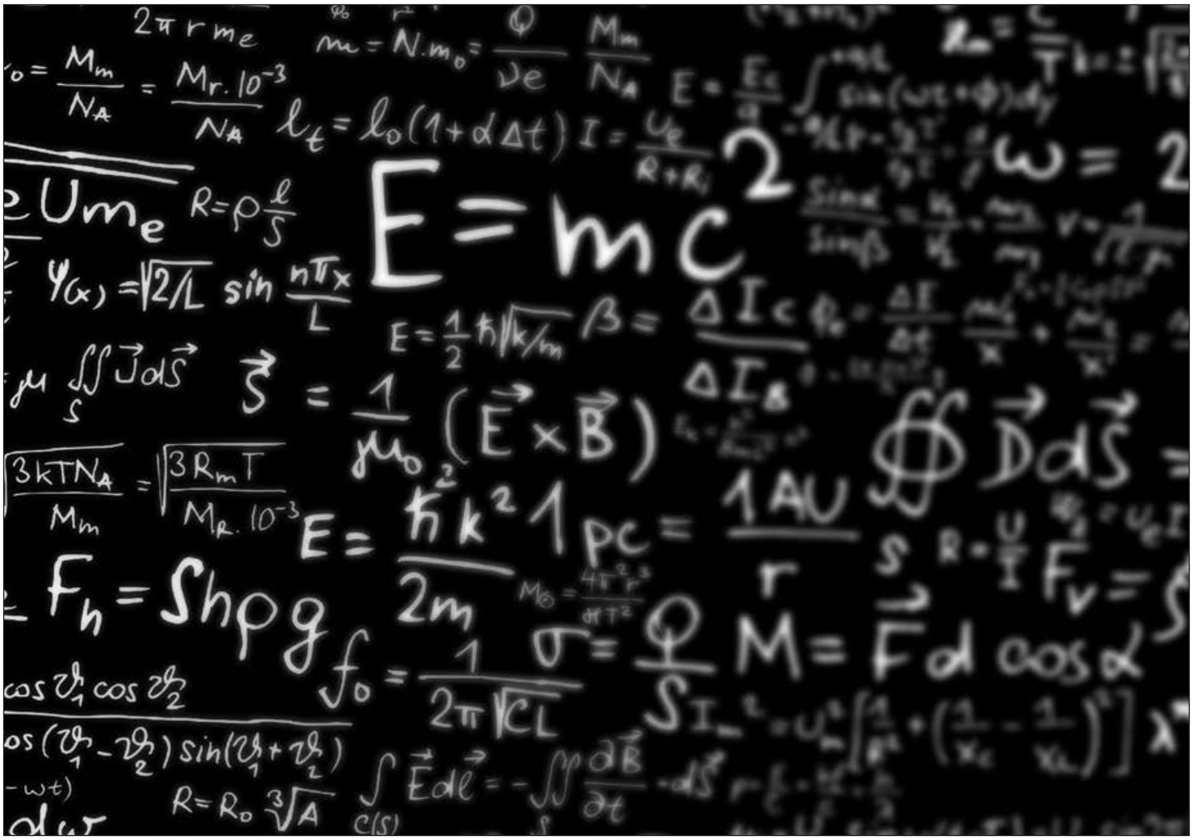

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Some Transformation Formulae of Basic Analogue of I-Function

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ABSTRACT

In this study we have introduced an alternative definition of the basic analogue of a generalization of well-known Fox's H-function in terms of I-function using q-Gamma function. This definition has been employed to obtain several transformation formulae. Some special cases have also been discussed.

Key words: Basic analogue of I-function and transformations

INTRODUCTION

Saxena and Kumar (1995), introduced the following basic analogue of I-function in terms of the Mellin-Barnes type basic contour integral as:

$$I^{m,n}_{A_j,B_i}\left[z;q\left|\begin{matrix}(a_j,\alpha_j)1,n;(a_{ji},\alpha_{ji})_{n+1,A_i}\\(b_j,\beta_j)1,n;(b_{ji},\beta_{ji})_{m+1,B_i}\end{matrix}\right.\right]$$

$$=\frac{1}{2\pi i}\int_L\frac{\prod^m_{j=1}G(q^{b_j-\beta_j s})\prod^n_{j=1}G(q^{1-a_j+a_j s})_{\Pi z^s ds}}{\sum^r_{i=1}[\prod^{B_i}_{j=m+1}G(q^{1-b_{ji}+\beta_{ji}s})\prod^{A_i}_{j=n+1}G(q^{a_{ji}-\alpha_{ji}s})G(q^{1-s})\sin\pi s]}\tag{1}$$

where, α_j, β_j, α_{ji}, β_{ji}, are real and positive, a_j, b_j, a_{ji}, b_{ji} are complex numbers

$$G(q\alpha)=\prod^{\infty}_{n=0}(1-q^{\alpha+n})^{-1}=\frac{1}{(q^{\alpha};q)\infty}$$

where, L is contour of integration running from - i∞ to i∞ in such a manner that all poles of G ($q^{bj-\beta js}$) lie to right of the path and those of G ($q^{1-aj+\alpha js}$) are to the left of the path.

Setting r = 1, A_i = A, B_i = B, we get q-analogue of H-function defined by Saxena *et al.* (1983) as follows:

$$H^{m,n}_{q,A,B}\left[z;q\left|\begin{matrix}(a_j,\alpha_j)_{1,A}\\(b_j,\beta_j)_{1,B}\end{matrix}\right.\right]$$

$$= \frac{1}{2\pi i}\int_L \frac{\prod_{j=1}^{m} G(q^{b_j-\beta_j s})\prod_{j=1}^{n} G(q^{1-a_j+\alpha_j s})\pi z^s ds}{\prod_{j=m+1}^{B} G(q^{1-b_j+\beta_j s})\prod_{j=n+1}^{A} G(q^{a_j-\alpha_j s})G(q^{1-s})\sin \pi s}$$

Further, for r = 1, A_i = A, B_i = B, $\alpha_j = \beta_i$ = 1, j = 1, 2, 3, ---- A, i = 1, 2, 3, --- B, Eq. 1. reduces to the basic analogue of Meijer's G-function given by Saxena *et al.* (1983).

MAIN RESULTS

In this we establish an alternative definition of basic analogue of I-function by using q-gamma function:

We shall make use of I_q (.) notation for basic analogue of I-function and the same is defined as:

$$I^{m,n}_{q,A_i,B_i;r}\left[z;q\left|\begin{matrix}(a_j,\alpha_j)1,n;(a_{ji},\alpha_{ji})_{n+1,A_i}\\(b_j,\beta_j)1,n;(b_{ji},\beta_{ji})_{m+1,B_i}\end{matrix}\right.\right]$$

$$=(1-q)\sum{}^{n}{}_{t=1}a_{t}+\sum_{t=1}^{m}b_t+m+n-1\sum_{i=1}^{r}\left[\sum_{t=n+1}^{A_i}a_{ti}-\sum_{t=m+1}^{B_i}b_{ti}+A_i\right]$$

$$G(q)\sum{}^{r}_{i=1}A_i+B_i-2(m+n-1)I^{m,n}q_{,A_i,B_i;r}$$

$$\left[z(1-q)\sum{}^{m}_{t=1}\beta_t-\sum{}^{n}_{t=1}\alpha_t+\sum{}^{r}_{i=1}\left\{\sum{}^{B_i}_{t=m+1}\beta_{ti}-\sum{}^{Ai}_{t=n+1}\alpha_{ti};\right.\right.$$

$$q\left[\begin{matrix}(a_j,\alpha_j)_{1,n};(a_{ji},\alpha_{ji})_{n+1,A_i}\\(b_j,\beta_j)_{1,n};(b_{ji},\beta_{ji})_{m+1,B_i}\end{matrix}(1,1)\right] \quad (2)$$

Proof: To prove (2) we consider the expression:

$$I^{m,n}_{q,A_i,B_{i+1}};r\left[z(1-q)\sum{}^{m_i}_{t=1}\beta_t-\sum{}^{m}_{t=1}\alpha_t+\Sigma^{r}_{i=1}\left\{\sum{}^{B_i}_{t=m+1}\beta_{ti}-\sum{}^{A_i}_{t=n+1}\alpha_{ti}\right];q\right.$$

$$q\left[\begin{matrix}(a_j,\alpha_j)_{1,n};(a_{ji},\alpha_{ji})_{n+1,A_i}\\(b_j,\beta_j)_{1,n};(b_{ji},\beta_{ji})_{m+1,B_i}\end{matrix}(1,1)\right]$$

$$=\frac{1}{2\pi i}\int_L\frac{\Pi^m_{j=1}G(q^{b_j-\beta_j s})\Pi^n_{j=1}G(q^{1-\alpha_j+\alpha_j s})\pi z^s(1-q)S\left[\Sigma^m_{t=1}\beta_t-\Sigma^n_{t=1}\alpha_t+\Sigma^r_{i=1}\left[\Sigma^{B_i}_{m+1}\beta_{ti}-\Sigma^{A_i}_{t=n+1}\alpha_{ti}\right]\right.}{\Sigma^r_{i=1}\{\Pi^{B_i}_{m+1}G(q^{1-b_{ji}+\beta_{ji}s})\Pi^{A_i}_{j=n+1}G(q^{\alpha_{ji}-\alpha_{ji}s})G(q^2)G(q^{1-s})\sin\pi s}ds$$

On multiplying above equation by:

$$(1-q)\sum{}^{n}_{t=1}\alpha_t-\sum{}^{m}_{t=1}b_t+m+n-1+\sum{}^{r}_{i=1}\left[\sum{}^{A_i}_{t=n+1}\alpha_{ti}-\sum{}^{B_i}_{m+1}b_{ti}-A_i\right]\times G(q)\sum{}^{r}_{i=1}A_i+B_i-2(m+n-1)$$

and making use of the following identity given by Askey (1978):

$$r_q(x)=\frac{G(q^x)}{(1-q)^{x-1}G(q)};|q|<1,$$

the left hand side takes the form:

$$\frac{1}{2\pi i}\int_L \frac{\prod_{j=1}^{m}\frac{G(q^{b_j-\beta_j s})}{(1-q)^{b_j-\beta_j s-1}G(q)}\prod_{j=1}^{n}\frac{G(q^{1-\alpha_j+\alpha_j s})}{(1-q)^{-\alpha_j+\alpha_j s}G(q)}\pi Z^s ds}{\sum_{i=1}^{r}\left[\prod_{j=m+1}^{B_i}\frac{G(q^{1-b_{ji}+\beta_{ji}s})}{(1-q)^{-1b_{ji}+\beta_{ji}s}G(q)}\times\prod_{j=1}^{A_i}\frac{G(q^{\alpha_{ji}+\alpha_{ji}s})}{(1-q)^{\alpha_{ji}+\alpha_{ji}s-1}G(q)}\times\frac{G(q^s)}{(1-q)^{s-1}G(q)}\times\frac{G(q^{1-s})\sin\pi s}{(1-q)^{-s}G(q)}\right.}$$

$$=\frac{1}{2\pi i}\int_L \frac{\prod_{j=1}^{m}\Gamma(q^{b_j-\beta_j s})\times\prod_{j=1}^{n}\Gamma(q^{1-a_j+a_j s})\,\pi Z^s ds}{\sum_{i=1}^{r}\left[\prod_{j=m+1}^{B_i}\Gamma(q^{1-b_{ji}+\beta_{ji}s})\times\prod_{j=1}^{A_i}\Gamma(q^{\alpha_{ji}+\alpha_{ji}s})\times\Gamma(q^s)\times\Gamma(q^{1-s})\sin\pi s\right.}$$

Hence, we have:

$$I_{q,A_i,B_i;r}^{m,n}\left[Z;q\left|\begin{matrix}(\alpha_j,\alpha_j)1,\ n;(\alpha_{ji},\alpha_{ji})n+1,\ A_i\\(b_j,\beta_j)1,\ m;(\beta_{ji},\beta_{ji})m+1,\ B_i\end{matrix}\right.\right]$$

$$=\frac{1}{2\pi i}\int_L \frac{\Pi_{j=1}^{m}\Gamma\left(q^{b_j-\beta_j s}\right)\times\Pi_{j=1}^{n}\Gamma\left(q^{1-a_j+\alpha_j s}\right)\pi Z^s\ ds}{\sum_{i=1}^{r}[\Pi_{j=m+1}^{B_i}\Gamma_q\,1-b_{ji}+\beta_{ji}s\times\Pi_{j=n+1}^{A_i}\Gamma_q\,a_{ji}-a_{ji}s\times\Gamma_q\,s\times\Gamma_q(1-s)\sin\pi s}$$

If we take r = 1, A_i = A, B_i = B, we get following well know basic analogue of Fox's H function [3]:

$$H_{q,A,B;r}^{m,n}\left[Z;q\left|\begin{matrix}(a_j,\alpha_j)_{1,A};\\(b_j,\beta_j)_{1,B};\end{matrix}\right.\right]$$

$$=\frac{1}{2\pi i}\int_L \frac{\Pi_{j=1}^{m}\Gamma_q\left(b_j-\beta_j s\right)\Pi_{j=1}^{n}\Gamma_q\left(1-\alpha_j+\alpha_j s\right)_{\pi z^s\ ds}}{\Pi_{j=m+1}^{B}\Gamma_q\ (1-a_j+a_j s)\ \Pi_{j=n+1}^{A}\Gamma_q\,(a_j-a_j s)\ \Gamma_q(s)\Gamma_q(1-s)\ \sin\pi s}$$

Transformation formulae of I_q -Function: In this section we derive number of transformation formulae for basic analogue of i-function.

Theorem 3:

$$I_{q,A_i,B_i;r}^{m,n}\left[Z;q\left|\begin{matrix}(a,o;)(a_j,\alpha_j)_{2,n};(a_{ji},\alpha_{ji})n+1,\ A_i\\(b_j,\beta_j)_{1,m;}\qquad(b_{ji},\beta_{ji})m+1,\ B_i\end{matrix}\right.\right]$$

$$=\Gamma q(1-\alpha)I_{A_i-1,B_i;r}^{m,n-1}\left[Z;q\left|\begin{matrix}(a_j,\alpha_j)_2,n;(\alpha_{ji},\alpha_{ji})_{n+1},\ A_{i-1}\\(b_j,\beta_j)_{1,m;}\ (b_{ji},\beta_{ji})_{m+1},\ B_i\end{matrix}\right.\right]\quad(3)$$

Proof: Consider the L.H.S. of (3):

$$=\frac{1}{2\pi i}\int_L \frac{\Pi_{j=1}^{m}\Gamma_q\left(b_j-\beta_j s\right)\Gamma_q(1-\alpha)\Pi_{j=2}^{n}\Gamma_q\left(1-\alpha_j+\alpha_j s\right)_{\pi z^s\, ds}}{\sum_{i=1}^{r}\{\Pi_{j=m+1}^{B_i}\Gamma_q(1-b_{ji}+\beta_{ji}s)\,\Pi_{j=n+1}^{A}\Gamma_q(\alpha_{ji}-\alpha_{ji}s)\,\Gamma_q(s)\,\Gamma_q(1-s)\sin\pi s}$$

$$zD_{Z,q}\left[z^{1-a1}\, I_{q,\,A_i,\,B_i;\,r}^{m,n}\left[z;q\middle|\begin{matrix}(\alpha_j,1)_{1,n};\ (\alpha_{ji},1)_{n+1,A_i}\\(b_j,1)_{1,m};\ (b_{ji},1)_{m+1,B_i}\end{matrix}\right]\right]$$

$$=\Gamma_q(1-\alpha)\frac{1}{2\pi i}\int_L \frac{\Pi_{j=1}^{m}\Gamma_q\left(b_j-\beta_j s\right)\,\Pi_{j=2}^{n}\Gamma_q\left(1-a_j-\alpha_j s\right)_{\pi z^s\, ds}}{\sum_{i=1}^{r}\{\Pi_{j=m+1}^{B_i}\Gamma_q(1-b_{ji}+\beta_{ji}s)\,\Pi_{j=n+1}^{A}\Gamma_q(a_{ji}-\alpha_{ji}s)\,\Gamma_q(s)\,\Gamma_q(1-s)\sin\pi s}$$

By definition of I_q- function, we get:

$$=\Gamma_q(1-\alpha)I_{A_i-1,\,B_i;r}^{m,n-1}\left[Z;q\middle|\begin{matrix}(\alpha_j,\ \alpha_j)_{2,n};\ (a_{ji},\ \alpha_{ji})_{n+1},\ _{Ai}\\(b_j,\ \beta_j)_{1,m};\ (b_{ji},\ \beta_{ji})_{\ m+1,\ B_i}\end{matrix}\right]=\text{R.H.S.}$$

This completes the proof of theorem (3).

Theorem 4:

$$I_{q,\,A_c,B_c;r}^{m,n}\left[Z;q\middle|\begin{matrix}(\alpha_j,\alpha_j)_{1,\,n};(\alpha_{ji},\alpha_{ji})_{n+1,A_{i-1}}(\alpha,o)\\(b_j,\beta_j)_{1,m};(b_{ji}\beta_{ji})_{m+1,\beta_i}\end{matrix}\right]$$
$$=\frac{1}{\Gamma_q(a)}I_{A_{j-1},\beta_{jr}}^{m,n}\left[\begin{matrix}z;q\big|(a_j,\alpha_j)_{1,n};(a_{ji},\alpha_{ji})_{n+1},A_{i-1}\\(b_j,\beta_j)_{1,m}\,(b_{ji},\beta_{ji})_{m+1},B_i\end{matrix}\right] \quad (4)$$

Theorem 5:

$$I_{q,\,A_i,B_j;r}^{m,n}\left[z;q\middle|\begin{matrix}(a_j,\alpha_j)_{1,\,n};(\alpha_{ji},\alpha_{ji})_{n+1,}\,A_i\\(b,o)\,(b_j\beta_j)_{2,m},(b_{ji},\beta_{ji})_{m+1,}\,B_i\end{matrix}\right]$$
$$=\Gamma_q(b)I_{q,A_i,B_i;r}^{m-1,n}\left[\begin{matrix}z;q\big|(\alpha_j,\alpha_j)_{1,n};(\alpha_{ji},\alpha_{ji})_{n+1},A_i\\(b_j,\beta_j)_{1,m};(b_{ji},\beta_{ji})_{m+1},B_i\end{matrix}\right] \quad (5)$$

Theorem 6:

$$I_{q,\,A_i,B_j;\,r}^{m,n}\left[z;q\middle|\begin{matrix}(\alpha_j,\alpha_j)_{1,\,n}; & (\alpha_{ji},\alpha_{ji})_{n+1,\,A_i}\\(b_j,\beta_j)_{1,m};\ (b_{ji},\beta_{ji})_{m+1,\ B_{i-1},} & (b,0)\end{matrix}\right]$$
$$=\Gamma_q(1-b)\,I_{q,A_i,B_{i-1};r}^{m-1,n}\left[z;q\middle|\begin{matrix}(\alpha_j,\alpha_j)_{1,n};\ (\alpha_{ji},\alpha_{ji})_{n+1,A_i}\\(b_j,\beta_j)_{1,m};\ (b_{ji},\beta_{ji})_{m+1,B_i}\end{matrix}\right] \quad (6)$$

Theorem 7:

$$zI_{q,\,A_i,B_j;\,r}^{m,n}\left[z;q\middle|\begin{matrix}(\alpha_j,\alpha_j)_{1,\,n};(\alpha_{ji},\alpha_{ji})_{n+1,A_i}\\(b_j,\beta_j)_{1,m};\ (b_{ji},\beta_{ji})_{m+1,\ B_i}\end{matrix}\right]$$
$$=\frac{1}{q}I_{q,A_i,B_i;r}^{m,n}\left[za;q\middle|\begin{matrix}(\alpha_j+\alpha_j,\alpha_j)_{1,n};\ (\alpha_{ji}+\alpha_{ji},\alpha_{ji})_{n+1,A_i}\\(b_j+\beta_j,\beta_j)_{1,m};\ (b_{ji}+\beta_{ji},\beta_{ji})_{m+1,B_i}\end{matrix}\right] \quad (7)$$

The proofs of theorem (4) to (7) are similar to that of theorem (3).

Special cases: If we take r = 1, A_i = A, B_i = B in theorems(3) to (7), we get the well-known results of basic analogue of Fox's H-function[2].

CONCLUSION

In this study we have obtained some transformation formulae for basic analogues for I-function. These results are quite general in nature and reduce to corresponding results for G and H functions and their several special cases. Thus these results can be applied to various problems of mathematical physics.

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