	15.86
		(100.0)	(100.0)	(100.0)	(100.0)	(100.0)	(100.0)	(100.0)	(100.0)	(100.0)
0.90	94.24	253.57	257.74	240.65	165.66	189.47	197.57	163.87	179.39	169.09
		(100.0)	(100.0)	(100.0)	(99.4)	(100.0)	(100.0)	(99.2)	(99.8)	(99.4)
0.97	219.50	534.33	548.46	486.05	185.10	259.20	290.85	182.54	205.38	190.10
		(100.0)	(100.0)	(100.0)	(28.7)	(85.6)	(95.6)	(26.9)	(41.0)	(31.8)
0.99	577.18	1272.14	1314.77	1125.88	190.09	353.45	452.80	187.28	205.65	195.66
		(100.0)	(100.0)	(99.9)	(0.1)	(0.3)	(13.2)	(0.1)	(0.1)	(0.1)
T = 30, ρ_Σ = 0.75										
0.75	131.62	309.43	312.04	298.52	157.84	238.26	248.66	159.93	210.99	151.93
		(100.0)	(100.0)	(100.0)	(78.2)	(100.0)	(100.0)	(80.3)	(98.6)	(71.1)
0.90	278.45	625.13	632.12	595.55	163.59	385.21	413.96	165.01	265.82	153.99
		(99.9)	(99.0)	(99.9)	(3.2)	(96.9)	(98.3)	(3.1)	(46.5)	(2.2)
0.97	847.05	1833.91	1857.96	1735.92	166.92	822.39	247.47	166.11	318.05	158.05
		(100.0)	(100.0)	(100.0)	(0.0)	(40.7)	(65.4)	(0.0)	(0.0)	(0.0)
0.99	2470.87	5448.49	5521.43	5152.42	181.31	1772.08	2286.40	179.50	328.34	176.84
		(99.8)	(99.9)	(99.7)	(0.0)	(7.2)	(36.2)	(0.0)	(0.0)	(0.0)
T = 100, ρ_Σ = 0.75										
0.75	63.10	203.26	204.04	201.14	182.25	190.70	190.73	181.67	190.21	183.21
		(100.0)	(100.0)	(100.0)	(100.0)	(100.0)	(100.0)	(100.0)	(100.0)	(100.0)
0.90	95.16	282.82	284.86	275.71	192.62	232.66	240.09	192.60	220.47	193.67
		(100.0)	(100.0)	(100.0)	(100.0)	(100.0)	(100.0)	(100.0)	(100.0)	(100.0)
0.97	219.25	530.41	537.34	505.79	178.56	313.12	345.29	178.64	232.41	178.81
		(100.0)	(100.0)	(100.0)	(24.6)	(99.2)	(99.7)	(23.9)	(62.6)	(24.5)
0.99	573.58	1296.63	1317.74	1223.63	202.61	532.80	625.17	201.15	261.26	205.13
		(100.0)	(100.0)	(100.0)	(0.0)	(30.3)	(70.8)	(0.0	(0.1)	(0.0)

The PR values (in percentage) are placed in parenthesis below the values of the corresponding estimators

True value of the regression coefficient β: The true values of β = (1, 1, 1, 1, 1)' and β = (1, 2, 3, 4, 5)

Distribution of X and collinearity among columns of X: Another factor that may affect the performance of the suggested SUR-ridge parameters is the strength and type of dependency among the explanatory variables. The explanatory variables were generated from a multivariate normal distribution, $MVN_4(0,\Sigma_x)$ and multivariate T distribution, T (6), respectively. The variance-covariance matrix Σ_x is defined as diag (Σ_x) = 1. and off-diag (Σ_x) = ñ$_x$. The strength of collinearity among these variables took on these values ñ$_x$ = 0.75, 0.90, 0.97 and 0.99, (for medium, high and very high).

Table 3: Estimated TMSEs and PRS for different methods, M = 3 equations T (6)

ρ_X	SGLS	SK	SHK	Sharm	Sarith	Sgeom	Skamed	Sqarith	Sqmax	Smax
T = 30, ρ_Σ = 0.35										
0.75	109.55	154.05	155.09	150.15	37.48	109.46	113.93	39.37	63.02	33.21
		(97.0)	(97.3)	(96.2)	(50.2)	(74.4)	(78.2)	(51.2)	(64.9)	(48.5)
0.90	271.56	381.65	384.36	369.85	44.81	212.84	231.44	45.78	82.00	37.81
		(98.0)	(98.4)	(95.5)	(25.0)	(57.4)	(57.4)	(24.4)	(35.9)	(23.5)
0.97	898.65	1414.34	1423.72	1370.78	59.61	540.78	493.29	54.50	117.68	42.49
		(95.6)	(97.5)	(91.3)	(3.8	(18.0)	(23.4)	(3.7)	(6.1)	(3.7)
0.99	2689.28	3764.09	3792.72	3629.72	59.59	1102.21	953.31	52.02	128.21	43.79
		(92.7)	(95.0)	(83.2)	(0.1)	(4.9)	(9.3)	(0.1)	(0.3)	(0.2)
T = 100, ρ_Σ = 0.35										
0.75	86.72	131.73	132.07	130.94	36.53	122.22	124.85	39.43	53.97	36.35
		(99.1)	(99.1)	(99.1)	(86.9)	(96.0)	(96.8)	(87.3)	(92.6)	(86.1)
0.90	216.03	427.21	428.21	424.38	41.28	368.93	395.73	44.35	69.34	40.10
		(99.4)	(99.5)	(99.4)	(73.2)	(89.7)	(91.9)	(74.3)	(83.3)	(72.6)
0.97	716.41	1707.49	1710.29	1696.38	45.88	1272.57	1546.07	47.92	92.97	42.30
		(98.4)	(98.4)	(07.8)	(32.7)	(57.4)	(61.4)	(32.6)	(42.5)	(32.9)
0.99	2145.16	5132.32	5140.85	5096.04	67.86	3655.04	4490.23	57.46	142.68	46.25
		(97.5)	(97.7)	(93.0)	(6.4)	(17.9)	(25.5)	(6.1)	(7.8)	(7.0)
T = 30, ρ_Σ = 0.75										
0.75	113.75	19.57	191.20	188.09	41.65	151.10	159.75	44.72	70.15	36.09
		(98.0)	(98.0)	(97.8)	(54.5)	(83.2)	(83.9)	(56.0)	(72.3)	(50.5)
0.90	282.16	463.45	465.09	456.15	54.75	339.34	352.55	56.42	105.45	41.42
		(97.9)	(98.0)	(97.5)	(28.4)	(66.2)	(66.3)	(27.5)	(45.7)	(25.1)
0.97	933.75	1310.35	1316.00	1282.48	72.29	820.48	818.11	65.88	167.84	45.93
		(96.3)	(97.0)	(94.1)	(5.7)	(30.7)	(37.3)	(5.4)	(10.8)	(4.8)
0.99	2794.25	4213.37	4239.65	4130.65	79.20	2322.33	2013.20	63.18	180.30	47.53
		(94.8)	(95.3)	(90.0)	(0.4)	(11.6)	(15.8)	(0.3)	(0.6)	(0.4)
T = 100, ρ_Σ = 0.75										
0.75	83.71	184.50	184.68	184.03	42.51	176.62	179.49	49.25	74.18	39.4
		(99.6)	(99.6)	(99.6)	(87.6)	(97.8)	(98.3)	(89.7)	(94.8)	(86.8)
0.90	201.78	310.27	310.74	308.61	50.00	279.15	284.76	57.98	99.81	44.00
		(99.4)	(99.4)	(99.4)	(75.3)	(95.2)	(95.4)	(77.7)	(89.0)	(72.2)
0.97	658.05	1103.05	1104.67	1096.71	59.91	958.27	983.56	67.97	170.21	46.92
		(98.2)	(98.3)	(97.9)	(35.8)	(74.3)	(73.7)	(35.6)	(53.0)	(32.2)
0.99	1960.66	2998.51	3003.42	2978.52	119.26	2485.17	2500.74	93.64	296.72	57.14
		(96.8)	(97.3)	(95.3)	(5.6)	(37.5)	(41.6)	(5.1)	(12.8)	(5.4)

The PR values (in percentage) are placed in parenthesis below the values of the corresponding estimators

Distribution of random errors and correlation among equations: The random errors were generate from a multivariate normal distribution $MVN_m(0, \Sigma_e)$, where m = 3 or 10 equations. The variance-covariance matrix Σ_e is defined as diag $\Sigma_e = 1$ and off-diag $(\Sigma_e) = \rho_\Sigma$. Two different degrees of interdependency among these equations were considered. These values are $\rho_\Sigma = 0.35$ and 0.75, for low and high interdependency, respectively.

For each model, 1000 replications using the statistical software S-plus version 6 were performed (Mardikyan and Darcan, 2006). Table 1-4, present the estimated TMSEs and PRs for different methods (SGLS estimator along with $\hat{\alpha}_{SUR}$ (R), R = R_{SK}, R_{SHK}, R_{Sharm}, R_{sarith}, R_{Sgeom}, R_{Skmed}, $R_{Sqarith}$, R_{Sqmax} and R_{Smax}), with M= 3 and 10 equations, using multivariate normal distribution, $MVN_4(0, \Sigma_x)$ and multivariate T distribution, T (6), respectively.

Table 4: Estimated TMSEs and PRs for different methods, M =10 equations T (6)

ρX	SGLS	SK	SHK	Sharm	Sarith	Sgeom	Skamed	Sqarith	Sqmax	Smax
T = 30, ρΣ = 0.35										
0.75	745.87	1386.51	1391.17	1367.7	172.41	1105.46	1130.3	174.94	509.93	129.15
		(96.9)	(97.1)	(96.5)	(5.9)	(75.7)	(81.7)	(5.7)	(38.2)	(4.0)
0.9	1937.55	3178.82	3190.73	3123.1	216.07	2346.6	2420.95	202.28	745.35	140.54
		(97.8)	(97.9)	(7.1)	(0.0)	(49.3)	(59.0)	(0.0)	(5)	(0.0)
0.97	6552.52	12730.1	12771.2	12519.8	263.49	8311.1	84540.64	211.89	1172.73	153.53
		(99.0)	(99.0)	(98.6)	(0.0)	(20.9)	(31.1)	(0.0)	(0.0)	(0.0)
0.99	19731.40	36585.7	36710.3	35920.1	283.16	20760.5	21546.8	209.22	1305.83	163.74
		(99.0)	(99.2)	(98.8)	(0.0)	(10.9)	(17.2)	(0.0)	(0.0)	(0.0)
T = 100, ρΣ = 0.35										
0.75	208.25	402.09	403.57	397.27	156.42	335.52	348.73	162.67	257.03	154.64
		(99.5)	(99.5	(99.5	(76.2	(96.1	(97.4	(77.1	(90.2	(75.5
0.9	485.51	839.78	843.68	824.91	171.81	627.59	661.03	180.98	363.6	163.95
		(98.7)	(98.8)	(98.3)	(38.7)	(84.7)	(87.8)	(39.2)	(64.9)	(39.4)
0.97	1557.64	2795	2808.74	2737.93	197.32	1680.67	1805.3	202	539.55	117.34
		(99.2)	(99.2)	(99.1)	(1.4)	(33.6)	(44.6)	(1.4)	(5.5)	(1.7)
0.99	4618.70	8077.39	8119.6	7892.4	212.02	4161	4441.68	210.02	656.81	186.25
		(99.4)	(99.4)	(99.2)	(0.0)	(6.2)	(13.2)	(0.0)	(0.0)	(0.0)
T = 30, ρΣ = 0.75										
0.75	1096.77	1743.12	1745.51	1732.38	263.56	1588.9	1616.7	281.09	946.39	153.9
		(97.7)	(97.7)	(97.3)	(14.2)	(91.6)	(93.3)	(14.2)	(64.8)	(7.8)
0.9	2929.46	4459.1	4465.57	4428.75	373.62	3859.91	3941.18	341.73	1627.85	167.97
		(98.3)	(98.3)	(98.1)	(0.9)	(76.3)	(82.0)	(0.6)	(18.5)	(0.5)
0.97	10036.80	17990.4	18012.8	17873.4	1039.92	14615	15041	530	3424.89	252.64
		(98.8)	(98.8)	(98.6)	(0.0)	(48.1)	(58.1)	(0.0)	(0.2)	(0.0)
0.99	30336.50	48478.5	48546.7	48128.8	1635.43	36591.7	37358.4	782.96	5506.74	284.75
		(98.9)	(98.9)	(98.2)	(0.0)	(27.9)	(41.7)	(0.0)	(0.0)	(0.0)
T = 100, ρΣ = 0.75										
0.75	215.34	415.65	416.28	413.39	189.18	380.11	387.47	207.24	322.81	176.27
		(99.4)	(99.4)	(99.4)	(83.1)	(98.8)	(98.9)	(85.0)	(96.5)	(80.5)
0.9	502.92	962.91	964.69	955.97	213.38	823.44	844.39	237.28	549.28	186.42
		(99.5)	(99.5)	(99.4)	(50.3)	(96.1)	(96.5)	(51.9)	(82.8)	(47.5)
0.97	1615.10	2994.24	3000.65	2966.35	235.89	2243.27	2328.69	251.75	913.76	186.74
		(99.1)	(99.1)	(99.1)	(1.8)	(72.1)	(76.1)	(1.8)	(18.3)	(1.7)
0.99	4790.60	8416.67	8436.51	8330.57	275.45	5580.19	571.38	258.04	1304.36	194.48
		(99.5	(99.5	(99.5	(0.0)	(29.9)	(41.7)	(0.0)	(0.1)	(0.0)

The PR values (in percentage) are placed in parenthesis below the values of the corresponding estimators

RESULTS AND DISCUSSION

This section is devoted to explain the output from the Monte Carlo experiment along with the main dominating factors effecting the properties of the different multivariate ridge parameters. These different estimators are:

- $\hat{\alpha}$ (SGLS), using Eq. 10 (Zellner, 1962)
- $\hat{\alpha}$ (R_{SK}), $\hat{\alpha}$ (R_{SHK}) and $\hat{\alpha}$ (R_{Sharm}) using Eq. 10,18 and 19 (Brown and Zidek, 1980; Srivastava and Giles,1987; Firinguetti, 1997)
- $\hat{\alpha}$ (R_{Sarith}), $\hat{\alpha}$ (R_{Sgeom}) and $\hat{\alpha}$ (R_{Skamed}) using Eq. 20 and 22 (Kibria, 2003; Alkhamisi *et al.*, 2006).
- The proposed estimators: $\hat{\alpha}$ ($R_{Sqarith}$), $\hat{\alpha}$ (R_{Sqmax}) and $\hat{\alpha}$ (R_{Smax}) using Eq. 23-25

The results of Table 1 pointed out that, a slight increase in the TMSE values for $\hat{\alpha}$ (R_{Sarith}), $\hat{\alpha}$ ($R_{sqarith}$) and $\hat{\alpha}$ (R_{Smax}), as the sample size increases. These multivariate ridge regression estimators have shown to have the best performance in terms of TMSE and PR criterion when compared with the remaining proposed multivariate ridge regression estimators. Moreover, multivariate ridge regression estimators based on R_{SK}, R_{SHK} and R_{Sharm} have produced the highest TMSE and the worst PR values among other estimators. In large samples, when the correlation among the explanatory variables (ρ_X) is low, the unbiased estimator $\hat{\alpha}$, SGLS of α, has occasionally shown to have the smallest TMSE among the remaining estimators, especially, when T = 100. In addition, when T = 30, 100, ρ_Σ = 0.35 and ρ_Σ =0.97, 0.99, the $\hat{\alpha}$ ($R_{Sqarith}$), have the best performance in terms of TMSE and PR criterion compared to the remaining proposed multivariate ridge regression estimators. While, the $\hat{\alpha}$ (R_{Smax}) have the best performance in terms of TMSE and PR criterion compared to the remaining proposed multivariate ridge regression estimators, for T = 30,when ρ_Σ =0.90, ρ_Σ = 0.35 and for T= 30, when ρ_Σ =0.90,0.97,0.99, ρ_Σ = 0.75.

From the results of Table 2, the unbiased estimator $\hat{\alpha}$, SGLS of α, has the smallest TMSE among the remaining estimators, when T = 100, ρ_Σ = 0.75 and ρ_Σ =0.75, 0.90. In addition, the $\hat{\alpha}$ (R_{Smax}) have the best performance in terms of TMSE and PR criterion compared to the remaining proposed multivariate ridge regression estimators, for T = 30, when ρ_Σ = 0.75, 0.90, ρ_Σ = 0.35 and for T= 30, when ρ_Σ = 0.90, 0.97, 0.99, ρ_Σ = 0.75 which, nearly the same result obtained from Table 1 when the number of equations (M = 3).

The results of Table 3 and 4 pointed out that the $\hat{\alpha}$ (R_{Smax}) have the best performance in terms of TMSE and PR criterion compared to the remaining proposed multivariate ridge regression estimators for all values of T, ρ_Σ and ρ_Σ.

To conclude, the results of Table 1 and 4, in all cases, pointed out that, the performance of the multivariate ridge parameters, based on terms of all factors (the number of equations (M), number of observations (T), correlation among the explanatory variable (ρ_Σ) and correlation among equations (ρ_Σ) can be summarized as:

- As M increases, the TMSE increases and PR increases
- As T increases, the TMSE decreases and PR increases
- As ρ_X increases, the TMSE increases and PR increases
- As ρ_Σ increases, the TMSE increases and PR increases

It is also noticed that the TMSEs of almost all of the different parameters have enormously increased when the dimension of the system of equations = 10, especially, the SUR regression estimators based on the R_{SK}, R_{SHK} and R_{Sharm} which have produced the highest TMSE and the worst proportion of replications, (PR), among the others.

SUMMARY AND CONCLUSION

This study presented a number of alternative procedures to develop some alternative multivariate biased estimators applicable to systems of regression equations. All in all, 10 multivariate parameters are studied and compared. This investigation used the TMSE and the PR criterion to measure the goodness of SUR ridge type estimators. The simulation results support the hypothesis that the number of equations, the number of observations per equation, the correlation among explanatory variables and equations are the main factors that affect the properties of SUR ridge estimators. It is noticed that the unbiased estimators, SGLS, has occasionally (in large sample and low correlation among explanatory variables) shown to have the smallest TMSE when compared with the others. However, for high correlation $\tilde{n}_X$, SUR ridge estimators based on R_{sarith}, $R_{Sqarith}$ and R_{Smax} perform better than the remaining estimators, in particular $\hat{\alpha}$ (R_{Smax}). It is evident from Table 3 and 4 that the estimators $\hat{\alpha}$ (R_{Smax}) performs quite well under all conditions or combination of factors discussed earlier. Clearly, SUR ridge estimators based on R_{SK}, R_{SHK} and R_{Sharm} perform very poorly when compared to the other estimators.

In conclusion, under certain conditions, the $\hat{\alpha}$ (R_{Smax}) estimator is recommended as one of the good estimators to estimate the multivariate ridge parameter R. however, this requires further considerations such as generating random errors from some non-normal distribution.

REFERENCES

Agunbiade, D.A. and J.O. Iyaniwura, 2010. Estimation under multicollinearity: A comparative approach using Monte Carlo methods. J. Math. Stat., 6: 183-192.

Alabi, O.O., K. Ayinde and T.O. Olatayo, 2008. Effect of multicollinearity on power rates of the ordinary least squares estimators. J. Math. Statist., 4: 75-80.

Alkhamisi, M., G. Khalaf and G. Shukur, 2006. Some modifications for choosing ridge parameter. Commun. Stat. Theory Methods, 35: 2005-2020.

Bagheri, A., H. Midi and A.H.M.R. Imon, 2010. The effect of collinearity-influential observations on collinear data set: A monte carlo simulation study. J. Applied Sci., 10: 2086-2093.

Batah, F.S.M., M.R. Ozkale and S.D. Gore, 2009. Combining unbiased ridge and principal component regression estimators. Commun. Stat. Theory Methods, 38: 2201-2209.

Brown, P.J. and J.V. Zidek, 1980. Adaptive multivariate ridge regression. Ann. Stat., 8: 64-74.

Camminatiello, I. and A. Lucademo, 2010. Estimating multinomial logit model with multicollinear data. Asian J. Math. Stat., 3: 93-101.

D'Ambra, A. and P. Sarnacchiaro, 2010. Some data reduction methods to analyze the dependence with highly collinear variables: A simulation study. Asian J. Math. Stat., 3: 69-81.

Fiebig, D.G. and J.H. Kim, 2000. Estimation and inference in SUR models when the number of equations is large. Econometric Rev., 19: 105-130.

Firinguetti, L., 1997. Ridge regression in the context of a system of seemingly unrelated regression equations. J. Stat. Comput. Simul., 56: 145-162.

Gujerati, D.N., 2002. Basic Econometrics. 4th Edn., McGraw-Hill, New York, USA., ISBN: 0071123431, pp: 1002.

Kibria, B.M.G., 2003. Performance of some new ridge regression estimators. Commun. Stat. Simul. Comput., 32: 419-435.

Mardikyan, S. and O.N. Darcan, 2006. A software tool for regression analysis and its assumptions. Inform. Technol. J., 5: 884-891.

Oluwayemisi, A.O., E.O. Olubusoye and S.O. Ojo, 2010. Efficiency of seemingly unrelated regression estimator over the ordinary least squares. Eur. J. Sci. Res., 39: 153-160.

Saleh, A.K.M.E. and B.M.G. Kibria, 1993. Performance of some new preliminary test ridge regression estimators and their properties. Commun. Stat. Theory Methods, 22: 2747-2764.

Srivastava, V.K. and D.E.A. Giles, 1987. Seemingly Unrelated Regression Equations Models. Marcel Dekker, New York.

Zellner, A., 1962. An efficient method of estimating seemingly unrelated regressions and tests of aggregation bias. J. Am. Stat. Assoc., 57: 348-368.