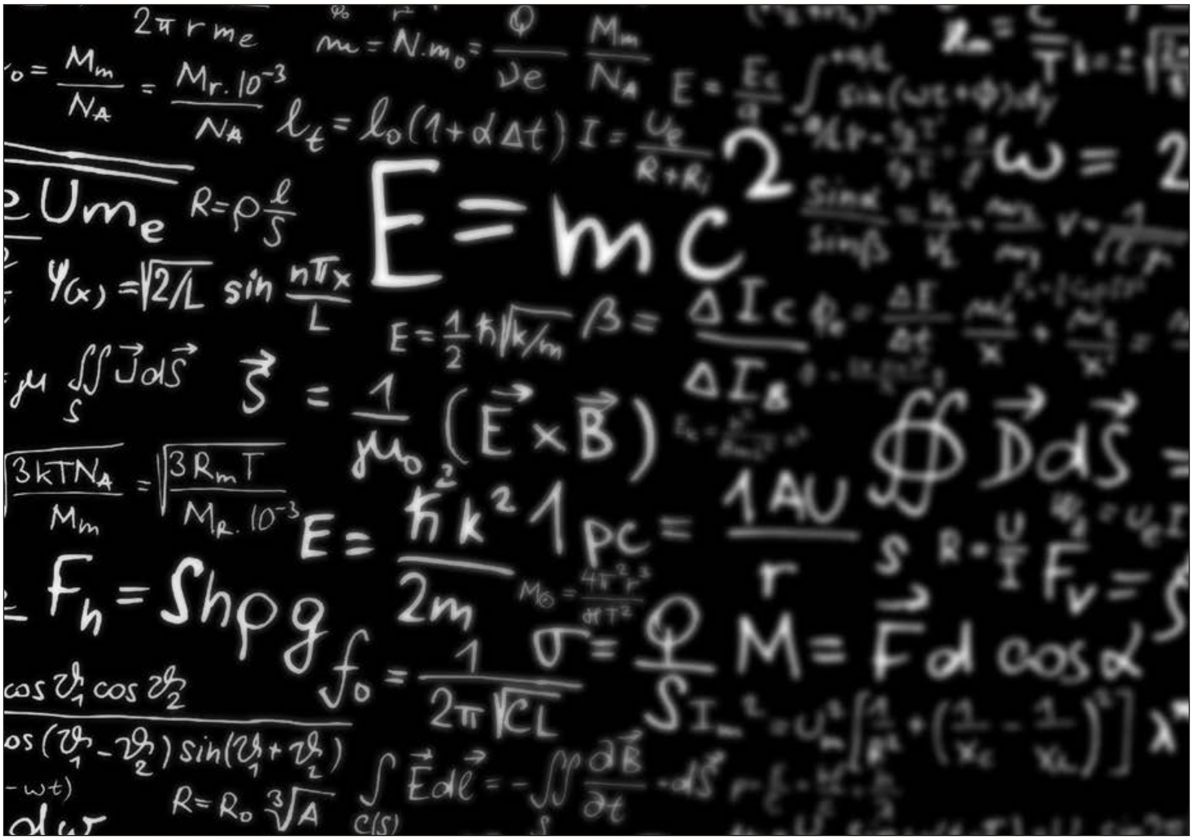

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Exponential Membership Function Evaluation based on Frequency

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ABSTRACT

The unknown parameters of membership function may be a problem in fuzzy works. In that case, frequency table of data set may be a key in the determination of membership function. Considering this, in the current study, the parameter formulas of exponential membership function are generated via a minimization problem in where the frequency table structures are used. The offered exponential membership function is performed on a classification problem of bispectral index data sets in order to observe whether they are more effective than previous one used in the literature. At the end of the analysis, it is concluded that, the mean of classification accuracies based on offered exponential membership functions is statistically greater than other one.

Key words: Exponential fuzzy number, frequency table, classification, bispectral index, time series, clustering

INTRODUCTION

In the literature, there are several classification and clustering methods which use exponential membership functions, patterns or fuzzy architecture. For example, Feng *et al*. (2009) proposed a new training algorithm for Hierarchical Hybrid Fuzzy-Neural Networks (HHFNN) based on gaussian membership function. Devillez (2004) introduced fuzzy pattern matching algorithm with exponential function in fuzzy supervised classification methods in order to design process monitoring of metal cutting with high-speed machining. McNicholas (2010) suggested a novel model based classification technique based on parsimonious Gaussian mixture models. Yang and Bose (2006) proposed automatic fuzzy membership generation with unsupervised learning in where the proper cluster is generated and then the fuzzy membership function is generated according to cluster. Yang and Wu (2006) offered a Possibilistic Clustering Algorithm (PCA) which results exponential membership function in order to be robust to noise and outliers. Chen and Chang (2005) proposed a new method to construct the membership functions of attributes and generate weighted fuzzy rules from training instances for handling fuzzy classification problems without any human experts' intervention. Agrawal *et al*. (2007) presented a supervised neural network classification model based on rough-fuzzy membership function, weak fuzzy similarity relation and back-propagation algorithm. El-Asir and Mamlook (2002) classified heart electrical axis by using fuzzy logic architecture.

Besides them, there are also some studies that utilize the data or distribution of data in the evaluation of membership functions, class labels or classification. For example, Liu *et al*. (2008)

designed a novel fuzzy membership function to represent the distribution of image samples in order to promote the classification performances of canonical correlation analysis. Mansoori *et al*. (2007) proposed an approach divides the covering subspace of each fuzzy rule into two subdivisions based on a α threshold. The splitting threshold for each rule was found by using distribution of patterns in the covering subspace of that rule. Teng *et al*. (2004) chosen a region-based exponential functions to construct fuzzy model and introduced an algorithm that partitions input space into several characteristic regions by using training data. Choi and Rhee (2009) suggested three novel interval type-2 fuzzy membership function (IT2 FMF) generation methods which based on heuristics, histograms and interval type-2 fuzzy C-means. Wu and Chen (1999) proposed a fuzzy learning algorithm based on the α-cuts of equivalence relations. Chang and Lilly (2004) suggested an evolutionary approach where rules and membership functions are automatically created and optimized in evolutionary process of compact fuzzy classification system. This system is derived from directly data without any a priori knowledge or assumptions on the distribution of the data. Au *et al*. (2006) presented a method to determine the membership functions of fuzzy sets directly from data. It maximizes the class-attribute interdependence in order to improve the classification results. Simpson (1992) handled a supervised learning neural network classifier that utilizes fuzzy sets as pattern classes where min-max points are used. The min-max points are determined by using the fuzzy min-max learning algorithm that can learn nonlinear class boundaries in a single pass through the data.

In the current study, the parameter formulas of exponential membership function based on a minimization problem are presented. In the minimization problem, it is tried to reach an exponential membership function such as it takes form regard to the shape of frequency table, in other words histogram of data.

PRELIMINARIES

A fuzzy number A is a fuzzy subset of the real line R with the membership function μ_A which is normal, fuzzy convex, upper semicontinuous, supp A is bounded, where supp A = cl $\{x\in R, \mu_A(x)>0\}$ and cl is the closer operator. The LR-parametric form of A can be written follows (Nasibov, 2002):

$$\forall\alpha\in(0,1]: A^{\alpha}=\left[L_A(\alpha), R_A(\alpha)\right]\subset(-\infty,\infty) \tag{1}$$

where, A^{α} is an α-level set of the fuzzy number, L: $[0,1]\to(-\infty,\infty)$ is monotic nondecreasing left continuous, R: $[0,1]\to(-\infty,\infty)$ is monotic nonincreasing right continuous functions of left and right hand sides of the fuzzy number, respectively.

In the current study, the following definition of parametric exponential fuzzy number is considered:

$$\mu(x)=\begin{cases} e^{-\left(\frac{M-x}{\sigma}\right)^{s_L}} & ;\quad x\le M \\ e^{-\left(\frac{x-M}{\beta}\right)^{s_R}} & ;\quad x>M \end{cases} \tag{2}$$

And, the satisfaction of:

Table 1: Frequency table

Class interval	Midpoint	Frequency	Percentage
X_1-X_2	M_1: $(X_1 + X_2)/2$	f_1	$p_1 = f_1/N$
X_2-X_3	M_2: $(X_2 + X_3)/2$	f_2	$p_2 = f_2/N$
...	...	...	...
X_{K-1}-X_K	M_k: $(X_{k-1} + X_k)/2$	f_k	$p_k = f_k/N$
Total		N	1

$$\lim_{\alpha\to 0} A^{\alpha} = \lim_{\alpha\to 0}\left[L_A(\alpha), R_A(\alpha)\right] = \left[L_A(0), R_A(0)\right] \tag{3}$$

$$L_A(0) > -\infty,\ R_A(0) < \infty \tag{4}$$

are supposed.

EXPONENTIAL MEMBERSHIP FUNCTION GENERATION BASED ON FREQUENCIES

Sometimes the data distribution may be skewed, so the usage of following exponential approach to data distribution may give more weak results which is used by Nasibov and Ulutagay (2010):

$$\mu_A(x) = e^{-\frac{1}{2}\left(\frac{x-\alpha_A}{\sigma_A}\right)^2} \tag{5}$$

$$\alpha_A = \frac{\sum x_i}{n_A},\ \sigma_A = \sqrt{\frac{\sum (x_i - \alpha_A)^2}{n_A}} \tag{6}$$

Considering this, parametric exponential approximation to data distribution is handled in the current study and the following frequency table is utilized to catch the shape of X_1, X_2, ... , X_N data. In Table 1, f values show the frequencies of each class where the number of classes is k.

In the evaluation of parametric exponential membership function, the LR parametric form in Eq. 1 is considered. For the left and right side shape of exponential membership function:

$$p_i = \exp\left(-\left(\frac{M - M_i}{\sigma}\right)^{s_L}\right),\ p_i = \exp\left(-\left(\frac{M_i - M}{\beta}\right)^{s_R}\right) \tag{7}$$

equalities are taken into account in objective function (8), respectively. In here, M denotes the midpoint of class interval with maximum percentage:

$$\begin{aligned} f(s_L, \sigma, s_R, \beta) = &\sum_{i=1}^{m-1}\left(\ln(-\ln(\tilde{p}_i)) - s_L \ln\left(\frac{M - M_i}{\sigma}\right)\right)^2 \\ &+ \sum_{i=m+1}^{k}\left(\ln(-\ln(\tilde{p}_i)) - s_R \ln\left(\frac{M_i - M}{\beta}\right)\right)^2 \end{aligned} \tag{8}$$

Before minimization of objective function in Eq. 8, the following prior procedure is applied in order to keep normal condition of fuzzy number at 1 level:

- Assign 1 membership level to midpoint of class interval with maximum percentage
- Normalize other percentage with:

$$\tilde{p}_i = \frac{p_i}{p_m} \tag{9}$$

where, p_m is maximum percentage.

After that the unknown parameters of exponential fuzzy number s_L, σ, s_R, β in Eq. 2 are found by minimization of Eq. 8.

Theorem: To obtain fuzzy parametric exponential number which minimize the objective function in Eq. 8 the parameters of membership function in Eq. 2 must be as following:

$$s_L = \frac{\sum_{i=1}^{m-1} \ln(-\ln(\tilde{p}_i))}{\sum_{i=1}^{m-1} \ln\left(\frac{M-M_i}{\sigma}\right)}$$

$$\sigma = \exp\left(\frac{\sum_{i=1}^{m-1} \ln(-\ln(\tilde{p}_i))\sum_{i=1}^{m-1}\left(\ln(M-M_i)\right)^2 - \sum_{i=1}^{m-1}\left(\ln(-\ln(\tilde{p}_i))\ln(M-M_i)\right)\sum_{i=1}^{m-1}\ln(M-M_i)}{\sum_{i=1}^{m-1} \ln(-\ln(\tilde{p}_i))\sum_{i=1}^{m-1}\ln(M-M_i) - (m-1)\sum_{i=1}^{m-1}\left(\ln(-\ln(\tilde{p}_i))\ln(M-M_i)\right)}\right)$$

$$s_R = \frac{\sum_{i=m+1}^{k} \ln(-\ln(\tilde{p}_i))}{\sum_{i=m+1}^{k} \ln\left(\frac{M_i-M}{\beta}\right)}$$

$$\beta = \exp\left(\frac{\sum_{i=m+1}^{k} \ln(-\ln(\tilde{p}_i))\sum_{i=m+1}^{k}\left(\ln(M_i-M)\right)^2 - \sum_{i=m+1}^{k}\left(\ln(-\ln(\tilde{p}_i))\ln(M_i-M)\right)\sum_{i=m+1}^{k}\ln(M_i-M)}{\sum_{i=m+1}^{k} \ln(-\ln(\tilde{p}_i))\sum_{i=m+1}^{k}\ln(M_i-M) - (k-m)\sum_{i=m+1}^{k}\left(\ln(-\ln(\tilde{p}_i))\ln(M_i-M)\right)}\right) \tag{10}$$

Note: In Theorem:

$$\sum_{i=m+1}^{k} \ln(-\ln(\tilde{p}_i))\sum_{i=m+1}^{k}\ln(M_i-M) \neq (k-m)\sum_{i=m+1}^{k}\left(\ln(-\ln(\tilde{p}_i))\ln(M_i-M)\right) \tag{11}$$

and

$$\sum_{i=1}^{m-1} \ln(-\ln(\tilde{p}_i))\sum_{i=1}^{m-1}\ln(M-M_i) \neq (m-1)\sum_{i=1}^{m-1}\left(\ln(-\ln(\tilde{p}_i))\ln(M-M_i)\right) \tag{12}$$

conditions must be satisfied.

Proof: To evaluate the unknown parameters of parametric exponential membership function which has got membership level close to normalized $\tilde{p}_i$ levels, the objective function in Eq. 10 can be minimized with respect to unknown s_L, $\boldsymbol{\sigma}$, s_R, $\boldsymbol{\beta}$ parameters. With this aim:

$$\frac{\partial f}{\partial s_L}=0,\ \frac{\partial f}{\partial \sigma}=0,\ \frac{\partial f}{\partial s_R}=0,\ \frac{\partial f}{\partial \beta}=0$$

equalities must be satisfied.

To find s_L and $\boldsymbol{\sigma}$, the following ones have to be solved sequentially.

$$\frac{\partial f}{\partial \sigma}=-2\sum_{i=1}^{m-1}\left(\ln(-\ln(\tilde{p}_i))-s_L\ln\left(\frac{M-M_i}{\sigma}\right)\right)\frac{s_L}{\sigma}=0$$

$$\frac{\partial f}{\partial s_L}=-2\sum_{i=1}^{m-1}\left(\ln(-\ln(\tilde{p}_i))-s_L\ln\left(\frac{M-M_i}{\sigma}\right)\right)\ln\left(\frac{M-M_i}{\sigma}\right)=0$$

From here:

$$\sum_{i=1}^{m-1}\ln(-\ln(\tilde{p}_i))-s_L\sum_{i=1}^{m-1}\ln\left(\frac{M-M_i}{\sigma}\right)=0$$

and

$$\sum_{i=1}^{m-1}\ln(-\ln(\tilde{p}_i))\ln\left(\frac{M-M_i}{\sigma}\right)-s_L\sum_{i=1}^{m-1}\left(\ln\left(\frac{M-M_i}{\sigma}\right)\right)^2=0$$

equalities can be received, respectively.

At the end:

$$s_L=\frac{\sum_{i=1}^{m-1}\ln(-\ln(\tilde{p}_i))}{\sum_{i=1}^{m-1}\ln\left(\frac{M-M_i}{\sigma}\right)}$$

and

$$\ln\sigma=\frac{\sum_{i=1}^{m-1}\ln(-\ln(\tilde{p}_i))\sum_{i=1}^{m-1}\left(\ln(M-M_i)\right)^2-\sum_{i=1}^{m-1}\left(\ln(-\ln(\tilde{p}_i))\ln(M-M_i)\right)\sum_{i=1}^{m-1}\ln(M-M_i)}{\left(\sum_{i=1}^{m-1}\ln(-\ln(\tilde{p}_i))\sum_{i=1}^{m-1}\ln(M-M_i)-(m-1)\sum_{i=1}^{m-1}\left(\ln(-\ln(\tilde{p}_i))\ln(M-M_i)\right)\right)}$$

can be reached. From here, $\boldsymbol{\sigma}$ can be written as following:

$$\sigma = \exp\left(\frac{\sum_{i=1}^{m-1}\ln(-\ln(\tilde{p}_i))\sum_{i=1}^{m-1}\left(\ln(M-M_i)\right)^2 - \sum_{i=1}^{m-1}\left(\ln(-\ln(\tilde{p}_i))\ln(M-M_i)\right)\sum_{i=1}^{m-1}\ln(M-M_i)}{\left(\sum_{i=1}^{m-1}\ln(-\ln(\tilde{p}_i))\sum_{i=1}^{m-1}\ln(M-M_i)-(m-1)\sum_{i=1}^{m-1}\left(\ln(-\ln(\tilde{p}_i))\ln(M-M_i)\right)\right)}\right)$$

In a similar way, by solving the following equalities sequentially:

$$\frac{\partial f}{\partial \beta} = -2\sum_{i=m+1}^{k}\left(\ln(-\ln(\tilde{p}_i)) - s_R\ln\left(\frac{M_i-M}{\beta}\right)\right)\frac{s_R}{\beta} = 0$$

$$\frac{\partial f}{\partial s_R} = -2\sum_{i=1}^{m-1}\left(\ln(-\ln(\tilde{p}_i)) - s_R\ln\left(\frac{M_i-M}{\beta}\right)\right)\ln\left(\frac{M_i-M}{\beta}\right) = 0$$

the following ones can be obtained in order:

$$s_R = \frac{\sum_{i=m+1}^{k}\ln(-\ln(\tilde{p}_i))}{\sum_{i=m+1}^{k}\ln\left(\frac{M_i-M}{\beta}\right)}$$

$$\ln\beta = \frac{\sum_{i=m+1}^{k}\ln(-\ln(\tilde{p}_i))\sum_{i=m+1}^{k}\left(\ln(M_i-M)\right)^2 - \sum_{i=m+1}^{k}\left(\ln(-\ln(\tilde{p}_i))\ln(M_i-M)\right)\sum_{i=m+1}^{k}\ln(M_i-M)}{\sum_{i=m+1}^{k}\ln(-\ln(\tilde{p}_i))\sum_{i=m+1}^{k}\ln(M_i-M)-(k-m)\sum_{i=m+1}^{k}\left(\ln(-\ln(\tilde{p}_i))\ln(M_i-M)\right)}$$

By transforming, **β** can be written as:

$$\beta = \exp\left(\frac{\sum_{i=m+1}^{k}\ln(-\ln(\tilde{p}_i))\sum_{i=m+1}^{k}\left(\ln(M_i-M)\right)^2 - \sum_{i=m+1}^{k}\ln(-\ln(\tilde{p}_i))\ln(M_i-M)\sum_{i=m+1}^{k}\ln(M_i-M)}{\sum_{i=m+1}^{k}\ln(-\ln(\tilde{p}_i))\sum_{i=m+1}^{k}\ln(M_i-M)-(k-m)\sum_{i=m+1}^{k}\ln(-\ln(\tilde{p}_i))\ln(M_i-M)}\right)$$

which ends proof.

BISPECTRAL INDEXES' CLASSIFICATION BASED ON PARAMETRIC EXPONENTIAL MEMBERSHIP FUNCTIONS

The Bispectral Index (BIS) is a parameter derived from the electroencephalograph (EEG) (Ozgoren *et al*., 2008). It correlate with increasing sedation and loss of awareness so its monitoring may help to assess the hypnotic component of anesthesia, reduce drug consumption and shorten recovery times (Kreuer *et al*., 2001; Gan *et al*., 1997). It is scaled from 100 (awake patient) to 0 (no cortical activity).

In the current section, BIS index classification is handled and the same data used in Nasibov and Ulutagay (2010) are performed, except one. To observe the efficiency of the proposed parametric exponential membership function definition in a classification problem, it is wanted to be compared with Eq. 5. Accordingly, all data sets are merged and the values of each sedation stages are regrouped for evaluation of fuzzy numbers. Then, the following membership functions of each sedation stage are calculated by using Eq. 5 for comparison:

Table 2: Frequency table of first sedation stage

Class interval	Mid-point	Frequency	Percent	Cumulative percent
0.15-0.20	0.175	8	1.0	1.0
0.20-0.25	0.225	65	7.8	8.8
0.25-0.30	**0.275**	**322**	**38.7**	**47.4**
0.30-0.35	0.325	266	31.9	79.4
0.35-0.40	0.375	130	15.6	95.0
0.40-0.45	0.425	28	3.4	98.3
0.45-0.50	0.475	7	0.8	99.2
0.50-0.55	0.525	3	0.4	99.5
0.55-0.60	0.575	3	0.4	99.9
0.60-0.65	0.625	1	0.1	100.0
Total		833	100.0	

Bold class denotes the class interval with maximum percentage

Table 3: Frequency table of second sedation stage

Class interval	Mid-point	Frequency	Percent	Cumulative percent
0.20-0.25	0.225	4	0.2	0.2
0.25-0.30	0.275	33	1.9	2.1
0.30-0.35	0.325	124	7.1	9.2
0.35-0.40	0.375	280	16.1	25.3
0.40-0.45	**0.425**	**355**	**20.4**	**45.7**
0.45-0.50	0.475	216	12.4	58.1
0.50-0.55	0.525	199	11.4	69.5
0.55-0.60	0.575	143	8.2	77.7
0.60-0.65	0.625	157	9	86.7
0.65-0.70	0.675	117	6.7	93.5
0.70-0.75	0.725	67	3.8	97.3
0.75-0.80	0.775	38	2.2	99.5
0.80-0.85	0.825	6	0.3	99.8
0.85-0.90	0.875	1	0.1	99.9
0.90-0.95	0.925	2	0.1	100.0
Total		1742	100.0	

Bold class denotes the class interval with maximum percentage

$$\mu_1 = e^{-\frac{1}{2}\left(\frac{0.31121-x}{0.055421}\right)^2},\ \mu_2 = e^{-\frac{1}{2}\left(\frac{0.49281-x}{0.122669}\right)^2},\ \mu_3 = e^{-\frac{1}{2}\left(\frac{0.69081-x}{0.128966}\right)^2},\ \mu_4 = e^{-\frac{1}{2}\left(\frac{0.83307-x}{0.088472}\right)^2},\ \mu_5 = e^{-\frac{1}{2}\left(\frac{0.88617-x}{0.084955}\right)^2}$$

After that, the one of the possible frequency tables of sedation stages can be generated as given in Table 2-6 to apply proposed theorem. In these tables, the frequencies and percentages of bispectral index classes for each sedation stages are shown. The highlighted class in each table denotes the class interval with maximum percentage (p_m). The midpoints of these classes are M values in parametric exponential fuzzy numbers of sedation stages.

Accordingly, the following parametric exponential membership functions are found by using proposed theorem:

$$\mu_1 = \begin{cases} e^{-\left(\frac{0.275-x}{0.03366153}\right)^{1.19058453}} & , \quad x \le 0.275 \\ e^{-\left(\frac{x-0.275}{0.10954908}\right)^{1.75612535}} & , \quad x > 0.275 \end{cases}$$

Table 4: Frequency table of third sedation stage

Class interval	Mid-point	Frequency	Percent	Cumulative percent
0.25-0.30	0.275	1	0.0	0.0
0.30-0.35	0.325	3	0.1	0.1
0.35-0.40	0.375	25	0.9	1.1
0.40-0.45	0.425	119	4.4	5.5
0.45-0.50	0.475	112	4.2	9.7
0.50-0.55	0.525	199	7.4	17.0
0.55-0.60	0.575	189	7.0	24.1
0.60-0.65	0.625	344	12.8	36.8
0.65-0.70	0.675	304	11.3	48.1
0.70-0.75	0.725	347	12.9	61.0
0.75-0.80	0.775	379	14.1	75.1
0.80-0.85	**0.825**	**499**	**18.5**	**93.6**
0.85-0.90	0.875	129	4.8	98.4
0.90-0.95	0.925	30	1.1	99.5
0,95-1	0.975	14	0.5	100.0
Total		2694	100.0	

Bold class denotes the class interval with maximum percentage

Table 5: Frequency table of fourth sedation stage

Class interval	Mid-point	Frequency	Percent	Cumulative percent
0.40-0.45	0.425	1	0.1	0.1
0.45-0.50	0.475	4	0.5	0.7
0.50-0.55	0.525	8	1.1	1.8
0.55-0.60	0.575	8	1.1	2.8
0.60-0.65	0.625	10	1.4	4.2
0.65-0.70	0.675	23	3.1	7.3
0.70-0.75	0.725	65	8.8	16.1
0.75-0.80	0.775	51	6.9	23.1
0.80-0.85	**0.825**	**219**	**29.7**	**52.8**
0.85-0.90	0.875	215	29.2	82.0
0.90-0.95	0.925	92	12.5	94.4
0,95-1	0.975	41	5.6	100.0
Total		737	100.0	

Bold class denotes the class interval with maximum percentage

Table 6: Frequency table of fifth sedation stage

Class interval	Mid-point	Frequency	Percent	Cumulative percent
0.35-0.40	0.375	3	0.4	0.4
0.40-0.45	0.425	4	0.6	1.0
0.45-0.50	0.475	3	0.4	1.4
0.50-0.55	0.525	11	1.5	2.9
0.55-0.60	0.575	2	0.3	3.2
0.60-0.65	0.625	4	0.6	3.7
0.65-0.70	0.675	2	0.3	4.0
0.70-0.75	0.725	4	0.6	4.5
0.75-0.80	0.775	7	1	5.5
0.80-0.85	0.825	46	6.3	11.8
0.85-0.90	0.875	260	35.8	47.7
0.90-0.95	**0.925**	**316**	**43.5**	**91.2**
0,95-1	0.975	64	8.8	100.0
Total		726	100.0	

Bold class denotes the class interval with maximum percentage

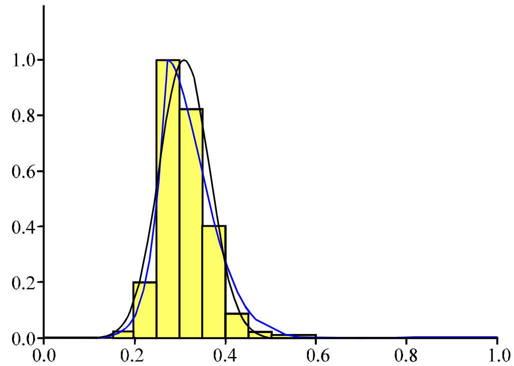

Fig. 1: The membership function and histogram of sedation stage 1 via Theorem 5

$$\mu_2 = \begin{cases} e^{-\left(\frac{0.425-x}{0.098304385}\right)^{2.12921635}} & , \quad x \le 0.425 \\ e^{-\left(\frac{x-0.425}{0.140004284}\right)^{0.97277506}} & , \quad x > 0.425 \end{cases}$$

$$\mu_3 = \begin{cases} e^{-\left(\frac{0.825-x}{0.23903161}\right)^{1.16559615}} & , \quad x \le 0.825 \\ e^{-\left(\frac{x-0.825}{0.03494202}\right)^{0.91429043}} & , \quad x > 0.825 \end{cases}$$

$$\mu_4 = \begin{cases} e^{-\left(\frac{0.825-x}{0.04244326}\right)^{0.67457267}} & , \quad x \le 0.825 \\ e^{-\left(\frac{x-0.825}{0.12082079}\right)^{4.33694334}} & , \quad x > 0.825 \end{cases}$$

$$\mu_5 = \begin{cases} e^{-\left(\frac{0.925-x}{0.08407821}\right)^{1.03766617}} & , \quad x \le 0.925 \\ 1-\left(\frac{x-0.925}{0.075}\right)^{0.55743699} & , \quad 0.925 < x \le 1 \\ 0 & , \quad \text{otherwise} \end{cases}$$

In the generation of membership function for 5th sedation stage, the constraints in Eq. 12 is not satisfied, so a curve that has 0.5 membership level at 85% percentages of data is fitted.

The shapes of membership functions evaluated by proposed theorem and Eq. 5 can be seen in Fig. 1-5, where blue curve lines represent exponential fuzzy numbers evaluated by offered theorem while black curve lines represent exponential fuzzy numbers evaluated by Eq. 5.

The classification procedures are applied on 21 data sets with maximum level criteria in order to see the usage of proposed parametric exponential fuzzy number in classification problem. Accordingly, the results of Classification Accuracies (CA) of data sets in Table 7 are obtained where classification accuracy denotes the ratio of correct estimated point number in each data set.

The paired-t test is applied in order to test whether the mean of classification accuracies based on membership functions evaluated by offered Theorem is greater than the one based on Eq. 5, or not.

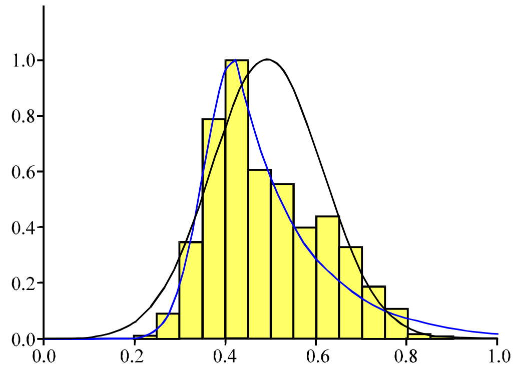

Fig. 2: The membership function and histogram of sedation stage 2 via Theorem 5

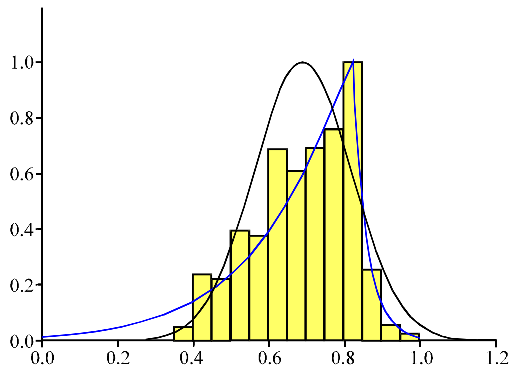

Fig. 3: The membership function and histogram of sedation stage 3 via Theorem 5

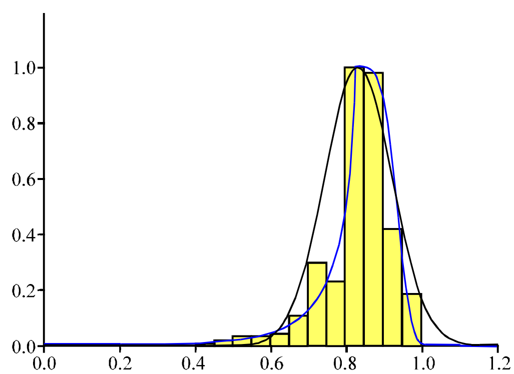

Fig. 4: The membership function and histogram of sedation stage 4 via Theorem 5

$$H_0 : \mu_{Theorem} = \mu_{3.1}$$
$$H_1 : \mu_{Theorem} > \mu_{3.1}$$

The results shown in Table 8 are obtained by using Minitab program. At the end of the analysis, it is concluded that the mean of classification accuracies of based on membership functions evaluated by offered theorem is greater than the one based on Eq. 5 in bispectral index data sets ($\alpha = 0.10$).

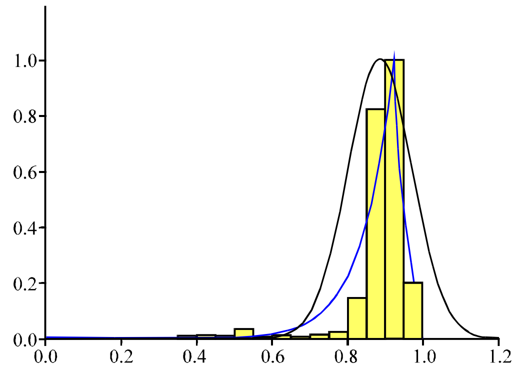

Fig. 5: The membership function and histogram of sedation stage 5 via Theorem 5

Table 7: The classification accuracies of the membership function approximations

	Normal		Teorem	
Data Set	Correct point No.	CA	Correct point No.	CA
1	190.00000	0.620915	212.0000	0.692810
2	165.00000	0.539216	199.0000	0.650327
3	154.00000	0.503268	191.0000	0.624183
4	217.00000	0.709150	233.0000	0.761438
5	196.00000	0.640523	215.0000	0.702614
6	215.00000	0.702614	237.0000	0.774510
7	175.00000	0.571895	230.0000	0.751634
8	167.00000	0.545752	145.0000	0.473856
9	233.00000	0.761438	202.0000	0.660131
10	232.00000	0.758170	233.0000	0.761438
11	148.00000	0.48366	136.0000	0.444444
12	131.00000	0.428105	146.0000	0.477124
13	222.00000	0.725490	165.0000	0.539216
14	53.00000	0.173203	68.0000	0.222222
15	118.00000	0.385621	106.0000	0.346405
16	262.00000	0.856209	216.0000	0.705882
17	255.00000	0.833333	280.0000	0.915033
18	124.00000	0.405229	191.0000	0.624183
19	68.00000	0.222222	153.0000	0.500000
20	131.00000	0.428105	158.0000	0.516340
21	192.00000	0.627451	200.0000	0.653595
Mean	173.71430	0.567694	186.4762	0.609399
SD	56.77336	0.185534	49.64536	0.162240

Table 8: The minitab output of paired-t test of correct point numbers

Results	N	Mean	SD	SE
Theorem	21	0,609399	0,162240	0,035404
Formula (5)	21	0,567694	0,185534	0,040487
Difference	21	0,041706	0,114136	0,024906

Paired T for Theorem-Formula (5). 95% lower bound for mean difference: -0,001251, T-test of mean difference = 0 (vs>0): T-value = 1.67, p-value = 0.055

CONCLUSIONS

In the literature, there are researches underlined the fact that the parameters of membership functions have important role on classification accuracies. Considering exponential membership functions in the classification problems, in the current study, the formulas of exponential membership functions are evaluated via a minimization problem. In the minimization problem, the objective function is defined regard to percentages of frequency table in order to form a fuzzy number similar with the histogram of data.

In the last part of the study, the efficiency of membership function is tested in the classification problems of bispectral indexes which are measurements of brain activity. At the end of the analysis, the offered parameter formulas are found useful in the increasing of classification accuracies in the data sets.

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