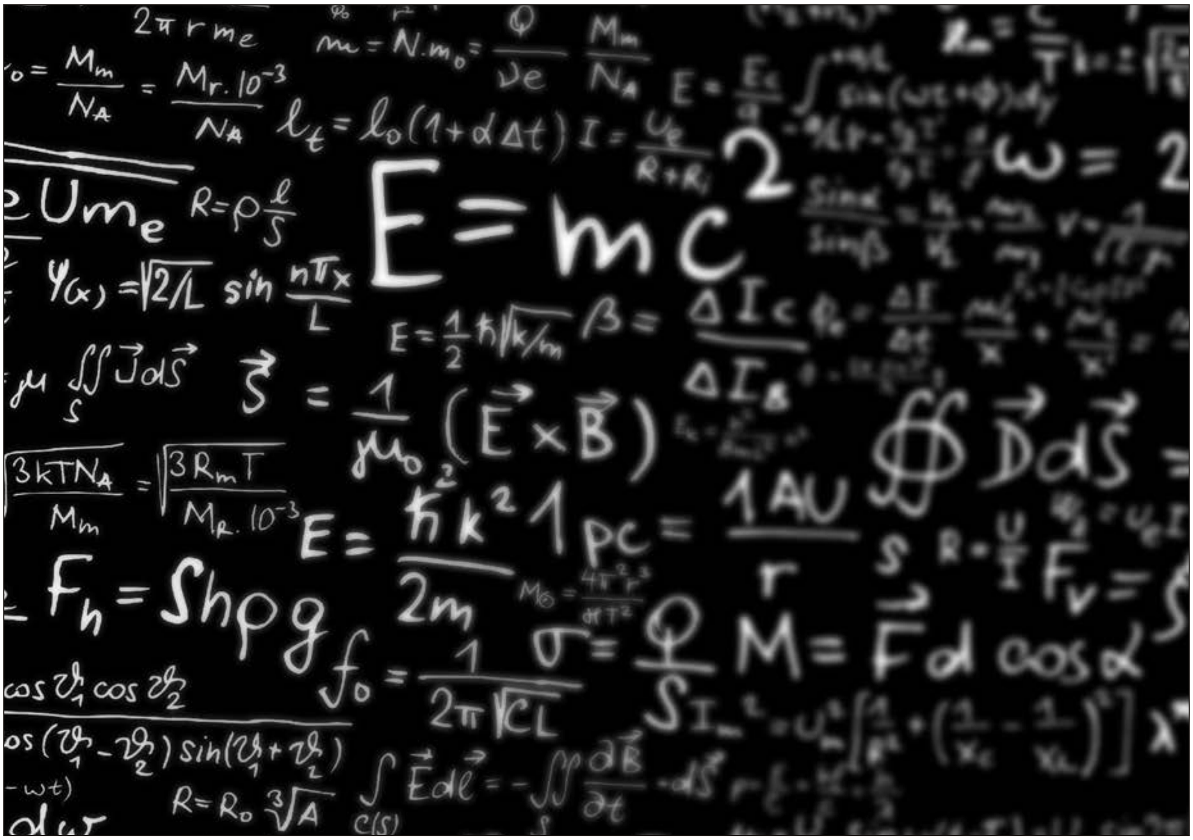

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The Application of Bernadelli Waves: A Demographic Theory

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ABSTRACT

This study has examined the concept of Bernadelli waves and discovered that the Leslie matrix approach can be used to predict the future population of a country. However, it is pertinent to mention that for this method to be useful, a reliable and precise census survey is a very important factor since a growth rate with maximum accuracy is highly needed. However, government should take the issue of census survey serious; it should not be seen from a political point of view but from economic and growth purposes since it can be use for policy making and implementation.

Key words: Bernadelli, population, waves, projection, leslie matrix

INTRODUCTION

Benradelli waves also known as population waves are important in so many ways some of these ways are:

- They are important in determining whether a population will persist
- They are also important as they are used to check the population age structure of most countries as these countries are under going cyclical, often irregular change that will persist for many decades

The primary goal of any forecasting strategy is to accurately predict magnitude of a variable at a specific time in the future (Bernadelli, 1941). If the variable of interest is population size, then prediction might be used in making decision in fields such as agriculture, to control the spread of disease, or in a campaign to prevent extinction of a species.

Accurate forecasts can sometimes be derived by modeling the pattern of part fluctuation in the variable or by associating the variable with change in other variables. Those approaches require little understanding of what caused the variable of interest to change. In contrast an objective of population ecology is to predict abundance accurately, based on an understanding of the system of births, deaths and the dispersal of individuals (Emlen, 1984). Predictions of abundance are made from mathematical models which very in complexity. Thus since population age distributions and sizes oscilate in waves as was given by Bernadelli in his article then the main purpose of population waves is to forecast or predict variables, such as population characteristics for future purpose. The utility of Leslie matrices for characterizing populations is the rationale for their inclusion in most basic population courses. In fisheries, the special attributes of a population assumed to be at equilibrium are used by Walters *et al.* (1980) to provide estimates of young-of-the-year mortality.

MODEL ILLUSTRATION

Bernadelli introduced a simple model known as population waves. For instance, taking an example from the population of salmon fishes. Say the female fish has a survival rate of 1/4 in the first year of life, a survival rate of 1/8 from the second to third year to an average of eight new females before drying at the end of the third year (Getz and Haight, 1989; Vaughan and Saila, 1976).

Thus, the contribution of an individual female fish, in a probabilistic sense, to the female population number can be summarized in the following Leslie matrix L:

$$L = \begin{pmatrix} 0 & 0 & 0 \\ 1/4 & 0 & 0 \\ 0 & 1/8 & 0 \end{pmatrix}$$

where, the row of L is the formulates (f_x) (Leslie, 1945), which are 0,0,8.

The diagonal below the main diagonal contains the survival rate or probabilities (S_x) which are 1/4 and 1/8.

Now, we have three age group 1, 2, 3 of salmon female fishes. So; Age group 1 has 4000 females. Age group 2 has 4000 females. Age group 3 has 4000 females.

Thus these population values given as our initial population vector n_t at the time t.
where, t = 0 we have the vector

$$n_0 = \begin{pmatrix} 4000 \\ 4000 \\ 4000 \end{pmatrix}$$

Illustrating this model, the number of female in the three age groups k years hence is obtained by calculating $n_o L^k$.

Calculating n_1 we have:

$$n_1 = n_o L = L n_o = \begin{pmatrix} 0 & 0 & 8 \\ 1/4 & 0 & 0 \\ 0 & 1/8 & 0 \end{pmatrix} \begin{pmatrix} 4000 \\ 4000 \\ 4000 \end{pmatrix}$$

$$= 0 \text{ x } 4000+0 \text{ x } 4000+8 \text{ x } 4000 = 32,000$$
$$¼ \text{ x } 4000+0 \text{ x } 4000+0 \text{ x } 4000 = 1,000$$
$$0 \text{ x } 4000+1/8 \text{ x } 4000+0 \text{ x } 4000 =500$$

$$n_1 = \begin{pmatrix} 32000 \\ 1000 \\ 500 \end{pmatrix}$$

$$n_2 = n_1 L = L n_1 = \begin{pmatrix} 0 & 0 & 8 \\ 1/4 & 0 & 0 \\ 0 & 1/8 & 0 \end{pmatrix} \begin{pmatrix} 32000 \\ 1000 \\ 500 \end{pmatrix}$$

= 0 x 32000+0 x 1000+8 x 500 = 4000
¼ x 32000+0 x 1000+0 x 500 = 8,000
0 x 32000+1/8 x 1000 + 0 x 500 = 125

$$n_2 = \begin{pmatrix} 4000 \\ 8000 \\ 125 \end{pmatrix}$$

$$n_3 = n_2L = Ln_2 = \begin{pmatrix} 0 & 0 & 8 \\ 1/4 & 0 & 0 \\ 0 & 1/8 & 0 \end{pmatrix}\begin{pmatrix} 4000 \\ 8000 \\ 125 \end{pmatrix}$$

= 0 x 4000+0 x 8000+8x 125 =1000
¼ x 4000+0 x 8000+0 x 125 = 1000
0 x 4000+1/8 x 8000+0 x 125 = 1000

Thus:

$$n_o = \begin{pmatrix} 4000 \\ 4000 \\ 4000 \end{pmatrix} n_1 = \begin{pmatrix} 32000 \\ 1000 \\ 500 \end{pmatrix} n_2 = \begin{pmatrix} 4000 \\ 8000 \\ 125 \end{pmatrix} n_3 = \begin{pmatrix} 1000 \\ 1000 \\ 1000 \end{pmatrix}$$

then for this example, n_1 indicates that one year there are 32000 female fishes of age group 1, 1000 of age group 2 500 of age group 3.

n_2, indicate that two years later there are 4000 female fishes of age group 1, 8000 of age group 2 and 125 of age group 3.

n_3, indicates that three years later there are 1000 female fishes of age group 1,1000 of age group 2 and 1000 of age group 3.

Thus, this can also be used to project the population of these salmon fishes for 4 years, 5 years ... up to k number of years (Leslie, 1948).

MODEL ILLUSTRATION, USING THE 1991 ENUGU STATE CENSUS DATA

Let's say the female population of Enugu State has a survival rate of ½ in the first years, a survival rate of 1/4 from the second to third years. The age specific fecundities are given as 1, 1, 1, (United Nations Population Fund, 1999). Thus the matrix is given as:

$$L = \begin{pmatrix} 1 & 1 & 1 \\ 1/2 & 0 & 0 \\ 0 & 1/4 & 0 \end{pmatrix}$$

let's use three age group from the census data of say:

- 20-24
- 25-29
- 30-34

The population vector for 1991 is obtained from this census data thus given as:

$$n_{91} = \begin{pmatrix} 108673 \\ 99802 \\ 77624 \end{pmatrix}$$

projecting these population to k no of year we use the Bernadelli model:

$$n_{92} = Ln_{91} = n_{92} = \begin{pmatrix} 1 & 1 & 1 \\ 1/2 & 0 & 0 \\ 0 & 1/4 & 0 \end{pmatrix} \begin{pmatrix} 108673 \\ 99802 \\ 77624 \end{pmatrix}$$

= 1 x 108673+1 x 99802+1 x 77624 = 286099
1/4 x 108673+0 x 99802+0 x 77624 =54336.5
0 x 108673+1/4 x 99802+0 x 77624 = 24950.5

$$n_{92} = \begin{pmatrix} 286{,}099 \\ 54{,}336\ .5 \\ 24{,}950.5 \end{pmatrix}$$

$$n_{93} = Ln_{92} = \begin{pmatrix} 1 & 1 & 1 \\ 1/2 & 0 & 0 \\ 0 & 1/4 & 0 \end{pmatrix} \begin{pmatrix} 286{,}099 \\ 54{,}336.5 \\ 24{,}950.5 \end{pmatrix}$$

= 1 x 286099+1 x 54336.5+1 x 24950.5 =365,386.
½ x 286099+0 x 54336.5+0 x 24950.5 = 143049.5
0 x 286099+1/4 x 54336.5+0 x 24950.5 =13584.125

$$n_{93} = \begin{pmatrix} 365{,}356 \\ 143{,}049.5 \\ 13584.125 \end{pmatrix}$$

$$n_{94} = Ln9_3 = \begin{pmatrix} 1 & 1 & 1 \\ 1/2 & 0 & 0 \\ 0 & 1/4 & 0 \end{pmatrix} \begin{pmatrix} 365386 \\ 143049.5 \\ 13584.125 \end{pmatrix}$$

= 1 x 365386+1 x 143049.5+1 x 13584.125 =522019.625
½ x 365386+0 x 143049.5+0 x 13584.125 =182693
0 x 365386+1/4 x 143049.5+0 x 13584.125 =35762.375

$$n_{94} = \begin{pmatrix} 5220193625 \\ 182693 \\ 35762.375 \end{pmatrix}$$

Interpreting this, n_{92} indicates that in 1992 there are 286,099 females of age group 20-24 years, 54, 337 females of age group 25-29 years and 24, 951 females of group 30-34 years etc. thus we can project to k no of years. We can also determine the eigen value and eigenvectors of L. If x is an n-element column vector, the matrix L x = λx has non trivial solutions s for x if and only if the characteristic matrix L-λI has a zero determinant.

Expanding/L-λI/ = 0 yields an nth-order characteristic polynomial equation for the eigenvalues x. corresponding to each of these n eigenvalues will be a nontrivial eigenvector x.

The characteristic matrix CM is generates for L; thus:

$$CM = \begin{pmatrix} \lambda - 1 & 1 & 1 \\ -1/2 & \lambda & 0 \\ 0 & -1/4 & \lambda \end{pmatrix}$$

Which is the characteristic polynomial. We should note that sometimes this model gives us a cyclic variation of the population as k increases (Searle, 1982).

CONCLUSION

Bernadelli introduced a matrix model which is associated with the Leslie matrix and is given as: $n_{t+1} = L\, n_t$ This Leslie metrics makes use of age-specific fecundities f_x and age-specific survival rates S_x. The model is used for demonstrating the ultimate effect of the present population, trends etc.

Thus, Bernadelli waves can be used to forecast future Population if the population at the base year is accurate to a reasonable extent. It is capable of generating figures for exact years without necessarily projecting the population for a specified interval of years, for this reason; it can be used for immediate policy making and implementation. However, for these to be achieved, the census figures used should be highly reliable.

RECOMMENDATIONS

- The Nigeria federal government should take census excises serious so that researcher will have reliable data to work with
- Bernadelli approach should be used as an alternative method of generating data needed for policy implementation without necessarily waiting for census to be conducted as successive census may take time to come

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